Answers & Explanatory notes

CONTENTS

Answers & Explanatory notes

UNIT 1 ERRORS IN MEASUREMENT

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17. B	18. D	18. D	20. C	21. D	22. D	23. A	24. A
25. B	26. B	27. A	28. C	29. A	30. A	31. D	

Explanatory Notes

1. Absolute error must be positive.
   $\therefore$ Absolute error = (369 – 368.72) mL = 0.28 mL
2. (A) Absolute error $= 0.3 - 0.2837 = 0.0163\,\text{g}$;
   (B) Absolute error $= 0.284 - 0.2837 = 0.0003\,\text{g}$
4. (A) Degree of accuracy = finest markings $= 5\,\text{mm}$;
   (B) Max. error $= 5\,\text{mm} \div 2 = 2.5\,\text{mm}$
6. A: Max. error $= 10000 \div 2 = 5000\,\text{g}$;
   B: Max. error $= 1000 \div 2 = 500\,\text{g}$;
   C: Max. error $= 10 \div 2 = 5\,\text{g}$;
   D: Max. error $= 1 \div 2 = 0.5\,\text{g}$. $\therefore$ The answer is C.
7. Relative error $= \dfrac{52.5 - 50}{52.5} = \dfrac{1}{21}$
8. Relative error $= \dfrac{5 \div 2}{375} = \dfrac{1}{150}$
9. % error $= \dfrac{300 - 257.54}{257.54} \times 100\% = 16.5\%$
10. % error $= \dfrac{0.01 \div 2}{15.15} \times 100\% = 0.03\%$
11. % error $= \dfrac{10}{140} \times 100\% = 7.14\%$
16. Lower limit $= 7.0 - (0.5 \div 2) = 6.75\,\text{m}^2$,
    upper limit $= 7.0 + (0.5 \div 2) = 7.25\,\text{m}^2$,
    i.e. the actual area lies between 6.75m^2 and 7.25m^2.
    $\therefore$ D (7.26m^2) lies outside this range.
17. Lower limit $= [(12.7 - 0.1 \div 2) + (9.4 - 0.1 \div 2)] \times 2 = 44$ cm
18. Upper limit $= (10 + 10 \div 2)^2 = 225\,\text{cm}^2$
19. Upper limit $= (30 + 1 \div 2) + (7.6 + 0.1 \div 2) + (18 + 1 \div 2) = 56.65$ cm
    Lower limit $= (30 - 1 \div 2) + (7.6 - 0.1 \div 2) + (18 - 1 \div 2) = 54.55\,\text{cm}$
20. Upper limit $= 180^\circ - (30.6^\circ - 0.1^\circ \div 2) - (53.9^\circ - 0.1^\circ \div 2) = 95.6^\circ$

21. Lower limit $= 96 - 96 \times \frac{1}{80} = 94.8$ beats/s
22. Max. error $= 72 \times 0.0375\% = 0.027$,
    $\therefore$ the actual speed lies between (72 ± 0.027) km/h.
23. Max. error $= 1 \div 2 \div 400 = 0.00125$ mm
24. Lower limit $= \left(150 - 150 \times \frac{3}{4}\%\right) \times 20 \div 1000 = 2.9775$ kg,
    upper limit $= \left(150 + 150 \times \frac{3}{4}\%\right) \times 20 \div 1000 = 3.0225$ kg,
    $\therefore$ actual weight of 20 *regular* packs lies between 2.9775 kg and 3.0225 kg. $\therefore$ A (2.977 kg) lies outside the range.
25. Measurement with the least relative error is the most accurate.
    Relative error of A $= \frac{1 \div 2}{8} = \frac{1}{16}$.
    Relative error of B $= \frac{0.2 \div 2}{36.4} = \frac{1}{364}$.
    Relative error of C $= \frac{0.1 \div 2}{10.2} = \frac{1}{204}$.
    Relative error of D $= \frac{150 \div 2}{1500} = \frac{1}{20}$. $\frac{1}{364}$ is the least.
26. Lower limit of $b - a = (30.0 - 0.2 \div 2) - (21.4 + 0.1 \div 2) = 8.45$
    Upper limit of $b - a = (30.0 + 0.2 \div 2) - (21.4 - 0.1 \div 2) = 8.75$
27. Lower limit of $\frac{x}{y} = \frac{56.5 - 0.5 \div 2}{17.1 + 0.1 \div 2} = \frac{56.25}{17.15}$
    Upper limit of $\frac{x}{y} = \frac{56.5 + 0.5 \div 2}{17.1 - 0.1 \div 2} = \frac{56.75}{17.05}$
28. Upper limit $= (38.1 + 0.1 \div 2) - (14.4 - 0.1 \div 2) = 23.8$ cm
29. Lower limit $= \frac{2000 - 10 \div 2}{100 + 2 \div 2} = \frac{1995}{101}$ cm
30. Lower limit $= (30 - 0.5)(25 - 0.5) - (20 + 0.5)(15 + 0.5) = 405\ \text{cm}^2$
31. Upper limit $= (75 + 0.5) - (22 - 0.5) - (16 - 0.5) = 38.5$ cm

UNIT 2 FACTORIZATION OF POLYNOMIALS (1)

1. A	2. B	3. D	4. C	5. C	6. B	7. A	8. C
9. A	10. C	11. D	12. B	13. A	14. C	15. B	16. D
17. C	18. B	19. C	20. C	21. C	22. D	23. A	24. B
25. A	26. C	27. A	28. A	29. D	30. C	31. D	32. C
33. B	34. A	35. B	36. B	37. A	38. B	39. B	40. A

41. A	42. C	43. D	44. B	45. C	46. B	47. B	48. B
49. C							

Explanatory Notes

10. $= (x-y)a-(x-y)b = (x-y)(a-b)$
13. $= 5(a+2)[2+(a+2)] = 5(a+2)(a+4)$
14. $= 12y(x-y)+18x(x-y) = 6(x-y)(2y+3x)$
15. $= 8(m-3)+4(m-3)^2 = 4(m-3)[2+(m-3)] = 4(m-3)(m-1)$
16. $= 12(p-q)^2-9p(p-q) = 3(p-q)[4(p-q)-3p]$
    $= 3(p-q)(p-4q)$
17. $= 2(x+y)^2[1-3(x+y)] = 2(x+y)^2(1-3x-3y)$
19. $= (m-n)(x-y)-(n+m)(x-y) = (x-y)[(m-n)-(n+m)]$
    $= (x-y)(-2n) = 2n(y-x)$
23. $= 2(a-b)^2+a(a-b) = (a-b)[2(a-b)+a] = (a-b)(3a-2b)$
24. $= 3(3x-1)-(3x-1)^2 = (3x-1)[3-(3x-1)] = (3x-1)(4-3x)$
27. $= 3a(a^2+5)-(5+a^2) = (a^2+5)(3a-1)$
28. $= 3(2-5k)-2k^2(2-5k) = (2-5k^2)(3-2k^2) = (5k^2-2)(2k^2-3)$
30. $= x(y-4)+7y(y-4) = (y-4)(x+7y)$
34. $= x(x+2z)+y(x+2z)+4(x+2z) = (x+2z)(x+y+4)$
35. $= ma(m-n)-nb(m-n)+c(m-n) = (m-n)(ma-nb+c)$
39. $= m(m-n)^2+(m-n)^3 = (m-n)^2[m+(m-n)] = (m-n)^2(2m-n)$
40. $= (a-b)^2[3a-(2a+b)] = (a-b)^2(a-b) = (a-b)^3$
41. $= (m+n)(m-n)[(m+n)-(m-n)] = 2n(m+n)(m-n)$
45. $= 5(p-2q)+q(p-2q) = (p-2q)(5+q)$
46. $= 6k(2-kt)-t(2-kt) = (2-kt)(6k-t)$
48. $= 3y(1-4x-2y)+w(1-4x-2y) = (3y+w)(1-4x-2y)$
49. $= c(1-c-5bc)-10b(1-c-5bc) = (c-10b)(1-c-5bc)$
    $= (10b-c)(5bc+c-1)$

UNIT 3 IDENTITIES

1. A	2. A	3. C	4. A	5. B	6. B	7. C	8. D
9. B	10. C	11. A	12. B	13. C	14. A	15. A	16. D
17. B	18. B	19. B	20. D	21. A	22. D	23. C	24. A
25. D	26. D	27. C	28. A	29. A	30. B	31. A	32. C
33. D	34. D	35. A	36. C	37. D	38. B	39. B	40. A
41. C	42. C	43. C	44. B	45. B	46. B	47. A	48. B

49. B 50. D 51. D 52. C 53. B 54. A 55. D 56. C
57. D 58. D 59. A 60. A

Explanatory Notes

3. $B=-6$; $A-(-6)=4$, $A=4-6=-2$
4. $(5-x)(2x+1)=-2x^2+9x+5$, $\therefore\ A=-2, B=9, C=-5$,
   $\therefore\ A+B+C=-2+9-5=2$
5. $Ax-(Ax+B)=Ax-Ax-B=-B=7$, $\therefore\ B=-7$
6. $A(x+6)(x-1)=A(x^2+5x-6)\ =Ax^2+5Ax-6A$
   $=3x^2-Bx+(C-1)$, $\therefore\ A=3$; $-B=5(3)=15$, $\therefore\ B=-15$;
   $C-1=-6(3)$, $\therefore\ C=-18+1=-17$
7. $(Ax-B)(x+2)=Ax^2+(2A-B)x-2B=2x^2-Cx-10$,
   $\therefore\ A=2$; $-2B=-10$, $\therefore\ B=5$; $-C=2(2)-5=-1$,
   $\therefore\ C=1$
12. $(Ax+2)^2=A^2x^2+4Ax+4=Bx^2+12x+4$, $4A=12$,
   $\therefore\ A=3$, $\therefore\ B=3^2=9$
13. $(x-C)^2=x^2-2Cx+C^2=x^2+8x-D$, $-2C=8$,
   $\therefore\ C=-4$; $-D=(-4)^2=16$, $\therefore\ D=-16$
14. $a^2x^2+2abx+b^2\equiv c^2x^2+2cdx+d^2$
   By comparing coefficients,
   $a^2=c^2$, $b^2=d^2$, $2ab=2cd$
   But a, c may have opposite signs [e.g. $3^2=(-3)^2$],
   $\therefore$ I may not be true.
   Similarly, II may not be true.
   $2ab=2cd$, $\therefore\ ab=cd$, $\therefore$ III is true.
20. $=3^2(a-b)^2=9(a^2-2ab+b^2)=9a^2-18ab+9b^2$
26. $=(200-0.1)^2=200^2-2(200)(0.1)+(0.1)^2$
27. $=(25+0.1)^2=25^2+2(25)(0.1)+(0.1)^2$
28. $(p-7)^2=p^2-14p+49=p(p-14)+49=-15+49=34$
38. $(8-3x)(8+3x)=64-9x^2=Px^2+Q$, $\therefore\ P=-9$, $Q=64$
39. $=(50+0.5)(50-0.5)=50^2-(0.5)^2$
40. $a:3=2:a$, $\dfrac{a}{3}=\dfrac{2}{a}$, $a^2=6$;
   $\therefore\ (a-2)(a+2)=a^2-4=6-4=2$
41. $Ax+B(x-3)=(A+B)x-3B=x+9$, $-3B=9$,
   $\therefore\ B=-3$; $A-3=1$, $\therefore\ A=4$
42. $(a-p)(b-p)\cdots(p-p)\cdots(y-p)(z-p)$
   $=(a-p)(b-p)\cdots(0)\cdots(y-p)(z-p)=0$

43. $=[a+(b+c)]^2 = a^2+2a(b+c)+(b+c)^2$
$=a^2+2ab+2ac+b^2+2bc+c^2$

44. $=[(p-q)-r]^2=(p-q)^2-2(p-q)r+r^2$
$=p^2-2pq+q^2-2pr+2qr+r^2$

45. $x+\frac{1}{x}=-5$, $\left(x+\frac{1}{x}\right)^2=(-5)^2$, $x^2+2+\frac{1}{x^2}=25$,
$\therefore\ x^2+\frac{1}{x^2}=23$

46. $a-b=6$, $(a-b)^2=6^2$, $a^2-2ab+b^2=36$, $20-2ab=36$,
$\therefore\ ab=-8$

47. $(1+x)^2=8$, $x^2+2x+1=8$, $x^2+2x=7$, $\therefore\ x^2-5x=7-7x$

48. $=(x^2-1)(x^2+1)(x^4+1)\cdots(x^{256}+1)$
$=(x^4-1)(x^4+1)(x^8+1)\cdots(x^{256}+1)=...$
$=(x^{256}-1)(x^{256}+1)=x^{512}-1$

49. $(m+n)(m-n)=m^2-n^2=5$, $n^2=m^2-5$;
$(m-2n)(m+2n)=m^2-4n^2=m^2-4(m^2-5)=20-3m^2$

50. $=[(x-y)+5][(x-y)-5]=(x-y)^2-5^2=x^2-2xy+y^2-25$

51. $=[(a+7)-2b][(a+7)+2b]$
$=(a+7)^2-(2b)^2=a^2+14a+49-4b^2$

52. $=[p-(q-3)][p+(q-3)]=p^2-(q-3)^2=p^2-q^2+6q-9$

53. $=[m-(6n+1)][m+(6n+1)]=m^2-(6n+1)^2=m^2-36n^2-12n-1$

54. $=[(a-b)(a+b)]^2=(a^2-b^2)^2=a^4-2a^2b^2+b^4$

55. $(a-b)^2=a^2-2ab+b^2=a^2-b^2+2b^2-2ab$
$=(a+b)(a-b)+2b^2-2ab$, $\therefore\ k=2b^2-2ab$

56. The number $=10a+b$, square of the number
$=(10a+b)^2=100a^2+20ab+b^2$

UNIT 4 FACTORIZATION OF POLYNOMIALS (2)

1. A	2. C	3. B	4. A	5. D	6. D	7. B	8. A
9. C	10. D	11. A	12. C	13. A	14. B	15. A	16. D
17. A	18. B	19. D	20. B	21. A	22. B	23. D	24. A
25. C	26. D	27. A	28. D	29. B	30. D	31. A	32. A
33. B	34. C	35. A	36. D	37. B	38. D	39. C	40. C
41. A	42. C	43. B	44. B	45. A	46. B	47. D	48. D
49. C	50. C	51. A	52. A	53. B	54. D	55. A	56. D

57. C 58. C 592. B 60. D

Explanatory Notes

2. $\because\ 25a^2 = (5a)^2$ and $30a = 2(5a)(3)$, $\therefore$ number needed $= 3^2 = 9$
3. $\because\ 4x^2 = (2x)^2$ and $9 = 3^2$, $\therefore$ term in the blank $= 2(2x)(3) = 12x$
13. $= \frac{1}{2}(4x^2 + 4x + 1) = \frac{1}{2}(2x+1)^2$
15. $= [3(x-y)+7]^2 = (3x-3y+7)^2$
16. $= [1+4(p-2)]^2 = (1+4p-8)^2 = (4p-7)^2$
17. $= [a-5(a-b)]^2 = (a-5a+5b)^2 = (5b-4a)^2$
18. $= 5[1-4(x+8)+4(x+8)]$
    $= 5[1-2(x+8)]^2 = 5(1-2x-16)^2 = 5(-15-2x)^2 = 5(15+2x)^2$
19. $= [(x+y)+3(x-2y)]^2 = (x+y+3x-6y)^2 = (4x-5y)^2$
20. $= [(3x-y)-(x+3y)]^2 = (3x-y-x-3y)^2 = (2x-4y)^2$
    $= [2(x-2y)]^2 = 4(x-2y)^2$
27. $= \frac{1}{3}(1-9k^2) = \frac{1}{3}(1+3k)(1-3k)$
30. $= [3(a-b)+x][3(a-b)-x] = (3a-3b+x)(3a-3b-x)$
31. $= [(p+5)+6p][(p+5)-6p] = (7p+5)(5-5p) = 5(7p+5)(1-p)$
33. $= [1+2(p+q)][1-2(p+q)] = (1+2p+2q)(1-2p-2q)$
34. $= [8x+(3x-y)][8x-(3x-y)] = (11x-y)(5x+y)$
35. $= [(2x-7)+(5x+3)][(2x-7)-(5x+3)]$
    $= (7x-4)(-3x-10) = (4-7x)(3x+10)$
36. $= 2[25(x+2y)^2 - (x-3y)^2]$
    $= 2[5(x+2y)+(x-3y)][5(x+2y)-(x-3y)]$
    $= 2(6x+7y)(4x+13y)$
37. $= [4(2p-q)+7(p-2q)][4(2p-q)-7(p-2q)]$
    $= (15p-18q)(p+10q) = 3(5p-6q)(p+10q)$
38. $= \left[\left(x+\frac{1}{x}\right)+\left(x-\frac{1}{x}\right)\right]\left[\left(x+\frac{1}{x}\right)-\left(x-\frac{1}{x}\right)\right] = (2x)\left(\frac{2}{x}\right) = 4$
40. $a^8 - 1 = (a^4+1)(a^4-1) = (a^4+1)(a^2+1)(a^2-1)$
    $= (a^4+1)(a^2+1)(a+1)(a-1)$, $\therefore$ The answer is C.
41. $251^2 - 249^2 = (251-249)(251+249) = 2\times 500$
42. $(105.5)^2 - (5.5)^2 = (105.5+5.5)(105.5-5.5) = 110\times 100$
45. $= (x-7)^2 - 6^2 = (x-7+6)(x-7-6) = (x-1)(x-13)$
46. $= (p+q)(p-q) - 3(p+q) = (p+q)(p-q-3)$

47. $= (2a+b)(2a-b) - a(2a-b)$
$= (2a-b)(2a+b-a) = (2a-b)(a+b)$

48. $= (4a)^2 - (a-9)^2 = [4a+(a-9)][4a-(a-9)]$
$= (5a-9)(3a+9) = 3(5a-9)(a+3)$

49. $= p^2 - (q^2+4q+4) = p^2 - (q+2)^2 = [p+(q+2)][p-(q+2)]$
$= (p+q+2)(p-q-2)$

50. $= y^2 - (25x^2+20xy+4y^2) = y^2 - (5x+2y)^2$
$= [y+(5x+2y)][y-(5x+2y)] = (5x+3y)(-5x-y)$
$= -(5x+3y)(5x+y)$

51. $= (x+3y)(x-3y) - (x-3y) = (x-3y)(x+3y-1)$

52. $= x^4 - (y^4 - 6y^2 + 9) = (x^2)^2 - (y^2-3)^2$
$= [x^2+(y^2-3)][x^2-(y^2-3)] = (x^2+y^2-3)(x^2-y^2+3)$

53. $= 2(c+2ab) - (4a^2b^2 - c^2) = 2(c+2ab) - (2ab+c)(2ab-c)$
$= (2ab+c)[2-(2ab-c)] = (2ab+c)(2-2ab+c)$

54. $= (y^2+1+2y)(y^2+1-2y) = (y+1)^2(y-1)^2$

55. $= (p^2-9)^2 = [(p+3)(p-3)]^2 = (p+3)^2(p-3)^2$

56. $= m^2(n-m) - n^2(n-m) = (n-m)(m^2-n^2)$
$= (n-m)(m+n)(m-n) = -(m+n)(m-n)^2$

57. $= x^2 + 4xy + 4y^2 - 8xy = x^2 - 4xy + 4y^2 = (x-2y)^2$

58. $\because\ 100a^2 + 60ab + 9b^2 = (10a+3b)^2$,
$\therefore$ length of square $= 10a + 3b$,
$\therefore$ perimeter $= 4(10a+3b) = (40a+12b)$ cm

59. Shaded area $= [(x-1)+(x+1)]^2 - (x+1)^2$
$= (2x)^2 - (x+1)^2 = [2x+(x+1)][2x-(x+1)]$
$= (3x+1)(x-1)$ cm^2

60. W + X + Y $= m^2 - (m-n)^2$
$= [m+(m-n)][m-(m-n)] = n(2m-n)$

UNIT 5 ALGEBRAIC FRACTIONS

1. B	2. B	3. D	4. A	5. B	6. B	7. B	8. D
9. A	10. C	11. D	12. A	13. B	14. C	15. C	16. D
17. B	18. A	19. C	20. C	21. A	22. D	23. A	24. B
25. C	26. D	27. A	28. A	29. C	30. D	31. A	32. B
33. A	34. C	35. B	36. A	37. B	38. D	39. D	40. C
41. D	42. A	43. B	44. C	45. C	46. A	47. A	48. D

49. A 50. C 51. C 52. A 53. D 54. A 55. B 56. B
57. B 58. C

Explanatory Notes

13. $=\dfrac{3a(a-2b)-7(a-2b)}{a(b+4)-2b(4+b)}=\dfrac{(a-2b)(3a-7)}{(b+4)(a-2b)}=\dfrac{3a-7}{b+4}$

26. $=\dfrac{m(1-n)}{a(n-1)+b(n-1)}\times\dfrac{b(m-1)+a(m-1)}{(1-m)^2}$

$=\dfrac{m(1-n)}{(n-1)(a+b)}\times\dfrac{(m-1)(a+b)}{(m-1)^2}=\dfrac{-m}{m-1}=\dfrac{m}{1-m}$

33. $=\dfrac{6}{t-3}-\dfrac{2t}{t-3}=\dfrac{6-2t}{t-3}=\dfrac{2(3-t)}{t-3}=-2$

37. $=\dfrac{x-1}{3(x+1)}-\dfrac{x^2}{(x+1)^2}=\dfrac{(x-1)(x+1)-3x^2}{3(x+1)^2}$

$=\dfrac{x^2-1-3x^2}{3(x+1)^2}=\dfrac{-2x^2-1}{3(x+1)^2}$

38. $=\dfrac{m+n}{3(3m-2n)}+\dfrac{m-n}{5(2n-3m)}=\dfrac{m+n}{3(3m-2n)}-\dfrac{m-n}{5(3m-2n)}$

$=\dfrac{5(m+n)-3(m-n)}{15(3m-2n)}=\dfrac{2m+8n}{15(3m-2n)}$

39. $=\dfrac{2a}{a(a+b)}+\dfrac{5b}{b(a+b)}=\dfrac{2ab+5ab}{ab(a+b)}=\dfrac{7ab}{ab(a+b)}=\dfrac{7}{a+b}$

42. $=\dfrac{x(xy-1)-y(xy-1)}{y(xy-1)+(xy-1)}=\dfrac{(xy-1)(x-y)}{(xy-1)(y+1)}=\dfrac{x-y}{y+1}$

47. $q(1-p)\div\dfrac{p(p-1)-q(p-1)}{p(p+1)-q(p+1)}=q(1-p)\div\dfrac{(p-1)(p-q)}{(p+1)(p-q)}$

$=q(1-p)\times\dfrac{p+1}{p-1}=-q(p+1)$

49. $=\dfrac{2y-1}{2(2y-1)}-\dfrac{2y-3}{3(2y-1)}=\dfrac{3(2y-1)-2(2y-3)}{6(2y-1)}=\dfrac{2y+3}{6(2y-1)}$

50. $=\dfrac{5(x-y)}{(x-y)^2}+\dfrac{3}{x-y}=\dfrac{5}{x-y}+\dfrac{3}{x-y}=\dfrac{8}{x-y}$

51. $=\dfrac{a(c-d)+b(c-d)}{a(c+d)+b(c+d)}-\dfrac{a(d-c)-b(d-c)}{a(d+c)-b(d+c)}$

$=\dfrac{(a+b)(c-d)}{(a+b)(c+d)}-\dfrac{(a-b)(d-c)}{(a-b)(d+c)}$

$=\dfrac{c-d}{c+d}-\dfrac{d-c}{d+c}=\dfrac{c-d-(d-c)}{d+c}=\dfrac{2(c-d)}{c+d}$

52. $=(1+\frac{1}{a})\div(1-\frac{1}{a})=\frac{a+1}{a}\div\frac{a-1}{a}=\frac{a+1}{a}\times\frac{a}{a-1}=\frac{a+1}{a-1}$

53. $=1\div(\frac{1}{a}-\frac{1}{b})=1\div(\frac{b-a}{ab})=\frac{ab}{b-a}$

54. $=1\div(\frac{1}{x+1}-1)=1\div[\frac{1-(x+1)}{x+1}]=1\div(\frac{-x}{x+1})=-\frac{x+1}{x}$

55. $\frac{A}{x-1}-\frac{B}{x}=\frac{Ax-B(x-1)}{x(x-1)}=\frac{(A-B)x+B}{x(x-1)}=\frac{5x-3}{x(x-1)}$,

$\therefore\ B=-3$; $A-(-3)=5$, $\therefore\ A=2$

56. $=\frac{3(4a^2-b^2)}{(2a+b)^2}=\frac{3(2a+b)(2a-b)}{(2a+b)^2}=\frac{3(2a-b)}{2a+b}$

57. $=\frac{(x+2+5)(x+2-5)}{2(9-6x+x^2)}=\frac{(x+7)(x-3)}{2(x-3)^2}=\frac{x+7}{2(x-3)}$

58. $=\frac{49-(x-5)^2}{(x^2+4)(x^2-4)}=\frac{[7+(x-5)][7-(x-5)]}{(x^2+4)(x+2)(x-2)}$

$=\frac{(x+2)(12-x)}{(x^2+4)(x+2)(x-2)}=\frac{12-x}{(x^2+4)(x-2)}$

UNIT 6 USE OF FORMULAE

1. A	2. C	3. A	4. B	5. B	6. D	7. B	8. D
9. D	10. D	11. A	12. C	13. B	14. A	15. D	16. B
17. C	18. C	19. B	20. A	21. B	22. A	23. D	24. D
25. C	26. A	27. B	28. B	29. D	30. D	31. A	32. B
33. C	34. B						

Explanatory Notes

8. $2=\frac{-6+3b}{b+6}$, $2b+12=-6+3b$, $\therefore\ b=18$

15. $ux+uy=wx-wz$, $uy+wz=wx-ux$, $\therefore\ x=\frac{uy+wz}{w-u}$

17. $b+abx=a-x$, $abx+x=a-b$, $\therefore\ x=\frac{a-b}{ab+1}$

18. $bx+ay=ab$, $bx=ab-ay$, $\therefore\ a=\frac{bx}{b-y}$

19. $3ax+6b=6px+2q$, $3ax-6px=2q-6b$, $\therefore\ x=\frac{2q-6b}{3a-6p}$

20. $\frac{D}{t}=1-\frac{1}{u}$, $\frac{1}{u}=1-\frac{D}{t}=\frac{t-D}{t}$, $\therefore\ u=\frac{t}{t-D}$

21. $A = P(1+\frac{nR}{100}) = P + \frac{PnR}{100}$, $A - P = \frac{PnR}{100}$, $\therefore\ R = \frac{100(A-P)}{Pn}$

22. Size of each interior angle $= \frac{(10-2)\times 180°}{10} = 144°$

30. $y(x+1) = x + 2 - 3(x+1)$, $xy + y = -2x - 1$,

$y + 1 = -2x - xy$, $y + 1 = -x(y+2)$, $\therefore\ x = -\frac{y+1}{y+2}$

31. $k(1+an+a) = 3an$, $k + kan + ka = 3an$, $k + ka = 3an - kan$,

$\therefore\ n = \frac{k+ka}{3a-ka}$

32. $= \frac{1}{6}(10)(10+1)(2\times 10+1) = 385$

33. $= (1^2 + 2^2 + ... + 20^2) - (1^2 + 2^2 + ... + 10^2)$

$= \frac{1}{6}(20)(20+1)(20\times 2+1) - 385 = 2485$

34. Distance travelled in the 4th second

= distance travelled in 4 seconds − distance travelled in 3 seconds

$= \frac{1}{2}(10)(4^2) - \frac{1}{2}(10)(3^2) = 35\text{ m}$

UNIT 7 GRAPHS OF LINEAR EQUATIONS IN TWO UNKNOWNS

1. B	2. D	3. C	4. A	5. C	6. B	7. C	8. C
9. A	10. A	11. B	12. A	13. C	14. D	15. A	16. D
17. B	18. A	19. C	20. C	21. A	22. D	23. A	24. D
25. B	26. C	27. D	28. A	29. B	30. B	31. D	32. A
33. B	34. C	35. D					

Explanatory Notes

6. $6(4) - k(-5) - 14 = 0$, $24 + 5k - 14 = 0$, $k = -2$

7. $5(\frac{2}{3}) = 4 - 2a$, $10 = 12 - 6a$, $6a = 2$, $a = \frac{1}{3}$

9. $8(m) - 3m(-2) + 7 = 0$, $14m = -7$, $m = -\frac{1}{2}$

11. y-coordinate = 5, $\therefore\ 5 = -4x - 3$, $x = \frac{8}{-4} = -2$

12. Put $(-1, 0)$, $\therefore\ 2(0) = 5(-1) + k$, $k = 5$

21. $7(y+4) = 7(-4+4) = 0$; $7(y-4) = 7(-4-4) = -56$;

$-6(x+1) = -6(1+1) = -12$; $-6(x-1) = -6(1-1) = 0$;

$\therefore\ 7(y+4) = -6(x-1)$

24. $6(p-1)-5(p+1)+8=0$, $6p-6-5p-5+8=0$, $p=3$
25. $B=(2a, a)$, $\therefore$ $a=-2(2a)-15$, $5a=-15$, $a=-3$, $\therefore$ $B=(-6, -3)$
26. $(a, b)=(a, 9-a)$, $\therefore$ $2(a)-3(9-a)+7=0$, $2a-27+3a+7=0$, $5a=20$, $a=4$, $\therefore$ $P=(4, 5)$
27. $P=(p, 0)$, $\therefore$ $p+0-8=0$, $p=8$; $Q=(q, 0)$, $\therefore$ $q+0+3=0$, $q=-3$; $\therefore$ $PQ=8-(-3)=11$ units
28. $P=(p, 0)$, $\therefore$ $p-2(0)=6$, $p=6$; $Q=(0, q)$, $\therefore$ $0-2(q)=6$, $q=-3$; $\therefore$ area $=\dfrac{6\times 3}{2}=9$ sq. units
29. $A=(a, 0)$, $\therefore$ $a+0=5$, $a=5$;
$B=(0, b)$, $0+b=5$, $b=5$;
$C=(c, 0)$, $\therefore$ $c-0=-5$, $c=-5$;
$\therefore$ area $=\dfrac{[5-(-5)]\times 5}{2}=25$ sq. units
30. $3(m)-(-2)+7=0$, $3m+9=0$, $m=-3$; $-2=k(-3)-5$, $3=-3k$, $\therefore$ $k=-1$
31. $1+k=4$, $k=3$; $1-2(3)+a=0$, $-5+a=0$, $\therefore$ $a=5$
32. y-coordinate of intersection point $=0$, $0=4x-6$, $x=\dfrac{6}{4}=\dfrac{3}{2}$; $0=6(\dfrac{3}{2})+d$, $0=9+d$, $\therefore$ $d=-9$
33. Dicky obtains the basic salary when number of sales $=0$. From the graph, when $x=0$, $y=40\ 000$, $\therefore$ basic salary = \$40 000

UNIT 8 SIMULTANEOUS LINEAR EQUATIONS IN TWO UNKNOWNS

1. A	2. B	3. D	4. B	5. A	6. D	7. D	8. C
9. C	10. A	11. C	12. B	13. B	14. C	15. D	16. D
17. D	18. C	19. C	20. C	21. A	22. A	23. A	24. C
25. B	26. A	27. C	28. C	29. D	30. B	31. D	32. A
33. C	34. B	35. A	36. D	37. A	38. B	39. B	40. B
41. D	42. A	43. A	44. D	45. B			

Explanatory Notes

10. $\dfrac{x}{2}-\dfrac{2y}{3}=-2$, $3x-4y=-12\cdots(1)$; $\dfrac{3x}{4}-\dfrac{y}{2}=1$, $3x-2y=4\cdots(2)$;
Solving (1) and (2), we have $x=6\dfrac{2}{3}, y=8$.

13. $4x-y=14x+y$, $10x+2y=0$, $5x+y=0\cdots(1)$; $4x-y=x+8$, $3x-y=8\cdots(2)$; Solving (1) and (2), we have $x=1, y=-5$.

14. $(x-2)+3(y-1)=-7$, $x+3y-5=-7$, $x+3y=-2\cdots(1)$;
$-4(x-5)-9(y+3)=-14$, $-4x-9y-7=-14$,
$4x+9y=7\cdots(2)$;
Solving (1) and (2), we have $x=13, y=-5$.

17. No. of hens $=x$, no. of rabbits $=y$; $x+y=32\cdots(1)$;
$2x+4y=100$, $x+2y=50\cdots(2)$;
Solving (1) and (2), we have $x=14, y=18$.

18. Fraction $=\dfrac{x}{y}$; $\dfrac{x}{y+5}=\dfrac{1}{2}$, $2x=y+5\cdots(1)$;
$\dfrac{x-1}{y}=\dfrac{2}{3}$, $3x-3=2y\cdots(2)$;
Solving (1) and (2), we have $x=7, y=9$.
$\therefore$ The fraction is $\dfrac{7}{9}$.

19. Larger number $=x$, smaller number $=y$; $\dfrac{2}{5}(x+y)=x-y$,
$2x+2y=5x-5y$, $3x-7y=0\cdots(1)$; $x=2y+2\cdots(2)$,
Solving (1) and (2), we have $x=14, y=6$.

20. Tens-digit $=x$, units-digit $=y$; $10x+y=4(x+y)$, $6x-3y=0$,
$2x-y=0\cdots(1)$; $10y+x=10x+y+36$, $9x-9y++36=0$,
$x-y+4=0\cdots(2)$;
Solving (1) and (2), we have $x=4, y=8$.
$\therefore$ The number is 48.

22. Present age of Vincent $=x$, present age of Winnie $=y$;
$x-2=3(y-2)$, $x-3y+4=0\cdots(1)$; $x+2=2(y+2)$,
$x-2y-2=0\cdots(2)$;
Solving (1) and (2), we have $x=14, y=6$.
$\therefore$ The difference is $14-6=8$.

23. Boat : x m/s, stream : y m/s; $30(x-y)=240$, $x-y=8\cdots(1)$;
$20(x+y)=240$, $x+y=12\cdots(2)$;
Solving (1) and (2),
we have $x=10, y=2$.

24. Train: x km/h, car: y km/h; $(x+y)\times\dfrac{40}{60}=144$, $x+y=216\cdots(1)$;
$1.5(x-y)=144$, $x-y=96\cdots(2)$;
Solving (1) and (2), we have $x=156, y=60$.

26. $A(2x+1)-B(x-1)=(2A-B)x+(A+B)=-x+7$;
$2A-B=-1\cdots(1)$, $A+B=7\cdots(2)$;
Solving (1) and (2), we have $A = 2$, $B = 5$.

27. $x+3y=8x-y$, $7x-4y=0\cdots(1)$;
$(x+3y)+(8x-y)+30=180$, $9x+2y=150\cdots(2)$;
Solving (1) and (2), we have $x=12, y=21$.

30. $7=m(-1)+c$, $-m+c=7\cdots(1)$;
$1=m(2)+c$, $2m+c=1\cdots(2)$;
Solving (1) and (2), we have $m=-2, c=5$.

31. $a(4)-b(-1)=8$, $4a+b=8\cdots(1)$;
$a(-4)-b(5)=8$, $4a+5b=-8\cdots(2)$;
Solving (1) and (2), we have $a=3, b=-4$.

32. $a(3)+b(1)=-5$, $3a+b=-5\cdots(1)$;
$b(3)+a(1)=1$, $a+3b=1\cdots(2)$;
Solving (1) and (2), we have $a=-2, b=1$.

34. $\frac{x+1}{y+1}=\frac{2}{3}$, $3x+3=2y+2$, $3x-2y=-1\cdots(1)$;
$\frac{x-1}{y-1}=\frac{1}{2}$, $2x-2=y-1$, $2x-y=1\cdots(2)$;
Solving (1) and (2), we have $x=3, y=5$.

35. Let $\frac{1}{x}=a, \frac{1}{y}=b$. $a-2b=2\cdots(1)$,
$3a-5b=7\cdots(2)$;
Solving (1) and (2), we have $a=4, b=1$. $\therefore\ x=\frac{1}{4}, y=1$.

36. $\frac{x}{y}=\frac{2}{3}$, $3x=2y\cdots(1)$;
$\frac{y-1}{x+6}=\frac{2}{3}$, $3y-3=2x+12$, $2x-3y=-15\cdots(2)$;
Solving (1) and (2), we have $x=6, y=9$.

37. $5x+3y=10\cdots(1)$, $3x+5y=190\cdots(2)$; $(1)+(2)$,
$(5x+3y)+(3x+5y)=10+190$, $8x+8y=200$,
$\therefore\ x+y=25$

38. $7x+4y=56\cdots(1)$, $x-3y=32\cdots(2)$; $(1)-(2)$,
$(7x+4y)-(x-3y)=56-32$, $\therefore\ 6x+7y=24$

39. Let $x+y=a, x-y=b$. $3a-2b=13\cdots(1)$,
$4a+3b=6\cdots(2)$;
Solving (1) and (2), we have $a=3, b=-2$. $\therefore\ x-y=-2$

40. Let $x-5=a, y+3=b$. $6a+b=8\cdots(1)$, $4a-3b=20\cdots(2)$;

Solving (1) and (2), we have $a=2, b=-4$. $\therefore\ \frac{x-5}{y+3}=\frac{2}{-4}=-\frac{1}{2}$

41. $6x-9y=15$, $2x-3y=5\cdots(1)$;

$6y=4x-10$, $3y=2x-5$, $2x-3y=5\cdots(2)$;

$(1)-(2)$, we have $0=0$,

$\therefore$ there are infinitely many solutions.

45. $\frac{x}{6}+\frac{3y}{4}=a$, $2x+9y=12a$, $\therefore\ 12a=-4, a=-\frac{1}{3}$

UNIT 9 RATES, RATIOS AND PROPORTIONS (1)

1. A	2. C	3. A	4. D	5. C	6. B	7. B	8. D
9. C	10. B	11. D	12. C	13. B	14. C	15. C	16. A
17. C	18. D	19. D	20. C	21. A	22. A	23. D	24. D
25. D	26. A	27. D	28. A	29. C	30. D	31. B	32. A
33. C	34. D	35. B	36. B	37. B	38. A	39. D	40. B
41. C	42. B	43. D	44. C	45. D	46. C	47. B	48. D
49. B	50. B	51. A	52. C	53. D	54. C	55. D	56. B
57. D	58. A	59. B	60. B	61. D	62. A	63. D	64. C
65. B	66. C	67. A	68. C	69. C	70. C		

Explanatory Notes

2. $\frac{7.5\text{m}}{1\text{s}}=\frac{7.5\times60\times60\text{m}}{1\times60\times60\text{s}}=\frac{27000\text{m}}{1\text{h}}=\frac{27\text{km}}{1\text{h}}=27\text{km/h}$

23. $\frac{x+3}{2x-5}=\frac{2}{3}$, $3x+9=4x-10$, $\therefore\ x=19$

24. $4x:3(x+2)=5:4$, $\frac{4x}{3x+6}=\frac{5}{4}$, $16x=15x+30$, $\therefore\ x=30$

31. No. of boys $=40\times\frac{3}{8}+40\times\frac{3}{4}=45$,

$\therefore$ no. of boys : no. of girls $=45:(80-45)=9:7$

35. Let age of man $=x$. $\frac{x}{x-25}=\frac{8}{3}$, $3x=8x-200$, $5x=200$,

$\therefore\ x=40$

36. Let weight of solution $=x$ g. $\frac{2}{11}x+28=\frac{9}{11}x$, $28=\frac{7}{11}x$,

$\therefore\ x=28\times\frac{11}{7}=44$

39. $= \frac{1}{a} \times abc : \frac{1}{b} \times abc : \frac{1}{c} \times abc = bc : ac : ab$

46. $\text{A} : \text{B} : \text{C} = 11000 : 27500 : 44000 = 2 : 5 : 8$; let share of B = \$$x$,

$\frac{x}{3600} = \frac{5}{8}, \quad \therefore \; x = 3600 \times \frac{5}{8} = 2250$

47. Total amount $= 0.2 \times 160 \times \frac{1}{10} + 0.5 \times 160 \times \frac{3}{10} + 2 \times 160 \times \frac{6}{10}$

$= \$219.2$

48. No. of red balls $= 144 \times \frac{3}{12} + 144 \times \frac{1}{6} = 60$,

no. of blue balls $= 144 \times \frac{2}{12} + 144 \times \frac{2}{6} = 72$,

$\therefore$ no. of red balls : no. of blue balls : no. of green balls

$= 60 : 72 : (288 - 60 - 72) = 5 : 6 : 13$

51. $a = 20\% b = \frac{b}{5}, \; \frac{a}{b} = \frac{1}{5}, \quad \therefore \; a : b = 1 : 5$

52. $q = p\left(1 - 33\frac{1}{3}\%\right) = \frac{2}{3}p, \; \frac{p}{q} = \frac{3}{2}, \quad \therefore \; p : q = 3 : 2$

53. $\frac{a}{b} = \frac{2}{1}, \; a = 2b; \; \frac{a - 3b}{2a + b} = \frac{2b - 3b}{2(2b) + b} = \frac{-b}{5b} = -\frac{1}{5}$

56. $\frac{5a - b}{a + 2b} = 4, \; 5a - b = 4a + 8b, \; a = 9b, \; \frac{a}{b} = \frac{9}{1}, \quad \therefore \; a : b = 9 : 1$

57. $\frac{b}{a + b} = \frac{2}{5}, \; 5b = 2a + 2b, \; 3b = 2a, \; \frac{a}{b} = \frac{3}{2}, \quad \therefore \; a : b = 3 : 2$

58. $\frac{x}{3} = \frac{y}{8}, \; x = \frac{3y}{8}, \; \therefore \; y : 3x = y : 3\left(\frac{3y}{8}\right) = 8y : 9y = 8 : 9$

59. $\frac{m + n}{3} = \frac{2m - n}{4}, \; 4m + 4n = 6m - 3n, \; 7n = 2m, \; \frac{m}{n} = \frac{7}{2}$,

$\therefore \; m : n = 7 : 2$

60. $\frac{x}{x + 1} = \frac{x + 3}{x + 5}, \; x^2 + 5x = x^2 + 4x + 3, \; \therefore x = 3$

61. $6x = 2y, \; \frac{x}{y} = \frac{2}{6} = \frac{1}{3}, \; x : y = 1 : 3; \; 2y = 9z, \; \frac{y}{z} = \frac{9}{2}, \; y : z = 9 : 2;$

Combining the two ratios, $x : y : z = 3 : 9 : 2$

62. $\frac{1}{a} : \frac{1}{b} : \frac{1}{c} = 2 : 3 : 4, \; a : b : c = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$

64. $\because \; a : b : c : d = 1 : 2 : 3 : 4$,

$\therefore \; (a + b) : (c + d) = (1 + 2) : (3 + 4) = 3 : 7$;

$\frac{c + d}{42} = \frac{7}{3}, \; c + d = 42 \times \frac{7}{3} = 98$

65. $\because a:b:c:d=2:3:4:6$, $\therefore$ let $a=k$, $b=2k$, $c=3k$, $d=4k$, where $k\neq 0$; $a+b+c+d=2k+3k+4k+6k=15k$, $15k=105$, $k=7$, $\therefore d-a=6k-k=5k=5(7)=35$

66. Hot dog: \$$x$, sandwiches: \$$y$, hamburger: \$$z$;
$5x=4y$, $\frac{x}{y}=\frac{4}{5}$, $x:y=4:5$; $3y=5z$, $\frac{y}{z}=\frac{5}{3}$, $y:z=5:3$;
combining the two ratios, $x:y:z=4:5:3$, $\therefore x:z=4:3$

67. Length $=(60\div 2)\times\frac{5}{6}=25$ cm, width $=60\div 2-25=5$ cm,
$\therefore$ area $=25\times 5=125$ cm^2

Unit 10 Rates, Ratios and Proportions (2)

1. A	2. C	3. D	4. B	5. C	6. C	7. B	8. C
9. A	10. B	11. C	12. C	13. B	14. C	15. B	16. C
17. B	18. C	19. C	20. A	21. D	22. B	23. A	24. C
25. C	26. D	27. C	28. A	29. C	30. B	31. B	32. B
33. A	34. C	35. B	36. D	37. D			

Explanatory Notes

8. Actual area $=(4\times 200)\times(5\times 200)=800000\ \text{cm}^2=80\ \text{m}^2$

9. Scale $=1:50000=1\text{cm}:0.5\text{ km}$,
$\therefore$ actual area $=(2\times 0.5)^2=1\ \text{km}^2$

10. Scale $=1:80000000=1\text{cm}:80\text{km}$,
$\therefore$ time taken $(5\times 80)\div 100=4$ h

13. Length of component $=36\times\frac{25}{450}=2$ mm

19. Choosing any pairs of x, their ratio is the same as the corresponding pair of y^2, e.g.

x	4	9	16	25
y^2	36	81	144	225

$x_1:x_4=4:25$, $(y_1)^2:(y_4)^2=36:225=12:75=4:25$

23. Let $PT=2a$, $QT=a$. Area of $\Delta PST=\frac{2a\times 3a}{2}=3a^2$,
area of $QRST=\frac{(a+3a)\times 3a}{2}=6a^2$,
$\therefore$ area of ΔPST : area of $QRST=3a^2:6a^2=1:2$

24. Let $DY = x$, $AD = y$. $\frac{(10+x)y}{2} : \frac{(10+20-x)y}{2} = 3:2$,

$(10+x):(30-x) = 3:2$, $\frac{10+x}{30-x} = \frac{3}{2}$,

$20 + 2x = 90 - 3x$, $5x = 70$, $x = 14$

25. Let area of $\Delta ADE = a$, then area of $\Delta CDE = 3a$,
area of ΔADC = area of $\Delta BCD = 4a$,
$\therefore$ area of ΔADE : area of $\Delta BCD = a : 4a = 1:4$

26. Area of $\Delta CDE = 2 \times 2 = 4\ \text{cm}^2$,
$\therefore$ area of $\Delta BCE = (4+2) \times \frac{1}{2} = 3\ \text{cm}^2$

27. Let area of $\Delta ABE = 2a$, then area of $\Delta BCE = 3a$,
area of $\Delta CDE = 3a$,
$\therefore$ area of ΔABC : area of $\Delta CDE = (2a+3a):3a = 5:3$

28. Let area of ΔABD = area of $\Delta ACD = x$ and
area of ΔBDE = area of $\Delta CDE = y$,
$\therefore$ area of ΔABE : area of $\Delta ACE = (x-y):(x-y) = 1:1$

30. Length of road $= 12 \times 9000 \times \frac{1}{7500} = 14.4\ \text{cm}$

31. Scale $= 1:1000 = 1\ \text{cm} : 10\ \text{m}$,
$\therefore$ actual area $= \frac{(70+90) \times 150}{2} = 12000\ \text{m}^2$

33. Total number of days required $= \frac{30 \times 20 - 10 \times 20}{16} + 10 = 35$,
number of days delayed = 35 – 30 = 5

34. $y_2 : y_1 \neq x_1 : x_2$ (e.g. 84: 140 = 3 : 5 ≠ 5 : 7), ∴ I is true.
$y_1 : y_2 \neq x_1 : x_2$ (e.g. 140: 84 = 5 : 3 ≠ 5 : 7), ∴ II is true.
Choosing any pair of $(x-2)$, their ratio is equal to the corresponding inverse pair of y, e.g. 3:7 = 60:140,
∴ III is true.

$x-2$	3	5	7	10
y	140	84	60	42

$y_1 : y_2 \neq (x_1+2):(x_2+2)$
e.g. (5 + 2) : (7 + 2) = 7 : 9 ; 40 : 84 = 10 : 21 ∴ IV is false.

35. Let area of $\Delta PQT = a$, then area of $\Delta QRT = 2a$,
area of $\Delta PRS = a + 2a = 3a$,
$\therefore$ area of ΔQRT : area of $PQRS = 2a : (a + 2a + 3a) = 1:3$

36. Area of $\Delta BCE = 4 \times 3 = 12\ \text{cm}^2$, area of $\Delta CDE = 12 \times \frac{3}{2} = 18\ \text{cm}^2$,
$\therefore$ area of $\Delta BCD = 12 + 18 = 30\ \text{cm}^2$

37. Let area of $\Delta ADE = a$, then area of $\Delta BDE = 3a$;
Let area of $\Delta BEF = 3b$, then area of $\Delta CEF = 5b$;
$a + 3a = 3b + 5b, 4a = 8b, a = 2b$;
$\therefore$ area of ΔABC : area of $BDEF = (4a + 8b):(3a + 3b)$
$= 16b : 9b = 16 : 9$

UNIT 11 SIMILAR TRIANGLES

1. C	2. B	3. B	4. C	5. A	6. C	7. A	8. C
9. D	10. B	11. B	12. A	13. B	14. C	15. A	16. C
17. D	18. B	19. A	20. D	21. C	22. D	23. C	24. A
25. A	26. D	27. B	28. D	29. B	30. C	31. A	32. A
33. C	34. C	35. C	36. C	37. A	38. D	39. B	40. D
41. A	42. C	43. C	44. B	45. B	46. B	47. C	48. B

Explanatory Notes

8. $\frac{3}{y} = \frac{1}{1+2}$, $\therefore\ y = 9$

10. $\frac{DE}{DG} = \frac{DF}{DH}$, $\frac{3}{4+FG} = \frac{4}{3+13}$, $16 + 4FG = 48$, $FG = 8$

12. $\frac{4}{5} = \frac{6}{6+a}$, $24 + 4a = 30$, $a = 1.5$

14. $\frac{ML}{KL} = \frac{KL}{NL}$, $\frac{2}{6} = \frac{6}{MN+2}$, $2MN + 4 = 36$, $MN = 16$

15. $x + 85° = 120°$, $x = 35°$; $y = x = 35°$

20. In ΔABC and ΔEDC, $\angle C = \angle C$ (common),
$\angle ABC = \angle EDC = 100°$ (given), $\angle A = \angle E$ ($\angle$ sum of Δ),
$\therefore \Delta ABC \sim \Delta EDC$ (A.A.A.)

21. In ΔABC and ΔBDC, $\frac{AB}{BD} = \frac{7}{10.5} = \frac{2}{3}$, $\frac{AC}{BC} = \frac{4}{6} = \frac{2}{3}$, $\frac{BC}{DC} = \frac{6}{9} = \frac{2}{3}$,
$\therefore \Delta ABC \sim \Delta BDC$ (3 sides proportional)

27. In ΔABC and ΔCDE, $\angle A = 180° - 90° - \angle ACB$ ($\angle$ sum of Δ),
$\angle ECD = 180° - 90° - \angle ACB$ (straight angle),
$\therefore \angle A = \angle ECD$; $\angle B = \angle D = 90°$ (given);
$\angle ACB = \angle CED$ ($\angle$ sum of Δ), $\therefore\ \Delta ABC \sim \Delta CDE$ (A.A.A.),
$\therefore\ \frac{3}{CD} = \frac{5}{15}$, $CD = 3 \times \frac{15}{5} = 9$, $\therefore\ BD = 4 + 9 = 13$

32. $\because \Delta FCD \sim \Delta FAB,\ \therefore\ \dfrac{FD}{FB} = \dfrac{k}{30}$;

$\because \Delta BCD \sim \Delta BEF,\ \therefore\ \dfrac{BD}{BF} = \dfrac{k}{20}$;

$\dfrac{FD}{FB} + \dfrac{BD}{BF} = \dfrac{k}{30} + \dfrac{k}{20},\quad \dfrac{FD+BD}{BF} = \dfrac{5k}{60},\quad \dfrac{k}{12} = \dfrac{BF}{BF} = 1,\ \therefore\ k = 12$

33. $\dfrac{y}{y+3} = \dfrac{AC}{AE} = \dfrac{y+3}{y+3+8},\ y^2 + 11y = y^2 + 6y + 9,\ \therefore\ y = 1.8$

34. $\because \Delta QRU \sim \Delta TSU$ (AAA), $\therefore\ \dfrac{QR}{TS} = \dfrac{QU}{TU} = \dfrac{4}{10} = \dfrac{2}{5}$.

$\because \Delta PQR \sim \Delta PST$ (AAA), $\therefore\ \dfrac{PQ}{PS} = \dfrac{QR}{TS},\quad \dfrac{y}{y+9} = \dfrac{2}{5}$,

$5y = 2y + 18,\ 3y = 18,\ \therefore\ y = 6$

39. There are 4 Δs in the figure, namely a, b, c and d. The 6 pairs of ~Δs are (a,b) , (a,c) , (a,d) , (b,c) , (b,d) and (c,d) .

41. $\because\ \Delta SRQ \sim \Delta TSR$ (AAA), $\therefore\ \dfrac{RS}{ST} = \dfrac{QR}{RS}$,

$RS^2 = 18 \cdot 8 = 144,\ \therefore\ RS = 12$

42. $\because\ \Delta PRQ \sim \Delta QSR$ (AAA), $\therefore\ \dfrac{PR}{QS} = \dfrac{QR}{RS},\quad \dfrac{PR}{18} = \dfrac{18}{12}$,

$PR = \dfrac{18}{12} \times 18 = 27.\ \therefore\ PS = 27 - 12 = 15$

45. Congruent Δs have the same shape, and the ratio of lengths = 1 : 1

46. $\dfrac{CE}{CD} = \dfrac{CA}{CB},\quad \dfrac{CD+DE}{CD} = \dfrac{CB+BA}{CB},\quad 1 + \dfrac{DE}{CD} = 1 + \dfrac{BA}{CB},\quad \dfrac{DE}{CD} = \dfrac{BA}{CB}$.

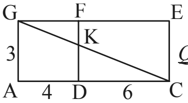
Similarly, $\dfrac{GE}{GF} = \dfrac{GK}{GH},\ \therefore\ \dfrac{FE}{GF} = \dfrac{HK}{GH}$.

Similarly, $\dfrac{CE}{DE} = \dfrac{GE}{FE},\ \therefore\ \dfrac{CD}{DE} = \dfrac{GF}{FE}$.

$\therefore\ \dfrac{CB}{BA} = \dfrac{CD}{DE} = \dfrac{GF}{FE} = \dfrac{GH}{HK},\quad \dfrac{CB}{BA} = \dfrac{GH}{HK},\quad \dfrac{a-1}{3} = \dfrac{8}{a+1}$,

$(a - 1)(a + 1) = 24,\ a^2 - 1 = 24,\ a^2 = 25,\ \therefore\ a = 5$

47. By flattening the two walls, the length of wire is minimum when *CKG* is a straight line.



Q.47

$\because \Delta CKD \sim \Delta CGA,\ \therefore\ \dfrac{DK}{3} = \dfrac{6}{6+4} = \dfrac{3}{5},\ \therefore\ DK = \dfrac{3}{5} \times 3 = 1.8$ m

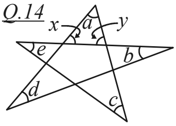
UNIT 12 ANGLES IN TRIANGLES AND POLYGONS

1. B	2. C	3. A	4. C	5. D	6. C	7. A	8. C
9. D	10. A	11. D	12. C	13. A	14. B	15. B	16. B
17. B	18. B	19. D	20. C	21. D	22. A	23. A	24. B
25. D	26. C	27. B	28. D	29. B	30. A	31. B	32. C
33. C	34. C	35. B	36. C	37. C	38. B	39. D	40. A
41. C	42. B	43. D	44. A	45. C	46. B	47. C	48. C
49. D	50. B	51. B	52. C	53. B	54. A	55. D	56. B
57. A	58. A	59. B	60. B	61. C	62. C	63. A	64. B
65. A	66. A	67. B	68. D	69. B	70. A	71. A	

Explanatory Notes

14. $x = b + d$, $y = c + e$, $\therefore\ a + x + y = 180°$,
$a + b + c + d + e = 180°$

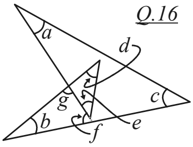
Q.14



15. $f = a + c$, $g = d + e$, $\therefore\ b + f + g = 180°$,
$a + b + c + d + e = 180°$

16. $\angle A + \angle B = \angle BOD$, $\angle C + \angle D = \angle DOF$,
$\angle E + \angle F = \angle FOB$,
$\therefore\ \angle A + \angle B + \angle C + \angle D + \angle E + \angle F$
$= \angle BOD + \angle DOF + \angle FOB = 360°$

Q.16



25. The ext. ∠s are $30°$, $24°$, $18°$ and $14°$.
$\because$ no. of sides $= \dfrac{360°}{\text{an ext.}\angle}$, $\dfrac{360°}{14°} = 25\dfrac{5}{7}$, $\therefore$ D is not possible.

26. The sum = 4 straight angles + sum of ext. ∠s of the quadrilateral
$4 \times 180° + 360° = 1080°$

29. n = number of sides, $(n-2)\times 180° = (6-2)\times 180° \times 2$, $n - 2 = 8$,
$\therefore\ n = 10$

30. $\dfrac{(n-2)\times 180°}{n} = \dfrac{(20-2)\times 180°}{20} = \dfrac{2}{3}$, $\dfrac{(n-2)\times 180°}{n} = 108°$,
$180°n - 360° = 108°n$, $72°n = 360°$, $\therefore\ n = 5$

34. n = number of sides, $(n-2)\times 180° = 3 \times 360°$, $n - 2 = 6$,
$\therefore\ n = 8$

35. n = number of sides, $\dfrac{(n-2)\times 180°}{n} = \dfrac{360°}{n} \times 5$,
$(n-2)\times 180° = 1800°$, $\therefore\ n = 12$

36. $360° \div 30° = 12$; $360° \div 40° = 9$; $360° \div 50° = 7.2$;
$360° \div 60° = 6$. $\therefore$ The answer is C.

39. The sum = 4 straight angles $+ 360° \times 2 -$ ∠sum of hexagon
$= 180° \times 4 + 720° - 180° \times (6-2) = 720°$

48. $\angle A = \angle ABD$, $\angle C = \angle CBD$, $\angle A + \angle ABD + \angle CBD + \angle C = 180°$,
$2\angle ABD + 2\angle CBD = 180°$, $2(\angle ABD + \angle CBD) = 180°$,
$\therefore\ \angle ABC = 90°$

49. $\angle A = \angle ABC = \angle CBD$, $\angle BCD = \angle D = \angle A + \angle ABC = 2\angle A$,
$\angle A + \angle ABC + \angle CBD + \angle D = 180°$, $\angle A + \angle A + \angle A + 2\angle A = 180°$,
$5\angle A = 180°$, $\therefore\ \angle A = 36°$

50. $\angle DAC = 180° - 110° - 60° = 10°$, $\angle BAD = 60° - 10° = 50°$,
$\therefore\ \angle ADB = (180° - 50°) \div 2 = 65°$

51. $\angle EAD = 60° - 40° = 20°$, $\angle ADE = (180° - 20°) \div 2 = 80°$,
$\therefore\ \angle CED = 80° - 60° = 20°$

53. DE = DC, and DC = DB, $\therefore$ DE = DB
$2 \times \angle EBD = \angle EDC$ (ext. $\angle$ of Δ)
$= 60°$ (equil. Δ)
$\therefore\ \angle$EBD = 30°.
$\angle$ABE = 60° + 30° = 90°

56. $x + y + z + 180° = 360°$, $\therefore\ x + y + z = 180°$

63. $k + 15° + 40° + [360° - (3k + 25°)] = 360°$,
$k + 55° + 360° - 3k - 25° = 360°$, $30° = 2k$, $\therefore\ k = 15°$

65. The sum = 4 straight angles + 360° − $\angle$sum of pentagon
$= 180° \times 4 + 720° - 180° \times (5 - 2) = 540°$

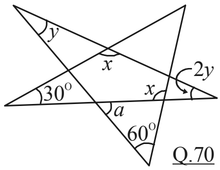
66. $(180° - a) + (180° - b) + c + d = 360°$, $360° - a - b + c + d = 360°$,
$\therefore\ a + b = c + d$

68. $\angle EAB = (5 - 2) \times 180° \div 5 = 108°$, $\angle EAF = 108° - 60° = 48°$,
$\therefore\ \angle AEF = (180° - 48°) \div 2 = 66°$

69. $\angle BAF = (6 - 2) \times 180° \div 6 = 120°$,
$\angle ABG = (180° - 120°) \div 2 = 30°$, $\angle BAG = (180° - 30°) \div 2 = 75°$,
$\therefore\ \angle GAF = 120° - 75° = 45°$

70. An exterior angle of an equilateral triangle equals $120°$, which is greater than $90°$ and also larger than its adjacent interior angle $(60°)$.

71. $a = y + 2y = 3y$, $x = a + 60° = 3y + 60°$;
$x + 2y + 30° = 180°$,
$(3y + 60°) + 2y = 150°$, $5y = 90°$,
$\therefore\ y = 18°$ and $x = 3(18°) + 60° = 114°$



Q.70

UNIT 13 RATIONAL AND IRRATIONAL NUMBERS

1. C	2. D	3. A	4. D	5. A	6. A	7. D	8. D
9. B	10. A	11. A	12. A	13. D	14. D	15. A	16. C
17. A	18. A	19. C	20. A	21. C	22. D	23. D	24. B
25. B	26. B	27. A	28. D	29. D	30. A	31. C	32. A
33. B	34. B	35. A	36. B	37. C	38. A	39. D	40. B
41. D	42. C	43. D	44. A	45. C	46. B	47. C	48. D
49. A	50. A	51. A	52. B	53. C	54. A	55. B	56. A
57. B	58. A	59. D	60. C	61. A	62. D	63. B	64. A
65. C							

Explanatory Notes

10. A square number with 0 as units-digit can only be obtained by squaring a number with 0 as units-digit.
$\therefore$ the units-digit of $\sqrt{10a}$ should be 0.

15. I. $\sqrt{2}+(-\sqrt{2})=0$ which is rational, $\therefore$ true.
II. $\sqrt{2}\times\sqrt{3}=\sqrt{6}$ which is irrational, $\therefore$ not true.
III. π^2 is irrational, $\therefore$ not true.
$\therefore$ The answer is A.

16. A. $\sqrt{2}-\sqrt{2}=0$ which is rational
B. $\sqrt{3}\times\sqrt{3}=3$ which is rational
D. $\dfrac{0}{\sqrt{5}}=0$ which is rational
$\therefore$ The answer is C.

28. $\sqrt{0.0343}=\sqrt{\dfrac{343}{10000}}=\dfrac{7\sqrt{7}}{100}=\dfrac{7p}{100}$

47. $\sqrt{18}x=1$, $3\sqrt{2}x=1$, $x=\dfrac{1}{3\sqrt{2}}$, $\therefore\ x=\dfrac{\sqrt{2}}{6}$

50. $\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{12}}=\dfrac{\sqrt{2}}{2}+\dfrac{1}{2\sqrt{3}}=\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{3}}{6}=\dfrac{3\sqrt{2}+\sqrt{3}}{6}=\dfrac{3x+y}{6}$

56. $\sqrt{24}=n$, $2\sqrt{2}\times\sqrt{3}=n$, $2m\sqrt{2}=n$, $\therefore\ \sqrt{2}=\dfrac{n}{2m}$

57. $x=\sqrt{45}=3\sqrt{5}$, $y=\sqrt{80}=4\sqrt{5}$, $\therefore\ \sqrt{5}=\dfrac{x}{3}=\dfrac{y}{4}$, $\therefore\ y=\dfrac{4x}{3}$

58. $(\sqrt{a}-\dfrac{1}{\sqrt{a}})^2-(\sqrt{a}+\dfrac{1}{\sqrt{a}})^2=(a-2+\dfrac{1}{a})-(a+2+\dfrac{1}{a})=-4$

59. $y-\frac{1}{y}=2\sqrt{6}$, $(y-\frac{1}{y})^2=(2\sqrt{6})^2$, $y^2-2+\frac{1}{y^2}=24$,

$\therefore\ y^2+\frac{1}{y^2}=26$

60. I. $m+n=\frac{\sqrt{5}-6}{2}+\frac{\sqrt{5}+6}{2}=\frac{2\sqrt{5}}{2}=\sqrt{5}$ which is irrational

II. $m^2+n^2=(\frac{\sqrt{5}-6}{2})^2+(\frac{\sqrt{5}+6}{2})^2$

$=\frac{5-12\sqrt{5}+36+5+12\sqrt{5}+36}{4}=\frac{41}{2}$ which is rational

III. $mn=(\frac{\sqrt{5}-6}{2})(\frac{\sqrt{5}+6}{2})=\frac{5-36}{4}=-\frac{31}{4}$ which is rational

$\therefore$ The answer is C.

61. $(\sqrt{7-3\sqrt{5}})(\sqrt{7+3\sqrt{5}})=\sqrt{(7-3\sqrt{5})(7+3\sqrt{5})}=\sqrt{49-45}=2$

62. $(\sqrt{7}+3)y=2$, $(\sqrt{7}-3)(\sqrt{7}+3)y=(\sqrt{7}-3)2$,

$(7-3^2)y=2(\sqrt{7}-3)$, $-2y=2(\sqrt{7}-3)$, $y=\frac{2(\sqrt{7}-3)}{-2}=3-\sqrt{7}$

63. $\frac{\sqrt{14}}{\sqrt{2}+\sqrt{7}}=\frac{\sqrt{14}(\sqrt{2}-\sqrt{7})}{2-7}=\frac{\sqrt{28}-\sqrt{98}}{-5}=\frac{7\sqrt{2}-2\sqrt{7}}{5}$

64. $\frac{\sqrt{6}+1}{7-3\sqrt{6}}=\frac{(\sqrt{6}+1)(7+3\sqrt{6})}{49-54}=\frac{10\sqrt{6}+25}{-5}=-5-2\sqrt{6}$

65. $a=\frac{1}{3-\sqrt{10}}=\frac{3+\sqrt{10}}{9-10}=-(3+\sqrt{10})$,

$b=\frac{1}{3+\sqrt{10}}=\frac{3-\sqrt{10}}{9-10}=\sqrt{10}-3$

I. $a-b=-(3+\sqrt{10})-(\sqrt{10}-3)=-2\sqrt{10}$ which is irrational

II. $a+b=-(3+\sqrt{10})+(\sqrt{10}-3)=-6$ which is rational

III. $ab=-(3+\sqrt{10})(\sqrt{10}-3)=-(10-9)=-1$ which is rational

IV. $\frac{a}{b}=\frac{-(3+\sqrt{10})}{\sqrt{10}-3}=\frac{-(3+\sqrt{10})^2}{10-9}=-(19+6\sqrt{10})$

which is irrational

$\therefore$ The answer is C.

Unit 14 Pythagoras' Theorem

1. A	2. C	3. B	4. C	5. C	6. D	7. A	8. B
9. B	10. A	11. C	12. D	13. D	14. C	15. B	16. C
17. B	18. B	19. D	20. C	21. C	22. A	23. B	24. B
25. B	26. A	27. A	28. B	29. A	30. A	31. B	32. B
33. D	34. C	35. D	36. A	37. A	38. B	39. B	40. C
41. A	42. B	43. B	44. A	45. A	46. B	47. B	48. A

Explanatory Notes

4. III. $\because \angle C = 90^\circ$, $\therefore \angle A + \angle B + \angle C = 180^\circ$,
$\angle A + \angle B = 180^\circ - 90^\circ = 90^\circ = \angle C$

13. Let h be the height. $h^2 + 6^2 = 12^2$, $h = \sqrt{144-36} = \sqrt{108} = 6\sqrt{3}$,
$\therefore$ area $= \dfrac{6\sqrt{3}\times 12}{2} = 36\sqrt{3}$ cm^2

14. Let y be the hypotenuse. $y^2 = 12^2 + 12^2$, $y = \sqrt{288} = 12\sqrt{2}$;
area $= \dfrac{12\times 12}{2} = \dfrac{12\sqrt{2}\times h}{2}$, $12 = \sqrt{2}h$, $h = \dfrac{12}{\sqrt{2}} = 6\sqrt{2}$

15. $(4x-1)^2 + (3x+2)^2 = (5x+1)^2$,
$16x^2 - 8x + 1 + 9x^2 + 12x + 4 = 25x^2 + 10x + 1$,
$4x + 5 = 10x + 1$, $4 = 6x$, $\therefore x = \dfrac{2}{3}$

26. Distance $= \sqrt{(5+16)^2 + 24^2} = \sqrt{1017} = 31.89$ m

29. Let a cm be the length of the base diagonal. $a^2 = 12^2 + 4^2 = 160$,
length of pencil $= \sqrt{a^2 + 3^2} = \sqrt{160+9} = \sqrt{169} = 13$ cm

31. $AB = \sqrt{(0.25\times 400)^2 + (0.2\times 400)^2} = \sqrt{100^2 + 80^2} = 128.06$ m

32. $\because 15^2 + 20^2 = 625 = 25^2$, $\therefore \angle BAC = 90°$.
$\therefore$ area $= \dfrac{20\times 15}{2} = \dfrac{25x}{2}$, $x = 12$

33. $AB^2 = [6-(-3)]^2 + [4-(-8)]^2 = 81 + 144$,
$\therefore AB = \sqrt{225} = 15$ units

34. $\because \Delta ABC \sim \Delta DCE$ (A.A.A.), $\therefore \dfrac{DE}{3} = \dfrac{2}{1}$, $DE = 2\times 3 = 6$.
$BC^2 = 1^2 + 3^2 = 10$, $CE^2 = 2^2 + 6^2 = 40$,
$\therefore BE = \sqrt{BC^2 + CE^2} = \sqrt{10+40} = \sqrt{50} = 5\sqrt{2}$

35. $OA^2 = AN^2 + ON^2$, $r^2 = \left(\dfrac{30}{2}\right)^2 + (r-9)^2$,

$r^2 = 225 + r^2 - 18r + 81$, $18r = 306$, $\therefore\ r = 17$

36. $RS^2 + (12-8)^2 = (12+8)^2$, $RS^2 = 400 - 16$,

$\therefore\ RS = \sqrt{384} = 8\sqrt{6}$

38. $OC = OA = OB = 8 + 2 = 10$ cm; $OD^2 + CD^2 = OC^2$,

$8^2 + CD^2 = 10^2$, $CD^2 = 100 - 64$, $CD = \sqrt{36} = 6$,

$\therefore\ AE = (8+2) - 6 = 4\,\text{cm}$

39. Length of square $= 2a$ cm, $a^2 + (2a)^2 = \left(\dfrac{40}{2}\right)^2$, $5a^2 = 400$,

$a = \sqrt{80} = 4\sqrt{5}$

40. Diameter of circle $= 50\,\text{cm}$,

$\therefore$ length of rectangle $= \sqrt{50^2 - 14^2} = \sqrt{2304} = 48\,\text{cm}$

41. $GE^2 = GH^2 + EH^2 = a^2 + a^2 = 2a^2$,

$BE^2 = BG^2 + GE^2 = a^2 + 2a^2 = 3a^2$, $\therefore\ BE = \sqrt{3a^2} = \sqrt{3}a$

42. $\angle SPR = 2\theta - \theta = \theta$ (ext.$\angle$ of Δ), $\therefore\ \angle SPR = \angle S$,

$\therefore\ PR = RS = 10$, $\therefore\ PQ = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$,

$\therefore\ PS = \sqrt{8^2 + (6+10)^2} = \sqrt{320} = 8\sqrt{5}$

43. $PQ^2 = (a^2 - b^2)^2 + (2ab)^2 = a^4 - 2a^2b^2 + b^4 + 4a^2b^2$

$= a^4 + 2a^2b^2 + b^4 = (a^2 + b^2)^2$, $\therefore\ PQ = a^2 + b^2$

44. $BD = \sqrt{12^2 + 9^2} = 15$, $\because\ \Delta BED \cong \Delta CED$ (S.A.S.),

$\therefore\ CD = BD = 15\,\text{cm}$. $BC^2 = 9^2 + (15-12)^2$,

$BC = \sqrt{90} = 3\sqrt{10}\,\text{cm}$

45. $18 \times 5\dfrac{40}{60} = 102\,\text{km}$,

$\therefore$ shortest distance $= \sqrt{160^2 - 102^2} = \sqrt{15196} = 123\,\text{km}$

46. $SY^2 + RS^2 = RY^2$, $SY^2 + 12^2 = (18 - SY)^2$,

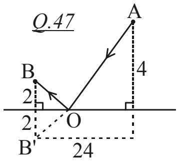
$SY^2 + 144 = 324 - 36SY + SY^2$, $36SY = 180$, $\therefore\ SY = 5\,\text{cm}$

47. Construct $\Delta OBC \cong \Delta OB'C$, then $OB = OB'$ and $AD + OB = AO + OB'$. $AO + OB'$ is minimum when AOB' is a straight line.

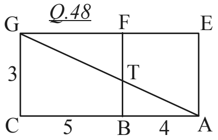
$AOB' = \sqrt{24^2 + (8+2)^2} = \sqrt{676} = 26$ m,

$\therefore$ the minimum distance is 26 m.

Q.47



48. When flattening the two walls, the length of wine is minimum when ATG is a straight line.



$\therefore$ Minimum length $= \sqrt{3^2 + (5+4)^2}$

$= \sqrt{90} = 9.487\text{ m}$

UNIT 15 TRIGONOMETRIC RATIOS

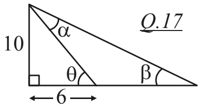
1. C	2. B	3. A	4. A	5. B	6. B	7. B	8. C
9. A	10. A	11. C	12. B	13. D	14. C	15. B	16. B
17. C	18. C	19. A	20. B	21. D	22. B	23. C	24. D
25. C	26. D	27. B	28. C	29. A	30. A	31. B	32. C
33. A	34. A	35. C	36. C	37. B	38. B	39. B	40. C
41. A	42. B	43. C	44. D	45. B	46. A	47. D	48. D
49. B	50. B	51. A	52. B	53. B	54. C	55. D	56. A
57. C	58. D	59. D	60. A	61. C	62. B	63. C	64. B
65. C	66. B	67. A					

Explanatory Notes

15. $\tan\angle AED = \dfrac{15}{6}$, $\tan\angle BEC = \dfrac{15}{15-6}$,

$\therefore\ \theta = 180^\circ - 68.1986^\circ - 59.036^\circ \approx 52.77^\circ$

17. $\tan\theta = \dfrac{10}{6}$, $\theta = 59.04^\circ$,



$\therefore\ \alpha + \beta = \theta = 59.04^\circ$

19. $\cos 3\theta = 0.66$, $3\theta = 48.70^\circ$, $\theta = 16.23^\circ$

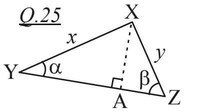
20. $\sin(\theta + 15^\circ) = 2\sin 25^\circ = 0.8452$, $\theta + 15^\circ = 57.70^\circ$, $\theta = 42.70^\circ$

21. $2\tan(80^\circ - \theta) = 5.1$, $\tan(80^\circ - \theta) = 2.55$,

$80^\circ - \theta = 68.59^\circ$, $\theta = 11.41^\circ$

22. $\tan(2\theta + 10^\circ) = 2.4$, $2\theta + 10^\circ = 67.38^\circ$, $\theta = 28.69^\circ$

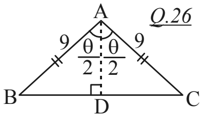
25. $\dfrac{AY}{x} = \cos\alpha$, $AY = x\cos\alpha$;



$\dfrac{AZ}{y} = \cos\beta$, $AZ = y\cos\beta$,

$\therefore\ YZ = AY + AZ = x\cos\alpha + y\cos\beta$

26. $\dfrac{BD}{9} = \sin\dfrac{\theta}{2}$, $BD = 9\sin\dfrac{\theta}{2}$,



$\therefore\ BC = 2BD = 18\sin\dfrac{\theta}{2}$

31. $\frac{a}{BC} = \tan 50°$, $BC = \frac{a}{\tan 50°}$; $\frac{a}{BD} = \tan 20°$, $BD = \frac{a}{\tan 20°}$;

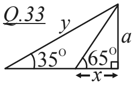
$\therefore\ CD = BD - BC = a\left(\frac{1}{\tan 20°} - \frac{1}{\tan 50°}\right)$

32. $\frac{WY}{16} = \cos 55°$, $WY = 16\cos 55°$; $\angle XWY = 90° - 55° = 35°$,

$\frac{XY}{WY} = \tan 35°$, $XY = WY \tan 35° = 16\cos 55° \tan 35°$

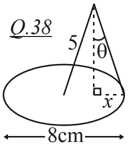
33. $\frac{a}{x} = \tan 65°$, $a = x\tan 65°$;

$\frac{a}{y} = \sin 35°$, $y = \frac{a}{\sin 35°} = \frac{x\tan 65°}{\sin 35°}$



34. $\angle BAC = 80° - 40° = 40° = \angle B$, $\therefore\ AC = BC = k$,

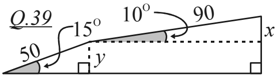
$\therefore\ \frac{AD}{k} = \sin 80°$, $AD = k\sin 80°$



38. $x = 8 \div 4 = 2$, $\sin\theta = \frac{2}{5}$, $\theta = 23.58°$,

$\therefore$ angle between legs is $23.58° \times 2 = 47.16°$.

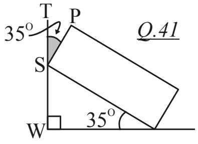
39. $\frac{x}{90} = \sin 10°$, $x = 90\sin 10°$;



$\frac{y}{50} = \sin 15°$, $y = 50\sin 15°$;

$\therefore$ Vertical distance $= x + y = 90\sin 10° + 50\sin 15° = 28.57$ m

41. $\frac{SW}{10} = \sin 35°$, $SW = 10\sin 35°$;

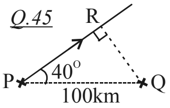


$\frac{TS}{4} = \cos 35°$, $TS = 4\cos 35°$;

$\therefore$ Height = SW + TS

= (10 sin 35° + 4 cos 35°) cm

45. The car is closest to town Q at R.



$\frac{PR}{100} = \cos 40°$, $PR = 100\cos 40°$,

$\therefore$ time taken $= \frac{100\cos 40°}{50} = 1.53$ h

48. A: $\tan\theta = \frac{5x}{6y}$; B: $\tan\theta = \frac{2x}{3y}$; C: $\tan\theta = \frac{5x}{4y}$; D: $\tan\theta = \frac{4x}{3y}$.

$\because\ \frac{4}{3} > \frac{5}{4} > \frac{5}{6} > \frac{2}{3}$, $\therefore\ \tan\theta$ is the greatest in D.

50. $\tan\alpha = \frac{AB}{BC}$, $\tan\beta = \frac{AB}{BD} = \frac{AB}{2BC} = \frac{1}{2}\tan\alpha$,

$\therefore\ \tan\alpha : \tan\beta = 2:1$

51. $\because\ \angle CAD = 2\theta - \theta = \theta = \angle C$, $\therefore\ AD = CD = 15.$

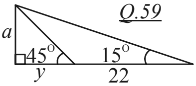
$\therefore\ AB = \sqrt{15^2 - 9^2} = 12$, $\therefore\ \tan 2\theta = \frac{12}{9} = \frac{4}{3}$

52. $(20-x)^2 + 24^2 = (x+12)^2$, $400 - 40x^2 + 576 = x^2 + 24x + 144$,

$832 = 64x$, $x = 13$. $\sin\theta = \frac{24}{13+12} = \frac{24}{25}$, $\therefore\ \theta = 73.74°$

53. $\because\ 20^2 + 21^2 = 841 = 29^2$, $\therefore\ \angle B = 90°$. $\sin\theta = \frac{21}{29}$, $\theta = 46.4°$

59. $\frac{a}{y} = \tan 45° = 1$, $a = y$; $\frac{a}{y+22} = \tan 15°$,



$y = (y+22)\tan 15°$,

$y - y\tan 15° = 22\tan 15°$, $y = \frac{22\tan 55°}{1 - \tan 15°}$

60. $\angle BAC = 180° - 30° - 60° = 90°$. In ΔABC, $\frac{AB}{25} = \sin 30°$,

$AB = 25\sin 30°$. In ΔABD, $\frac{h}{AB} = \sin 60°$,

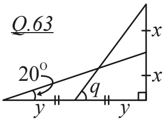
$h = AB\sin 60° = 25\sin 30°\sin 60°$

62. $\frac{AB}{6} = \tan 30°$, $AB = 6\tan 30°$;

$\tan\angle ADC = \frac{AC}{6} = \frac{2AB}{6} = \frac{2(6\tan 30°)}{6} = 2\tan 30°$,

$\angle ADC = \theta + 30° = 49.11°$, $\therefore\ \theta = 19.11°$

63. $\frac{x}{2y} = \tan 20°$, $\frac{x}{y} = 2\tan 20°$;



$\tan\theta = \frac{2x}{y} = 2(2\tan 20°) = 4\tan 20°$,

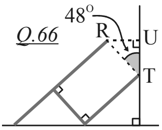
$\therefore\ \theta = 55.5°$

65. $\frac{30}{PS} = \sin 48°$, $PS = \frac{30}{\sin 48°}$; $\angle TSK = 48°$, $\frac{SK}{60} = \cos 48°$,

$SK = 60\cos 48°$;

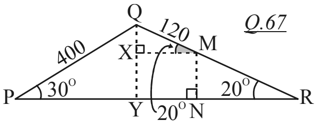
$\therefore\ PK = PS + SK = \frac{30}{\sin 48°} + 40\cos 48° = 80.5$ cm

66. $\frac{RU}{30} = \sin 48°$, $RU = 30\sin 48° = 22.3$,



$\therefore$ shortest distance is 22.3cm.

67. $\frac{QX}{120} = \sin 20°$, $QX = 120\sin 20°$;



$\frac{QY}{400} = \sin 30°$, $QY = 400\sin 30°$;

$\therefore\ MN = QY - QX = 400\sin 30° - 120\sin 20° = 159.0\,\text{m}$

UNIT 16 AREAS AND VOLUMES (2)

1. B	2. D	3. C	4. C	5. D	6. C	7. A	8. B
9. B	10. C	11. B	12. D	13. B	14. A	15. C	16. A
17. A	18. C	19. D	20. B	21. C	22. B	23. D	24. C
25. D	26. D	27. C	28. B	29. D	30. C	31. A	32. B
33. B	34. B	35. C	36. B	37. B	38. D	39. C	40. C
41. A	42. B	43. D	44. B	45. D	46. B	47. D	48. C
49. B	50. D	51. C	52. D	53. A	54. C	55. A	56. D
57. B	58. A	59. C	60. B	61. A	62. C	63. B	64. C
65. C	66. B	67. C	68. C	69. C			

Explanatory Notes

8. Radius $= r$ cm, $2r + 2\pi r \times \frac{1}{2} = 250$, $r(2+\pi) = 250$,

$\therefore\ r = 250 \div (2+\pi) = 48.6$

15. Shaded area = 2 × (sector area) – area of square

$$= \pi(9)^2 \times \frac{1}{4} \times 2 - 9^2 = 46.2\,\text{m}^2$$

16. Shaded area $= \pi\left(\frac{8}{2}\right)^2 - \left(\frac{1}{2} \times 8 \times \frac{8}{2}\right) \times 2 = 18.3\,\text{cm}^2$

18. Shaded area $= \pi\left(\frac{16}{2}\right)^2 \times \frac{3}{4} \times 2 = 301.6\,\text{cm}^2$

19. Originally, $A = \pi r^2$;

when $R = 2r$, area $= \pi R^2 = \pi(2r)^2 = 4\pi r^2 = 4A$.

$\therefore$ Area is 4 times that of the original one.

20. Radius of new ring $= r$ cm, $2\pi r = 2\pi(4) + 2\pi(6) = 20\pi, r = 10$.

$\therefore$ Area $= \pi(10)^2 = 100\pi\ \text{cm}^2$

22. New radius $= r$ cm, $\pi r^2 = \pi(15)^2(1 - 64\%) = 81\pi$, $r = \sqrt{81} = 9$.

$\therefore$ Decrease $= 15 - 9 = 6\,\text{cm}$

33. $\pi r^2 \times \frac{\theta_1}{360} = \pi(2r)^2 \frac{\theta_2}{360}$, $r^2\theta_1 = 4r^2\theta_2$, $\frac{\theta_1}{\theta_2} = \frac{4}{1}$, $\therefore\ \theta_1 : \theta_2 = 4:1$

34. $BC = 10\tan 48°$,

$\therefore$ shaded area $= \frac{10 \times 10\tan 48°}{2} - \pi(10)^2 \times \frac{48°}{360°} = 13.6\text{ cm}^2$

42. Volume $= \pi(14)^2 \times 25 \times (1 - 60\%) \div 200 = 9.8\pi \text{ cm}^3$

43. New water level $= (5^3 \times 30) \div (\pi \times 16^2) + 9 = 13.7$ cm

46. Height $= h$ cm, $2\pi(5)(h) + 2\pi(5)^2 = 350\pi$, $10\pi h = 300\pi$,

$\therefore\ h = 30$

48. Radius $= r$ cm, $\pi r^2(5) = 125\pi$, $r^2 = 25$, $r = 5$.

$\therefore$ Total surface area $= 2\pi(5)(5) + 2\pi(5)^2 = 100\pi \text{ cm}^2$

49. Radius $= r$ cm, $2\pi r = 30$, $r = \frac{15}{\pi}$.

$\therefore$ Volume $= \pi\left(\frac{15}{\pi}\right)^2(21) = 1504\text{ cm}^3$

50. Total surface area $= \pi(8)^2 \times \frac{40°}{360°} \times 2 + \left[2\pi(8) \times \frac{40°}{360°} + 8 \times 2\right] \times 5$

$= 152.6\text{ cm}^2$

52. $2\pi rh = \frac{1}{2}(\pi r^2)$, $4h = r$, $\frac{r}{h} = \frac{4}{1}$, $\therefore\ r : h = 4:1$

53. Let radius $= 2a$ and height $= 5a$.

Base area: curved surface area

$= \pi(2a)^2 : 2\pi(2a)(5a) = 4a^2 : 20a^2 = 1:5$

54. Original volume $= \pi r^2 h$, new volume $= \pi(2r)^2\left(\frac{h}{2}\right) = 2\pi r^2 h$,

$\therefore$ the volume is doubled.

55. Volume of A : volume of B

$= \pi(2r)^2(3h) : \pi(3r)^2(h) = 12\pi r^2 h : 9\pi r^2 h = 4:3$

56. Let radius of big wheel $= R$,

and radius of small wheel $= r$.

$2\pi R \times 16 = 2\pi r \times 20$, $4R = 5r$, $R = \frac{5r}{4}$.

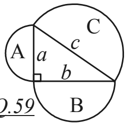
$\therefore$ Area of big wheel : area of small wheel

$= \pi R^2 : \pi r^2 = \left(\frac{5r}{4}\right)^2 : r^2 = 25:16$

58. Width of rectangle $= \sqrt{(8+2)^2 - 8^2} = 6$ cm,

$\therefore$ shaded area $= \pi(10)^2 \times \frac{1}{2} - (8+8) \times 6 = 61.1\text{ cm}^2$

59. $\pi\left(\frac{a}{2}\right)^2 \times \frac{1}{2} = 18\pi$, $\left(\frac{a}{2}\right)^2 = 36$, $\frac{a}{2} = 6$, $a = 12$;



Q.59

$\pi\left(\frac{b}{2}\right)^2 \times \frac{1}{2} = 32\pi$, $\left(\frac{b}{2}\right)^2 = 64$, $\frac{b}{2} = 8$, $b = 16$;

$c = \sqrt{12^2 + 16^2} = 20$, $\therefore$ area of $c = \pi\left(\frac{20}{2}\right)^2 \times \frac{1}{2} = 50\pi \text{ cm}^2$

60. Angle at centre $= \theta$, $2\pi(36) \times \frac{\theta}{360°} = 20\pi$, $\theta = 100°$,

$\therefore$ Area $= \pi(36)^2 \times \frac{100°}{360°} = 360\pi \text{ cm}^2$

61. Height of triangle $= 18\sin 30° = 9$ cm,

$\therefore$ shaded area $= \pi(18)^2 \times \frac{30°}{360°} - \frac{9 \times 18}{2} = 3.8 \text{ cm}^2$

62. Base of triangle $= 25\cos 30° \times 2 = 43.3$ cm,
height of triangle $= 25\sin 30° = 12.5$ cm,
angle at centre $= 180° - 30° \times 2 = 120°$,

$\therefore$ shaded area $= \pi(25) \times \frac{120°}{360°} - \frac{43.3 \times 12.5}{2} = 383.9 \text{ cm}^2$

63. $\because$ $\angle COB = 30° + 30° = 60°$ and $OC = 10$ cm,

$\therefore$ height of $\Delta OAC = 10\sin 60°$ cm,

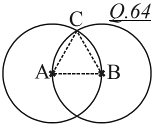
$\therefore$ shaded area $= \frac{10 \times 10\sin 60°}{2} + \pi(10)^2 \times \frac{60°}{360°} = 95.7 \text{ cm}^2$

64. ΔABC is an equilateral Δ.
Height of $\Delta ABC = 5\sin 60°$ cm,

area of $\Delta ABC = \frac{1}{2} \times 5 \times 5\sin 60° = 10.825 \text{ cm}^2$,

$\therefore$ shaded area

Q.64



$= \left[\pi(5)^2 \times \frac{60°}{360°} - 10.825\right] \times 4 + 10.825 \times 2 = 30.7 \text{ cm}^2$

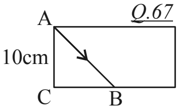
65. Volume of wood used $= \pi\left(\frac{12}{2}\right)^2(18) - \pi\left(\frac{12}{2} - 1\right)^2(18 - 1)$

$= 700.6 \text{ cm}^3$

66. Level of orange juice $= h$ cm, $\pi\left(\frac{8}{2}\right)^2 h = 300$, $h = 5.968$.

$\therefore$ Area in contact $= 2\pi\left(\frac{8}{2}\right)(5.968) + \pi\left(\frac{8}{2}\right)^2 = 200.3 \text{ cm}^2$

67. When the curved surface is flattened, it becomes a rectangle.



$BC = 2\pi(4)\times\frac{1}{2} = 4\pi$ cm,

$\therefore$ shortest distance $= AB = \sqrt{(4\pi)^2 + 10^2} = 16.1$

68. Time required $= (7\times 12\times 15)\div(\pi\times 0.11^2)\div 10\div 60 = 55.2$ min

69. Depth of water $= d$ cm,

$(d-50)\times 100\times 160 + \pi(50)^2(160)\times\frac{1}{2} = 1500000$,

$(d-50)\times 16000 = 871681.47$, $d-50 = 54.5$, $d = 104.5$ cm

UNIT 17 SIMPLE STATISTICAL GRAPHS (2)

1. B	2. D	3. A	4. C	5. B	6. C	7. D	8. A
9. C	10. B	11. D	12. D	13. B	14. C	15. D	16. C
17. A	18. B	19. B	20. B	21. C	22. C	23. D	24. C
25. B	26. A	27. D	28. A	29. C	30. D	31. A	32. C
33. C	34. B	35. A	36. D	37. A	38. B	39. A	

Explanatory Notes

12. Since actual values are not known in grouped data, we cannot determine the maximum temperature.

14. Least possible amount = lower class boundary of the 1st class

$$= \frac{15+20}{2} = \$17.5$$

16. The 3rd class has the highest frequency.

Lower class limit $= \frac{25+30}{2} + 0.5 = \$28$,

upper class limit $= \frac{30+35}{2} - 0.5 = \$32$

17. $m = 11 - 4 = 7$, $n = 35 - 32 = 3$

18. $x = 29.5 - (34.5 - 29.5) = 24.5$, $y = 11 + 13 = 24$

21. From the graph, there are 18 students whose marks are less than 50.

$\therefore$ Passing % $= \frac{40-18}{40}\times 100\% = 55\%$

22. No. of students who fail $= 40\times\frac{3}{8} = 15$, from the graph,

the passing mark is 44.

23. The 4th class has the highest frequency $(35-23=12)$.

Lower class limit $= 60 + 0.5 = 60.5$,
upper class limit $= 80 - 0.5 = 79.5$

24. Percentage $= \dfrac{37-26}{40} \times 100\% = 27.5\%$
25. III. Since actual values are not known in grouped data, we cannot determine the age of the youngest employee.
26. II and III. Since actual values are not known in grouped data, we cannot determine the maximum daily sales.
30. A frequency polygon is formed by joining the middle of the bars in the histogram. The end points of a frequency polygon should be zero.
31. A frequency curve should start from zero and end at zero.
32. I. A cumulative frequency curve is always non-decreasing.
    II. A cumulative frequency curve should start from zero.
33. From the frequency polygon, the frequencies of all classes are equal. Therefore, the cumulative frequency increases at a steady rate.
34. From the frequency polygon, the frequencies in the middle classes are less than that of others.
37. III. A cumulative frequency polygon is non-decreasing.
39. When a set of data is concentrated in the middle, the cumulative frequency curve will be increasing with the greatest slope in the middle part.