Answers & Explanatory notes

Contents

Answers & Explanatory notes

UNIT 1 FACTORIZATION OF SIMPLE POLYNOMIALS (3)

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33. D	34. C						

Explanatory Notes

1. $\because$ Coefficient of $x = -b$ which is negative
   and the constant term $= c$ which is positive,
   $\therefore$ we have $(x-p)(x-q) = x^2 - px - qx + pq = x^2 - (p+q)x + pq$

2. $\because$ Constant term $= pq = -c$ which is negative,
   $\therefore$ either p or q is negative,
   i.e. I and II are not necessarily true.
   $\because$ Coefficient of $x = p + q = b$ which is positive,
   $\therefore$ $p + q > 0$, i.e. III is true.
   $\therefore$ The answer is B.

9. A. $x^2 + 17x + 60 = (x+12)(x+5)$;
   B. $x^2 - 17x - 60 = (x-20)(x+3)$;
   C. $x^2 + 17x - 60 = (x+20)(x-3)$;
   D. $x^2 - 17x + 60 = (x-12)(x-5)$

11. I. $10y^2 - y - 2 = (5y+2)(2y-1)$;
    II. $2 - y - 10y^2 = (2-5y)(1+2y)$;
    III. $10y^2 - 9y + 2 = 2 - 9y + 10y^2 = (2-5y)(1-2y)$;
    $\therefore$ The answer is C.

14. $x^4 - 5x^2 - 36 = (x^2-9)(x^2+4) = (x+3)(x-3)(x^2+4)$

15. $12k(k+1) - 5(k+2) = 12k^2 + 7k - 10 = (4k+5)(3k-2)$

16. $(7t+3)(2t+1) - 15 = 14t^2 + 7t + 6t + 3 - 15 = 14t^2 + 13t - 12$
    $= (2t+3)(7t-4)$

21. A. $6x^2 + x - 7 = (6x+7)(x-1)$;
    B. $6x^2 + 11x - 7 = (2x-1)(3x+7)$;
    D. $6x^2 + 19x - 7 = (2x+7)(3x-1)$;
    $\therefore$ The answer is C.

22. I. $5a^2 - 3a + 1 - 3 = 5a^2 - 3a - 2 = (5a + 2)(a - 1)$;
II. $5a^2 - 3a + 1 - 3a = 5a^2 - 6a + 1 = (5a - 1)(a - 1)$;
III. $5a^2 - 3a + 1 - 3a^2 = 2a^2 - 3a + 1 = (2a - 1)(a - 1)$;
$\therefore$ The answer is D.

24. $x^4 - 3x^2 - 4 = (x^2 - 4)(x^2 + 1) = (x + 2)(x - 2)(x^2 + 1)$

25. $y^4 - 10y^2 + 9 = (y^2 - 9)(y^2 - 1) = (y + 3)(y - 3)(y + 1)(y - 1)$

28. $= \dfrac{1}{(y+3)(y-2)} + \dfrac{1}{(y+3)(y+8)} = \dfrac{y+8+y-2}{(y+3)(y-2)(y+8)}$
$= \dfrac{2(y+3)}{(y+3)(y-2)(y+8)} = \dfrac{2}{(y-2)(y+8)}$

29. $= \dfrac{1}{(4-m)(1-m)} + \dfrac{2}{(4-m)(2+m)} = \dfrac{2+m+2(1-m)}{(4-m)(1-m)(2+m)}$
$= \dfrac{4-m}{(4-m)(1-m)(2+m)} = \dfrac{1}{(1-m)(2+m)}$

30. Area = $56 + 10x - x^2 = (4 + x)(14 - x)$,
$\therefore$ perimeter = $2[(4 + x) + (14 - x)] = 36$ cm

31. $x^2 + 24x + 80 = (x + 20)(x + 4)$;
$x + 4 = 9$, $x = 5$;
$\therefore$ The larger number = 5 + 20 = 25

32. $= [(a^2 - 2a) + 1]^2 = [(a - 1)^2]^2 = (a - 1)^4$

33. $= [(x^2 + 3x) + 2][(x^2 + 3x) - 10] = (x + 1)(x + 2)(x + 5)(x - 2)$

34. $= (x^2 + 4x + 4) - (y^2 - 2y + 1) = (x + 2)^2 - (y - 1)^2$
$= [(x + 2) + (y - 1)][(x + 2) - (y - 1)] = (x + y + 1)(x - y + 3)$

UNIT 2 LAWS OF INDICES (2)

1. B	2. B	3. D	4. C	5. C	6. A	7. D	8. A
9. D	10. B	11. B	12. C	13. A	14. C	15. B	16. A
17. C	18. D	19. C	20. D	21. C	22. B	23. A	24. B
25. D	26. A	27. C	28. B	29. D	30. D	31. B	32. C
33. A	34. D	35. A	36. D	37. C	38. B	39. C	40. B
41. B	42. B	43. D	44. A	45. B	46. A	47. C	48. B
49. C	50. D	51. A	52. D	53. B	54. B	55. C	56. A
57. B	58. C	59. C	60. A	61. D	62. A	63. C	64. D
65. D	66. B	67. A	68. C	69. D	70. C	71. B	72. A
73. B	74. A	75. B	76. C	77. D	78. A	79. C	

Explanatory Notes

11. $=[-(a^{-1})^{-2}]^{-3}=(-a^{2})^{-3}=-a^{-6}=-\frac{1}{a^{6}}$

12. $=\frac{(2^{3})^{-3}}{(2^{2})^{6}}\times\frac{2^{7}}{(2^{5})^{-1}}=\frac{2^{-9}}{2^{12}}\times\frac{2^{7}}{2^{-5}}=2^{-9}$

15. $=6x^{-3}y^{2}\times(2x^{-1}y^{2})=12x^{-4}y^{4}=\frac{12y^{4}}{x^{4}}$

19. $=(a^{-5}b)^{2}(ab^{-1})^{-4}=(a^{-10}b^{2})(a^{-4}b^{4})=a^{-14}b^{6}=\frac{b^{6}}{a^{14}}$

20. $=(\frac{1}{m}-\frac{1}{n})^{-1}=(\frac{n-m}{mn})^{-1}=\frac{mn}{n-m}$

27. $27^{x}=(3^{3})^{x}=(3^{x})^{3}=y^{3}$

28. $4^{x+2}=4^{x}\cdot 4^{2}=16y$

29. $=4\cdot 2^{x}=2^{2}\cdot 2^{x}=2^{x+2}$

32. $5^{2x+1}=5^{2x}\cdot 5=(5^{x})^{2}\cdot 5=5y^{2}$

34. $=\frac{3^{2n-1}\cdot 3^{3n+3}}{3^{6n}}=3^{2-n}$

35. $=\frac{3^{n+2}\cdot 5^{n+2}}{3^{n+1}\cdot 5^{n-1}}=3\cdot 5^{3}$

45. $49\cdot 7^{4y-1}=(2006y)^{0}$, $7^{2}\cdot 7^{4y-1}=1$, $7^{4y+1}=7^{0}$, $4y+1=0$,

$\therefore\ y=-\frac{1}{4}$

46. $32^{m}\cdot 8^{m+2}=\frac{1}{16}$, $2^{5m}\cdot 2^{3m+6}=2^{-4}$, $8m+6=-4$, $\therefore\ m=-\frac{5}{4}$

49. $2^{n+2}-2^{n}=48$, $2^{n}(2^{2}-1)=48$, $2^{n}=16$, $\therefore\ n=4$

50. $10^{k-2}-10^{k+1}+999=0$, $10^{k}(10^{-2}-10)=-999$, $10^{k}(-\frac{999}{100})=-999$,

$10^{k}=100$, $\therefore\ k=2$

61. $a^{2}=2^{-1}$, $(a^{2})^{-3}=(2^{-1})^{-3}$, $\therefore\ a^{-6}=2^{3}=8$

62. $4a=3b=y$, $\therefore\ a=\frac{y}{4}$ and $b=\frac{y}{3}$,

$\therefore\ a^{-2}b^{3}=(\frac{y}{4})^{-2}(\frac{y}{3})^{3}=(\frac{16}{y^{2}})(\frac{y^{3}}{27})=\frac{16y}{27}$

63. $=1\div(\frac{2}{a}+\frac{1}{b})=1\div\frac{2b+a}{ab}=\frac{ab}{a+2b}$

64. $=(x+y)\div(\frac{1}{x^{2}}-\frac{1}{y^{2}})=(x+y)\div\frac{y^{2}-x^{2}}{x^{2}y^{2}}=(x+y)\times\frac{x^{2}y^{2}}{(y-x)(y+x)}$

$=\frac{x^{2}y^{2}}{y-x}$

65. $x-\dfrac{1}{x}=3$, $(x-\dfrac{1}{x})^2=3^2$, $x^2-2+\dfrac{1}{x^2}=9$, $\therefore\ x^2+\dfrac{1}{x^2}=11$

68. $=4^{n-1}(3\cdot4^2-5)=43\cdot4^{n-1}$

69. $=3^{2n-2}+3^{2n}=3^{2n-2}(1+3^2)=10\cdot3^{2n-2}$

70. $=\dfrac{3^n(7+6\cdot3)}{3^n\cdot3^{-2}}=25\cdot3^2=225$

71. $=\dfrac{4\cdot5^{2n-2}-6\cdot5^{2n-1}}{5^{2n}+5^{2n}}=\dfrac{5^{2n-2}(4-6\cdot5)}{2\cdot5^{2n}}=\dfrac{5^{-2}(-26)}{2}=-\dfrac{13}{25}$

73. $5^k+5^{k-1}=0.24$, $5^k(1+5^{-1})=\dfrac{6}{25}$, $5^k(\dfrac{6}{5})=\dfrac{6}{25}$, $5^k=\dfrac{1}{5}$,
$\therefore\ k=-1$

74. $5\cdot3^{y-1}+3^{y+2}-\dfrac{6^{-2}}{2^{-7}}=0$, $3^y(5\cdot3^{-1}+3^2)=\dfrac{6^{-2}}{2^{-7}}$, $3^y(\dfrac{32}{3})=\dfrac{2^7}{6^2}$,
$3^y=\dfrac{2^7}{2^2\cdot3^2}\times\dfrac{3}{2^5}=\dfrac{1}{3}$, $\therefore\ y=-1$

75. $9^{x+1}=16$, $3^{2x+2}=16$, $3^{2x}\cdot3^2=16$, $(3^x)^2=\dfrac{16}{9}$, $\therefore\ 3^x=\dfrac{4}{3}$

76. $4^{2x}\cdot2^{3y-5}=1$, $2^{4x}\cdot2^{3y-5}=2^0$, $4x+3y-5=0\ldots\ldots(1)$;
$3^{2x}\cdot9^{y-1}=27$, $3^{2x}\cdot3^{2y-2}=3^3$, $2x+2y-2=3\ldots\ldots(2)$;
Solving (1) and (2), we have $x=-2.5$, $y=5$.

77. $=(\dfrac{1}{x}-\dfrac{1}{y})^{-2}=(\dfrac{y-x}{xy})^{-2}=\dfrac{x^2y^2}{(y-x)^2}=\dfrac{x^2y^2}{(x-y)^2}$

UNIT 3 NUMERAL SYSTEMS

1. D	2. B	3. A	4. C	5. C	6. B	7. D	8. A
9. B	10. A	11. A	12. C	13. C	14. D	15. D	16. B
17. A	18. D	19. B	20. A	21. D	22. D	23. C	24. B
25. A	26. B						

Explanatory Notes

25. The given expression
$= (2^3 + 2^1 + 1) \times 2^8 + 2^7 + (5 - 1) \times 2^4$
$= 2^{11} + 2^9 + 2^8 + 2^7 + 2^2 \times 2^4$
$= 2^{11} + 2^9 + 2^8 + 2^7 + 2^6$
$= 101111000000_2$

26. Difference = $(10b + a) - (10a + b) = 9b - 9a$

UNIT 4 LINEAR INEQUALITIES IN ONE UNKNOWN

1. D	2. A	3. D	4. D	5. B	6. A	7. D	8. D
9. A	10. C	11.C	12. B	13. B	14. A	15. C	16. B
17. A	18. A	19. D	20. C	21. A	22. D	23. D	24. B
25. C	26. A	27. B	28. B	29. A	30. D	31. C	32. B
33. B	34. D	35. D	36. A	37. B	38. B	39. A	40. C
41. D	42. D	43. A	44. B	45. C	46. B	47. A	48. A
49. B	50. A	51. C	52. D	53. A	54. D	55. A	56. C
57.C	58. D	59. C	60. C	61. C	62. D	63. C	64. C
65. B	66. D	67. B	68. D				

Explanatory Notes

9. When $x=-1$, $y=-3$, $z=-9$,
$\because -1-(-3)=2<6=-3-(-9)$, $\therefore$ II is not true.
$\because 1=(-1)^2<(-9)^2=81$, $\therefore$ III is not true.

13. $\because \dfrac{m}{-5}$ is positive and $\dfrac{n}{5}$ is negative, $\therefore$ II is true.

29. $7x-4y<-1$, $7x+1<4y$, $\therefore y>\dfrac{7x+1}{4}$

34. Larger number = *x*, smaller number = $x-2$; $x+(x-2)\geq 30$,
$2x\geq 32$, $x\geq 16$. $\because$ *x* is odd, $\therefore$ minimum value = 17

35. Smaller number = *x*, larger number = $x+4$;
$x>\dfrac{x+4}{2}$, $2x>x+4$, $x>4$.
$\because$ *x* is a multiple of 4, $\therefore$ least value = 8

41. Selling price of each apple = \$*x*; $\dfrac{(80-20)x-80}{80}\times 100\%\geq 20\%$,
$\dfrac{60x-80}{80}\geq\dfrac{1}{5}$, $60x-80\geq 16$, $x\geq 1.6$. $\therefore$ Minimum price = \$1.6

42. $\because x<-3$, $\therefore x-1<-4<-3<-2$, $\therefore$ I, II and III are true.

43. $\because x\geq 15$, $\therefore x+1\geq 16$.
I is true because 16 > 15;
II is not true when *x* = 15;
III is not true because 16 < 17.

45. $y>-5$, $1-\dfrac{x}{3}>-5$, $-\dfrac{x}{3}>-6$, $x<18$. $\because$ *x* is non-negative,
$\therefore$ no. of possible values = 18 (from 0 to 17 inclusive).

46. $2a-b+10=0$, $2a=b-10$, $a=\dfrac{b-10}{2}$; $\because a\leq 0$, $\therefore \dfrac{b-10}{2}\leq 0$,
$b-10\leq 0$, $b\leq 10$. $\therefore$ Greatest value = 10

47. I. $\because\ 4a < a < b$, $\therefore$ true.
II. $\because\ -4b > 0 > a$, $\therefore$ true.
III. When $a = -3$, $b = -2$, $-3 > -8 = 4(-2)$, $\therefore$ not true.
$\therefore$ The answer is A.

48. When $m = 1.5$, $n = 1$, $\because\ 1.5 - 1 = 0.5 < 1$, $\therefore$ A is not always true.

49. I. $\because\ a > 0$ and $a > b$, $\therefore a^2 > ab$, n.$\therefore$ true.
II. $\because\ a^3$ is positive and b^3 is negative, $\therefore\ a^3 > b^3$, $\therefore$ true.
III. When $a = 1$, $b = -4$, $1^2 = 1 < 16 = (-4)^2$, $\therefore$ not true.
$\therefore$ The answer is B.

51. $\because\ ab < c$, $\therefore\ ab - c < 0 < 1$

54. I and II are not true when x is negative.
III is not true when $0 < x < 1$.
$\therefore$ The answer is D.

56. I. When $m = -4$, $n = 24$, $\dfrac{24}{-4} = -6 < -3$, $\therefore$ not true.
II. $m < -3$, $mn < -3n$ ……(i); $n > 9$, $-3n < 27$ ……(ii);
Combining (i) and (ii), we have $mn < -27$, $\therefore$ true.
III. $\because\ m < -3$ and $n > 9$, $\therefore m^2 > 9$ and $n^2 > 81$,
$\therefore m^2 + n^2 > 90$, $\therefore$ true.
$\therefore$ The answer is C.

57. I. If y is a positive integer, $\dfrac{1}{y}$ is a proper fraction less than or equal to 1, i.e. $\dfrac{1}{y} \le 1 < 10$, $\therefore$ true.
II. $\because\ \dfrac{1}{y}$ is negative when y is negative, $\therefore\ \dfrac{1}{y} < 0 < 10$, $\therefore$ true.
III. When $y = \dfrac{1}{20}$, $1 \div (\dfrac{1}{20}) = 20 > 10$, $\therefore$ not true.
$\therefore$ The answer is C.

61. $ay + 9a \le 2y - a$, $10a \le 2y - ay$, $(2 - a)y \ge 10a$,
$\therefore\ y \ge \dfrac{10a}{2 - a}$ ($\because\ a < 2$)

62. $mx + m^2 > nx + n^2$, $mx - nx > n^2 - m^2$,
$(m - n)x > (n - m)(n + m)$, $(m - n)x > -(m - n)(n + m)$
$\because\ m - n$ is negative, $\therefore\ x < -(n + m)$, $x < -m - n$

64. Smallest possible value $= -3 - (-1) = -2$

66. Greatest possible value $= (-8)^2 + (-3)^2 = 73$

67. Smallest possible value $= (-8)(2) = -16$;
greatest possible value $= (-8)(-3) = 24$; $\therefore\ -16 \le ab \le 24$

68. Greatest possible value $= \frac{-3}{-1} = 3$

UNIT 5 PERCENTAGES (2)

1. C	2. D	3. A	4. A	5. B	6. D	7. A	8. C
9. C	10. C	11. D	12. A	13. B	14. A	15. A	16. D
17. C	18. D	19. D	20. C	21. D	22. C	23. A	24. B
25. B	26. B	27. C	28. B	29. C	30. D	31. A	32. A
33. A	34. A	35. B	36. C	37. B	38. A	39. D	40. C
41. A	42. B	43. B	44. A	45. B	46. D		

Explanatory Notes

8. Amount $= 5000(1 + 4\% \times 2 + 5\% \times 3) = \$6150$

17. Compound interest $= 18000(1 + \frac{2\%}{4})^4 (1 + \frac{2.8\%}{4})^8 - 18000 = \$1416.6$

18. $P[(1 + 6\%)^2 - 1] \geq 4000$, $0.1236P \geq 4000$, $P \geq 32362.46$.
$\because$ P is a multiple of 10, $\therefore$ $P = 32370$

19. Difference $= 90000[(1 + \frac{9\%}{12})^{18} - 1] - 90000 \times 9\% \times \frac{18}{12}$
$= 12956.4 - 12150$ $= \$806.4$

20. Amount owed after the 1st payment $= 95000(1 + \frac{15\%}{12}) - 25000$
$= \$71187.5$, amount owed after the 2nd payment
$= 71187.5(1.0125) - 25000 = \$47077$

21. Amount owed at the end of 1st month $= 18000(1 + \frac{24\%}{12}) = \$18360$,
amount owed at the end of 2nd month $= (18360 - 5000)(1.02)$
$= \$13627.2$,
amount owed at the end of 3rd month $= (13627.2 - 5000)(1.02)$
$= \$8800$

22. Amount owed after 2 months $= 26000(1 + \frac{16\%}{12})^2 - 6000$
$= \$20697.96$,
amount owed after 4 months $= 20697.96(1 + \frac{16\%}{12})^2 - 6000$
$= \$15254$

23. Interest = amount in 4 years – amount in 3 years
$= 44000(1+\frac{8\%}{2})^8 - 44000(1+\frac{8\%}{2})^6$
$= 60217.04 - 55674.04 = \$4543$

32. Decay factor = r; $32000r^2 = 23120$, $r = \sqrt{\frac{23120}{32000}} = 0.85$;
$\therefore$ Value in 2006 $= 23120(0.85)^5 = \$10258$

33. Increase in book collection $= 74000[(1+8\%)^3(1+5\%)^2 - 1]$
$= 28774$

34. Sales figure $= 35000 \div (1+4\%)^4(1+8\%)^6 = 18854$

35. Let principal = $\$P$, no. of years = n.
$P(1+10\%n) = P(1+150\%)$, $1 + 0.1$, $n = 2.5$, $\therefore n = 15$

36. Let principal = $\$P$, interest rate = r.
$P(1+16r) = 2P$, $1 + 16r = 2$,
$\therefore r = 0.0625 = 6.25\%$

37. Principal $= 3200 \div (1+6\% \times 4\frac{2}{3}) = \$2500$,
$\therefore$ required amount $= 2500(1+3\frac{1}{3}\% \times 6) = \$3000$

38. $\because 6\% \times 5 = 2.4\% \times 12.5 = 0.3$, $\therefore$ the answer is A.

39. A. $(1+\frac{18\%}{12})^{12} = 1.1956$; B. $(1+\frac{18.2\%}{4})^4 = 1.1948$;
C. $(1+\frac{18.8\%}{2})^2 = 1.1968$; D. $(1+19\%) = 1.19$;
$\therefore$ Kelvin should choose D.

40. Let principal = $\$P$. $P[(1+r\%)^3 - 1] = P \times 20\%$, $(1+r\%)^3 = 1.2$,
$1 + r\% = \sqrt[3]{1.2} = 1.063$, $r\% = 0.063$, $\therefore r = 6.3$

41. Interest earned in the 3rd year
$= 65000[(1+\frac{4\%}{4})^{12} - (1+\frac{4\%}{4})^8] = \$2857.9$,
interest earned in the 2nd year $= 65000[(1.01)^8 - (1.01)^4] = \$2746.4$,
$\therefore$ difference $= 2857.9 - 2746.4 = \$112$

42. Amount at the end of 2003
$= 6800(1+5\%)^4 + 6800(1+5\%)^3 + 6800(1+5\%)^2 + 6800(1+5\%)$
$= \$30774$

43. Let monthly installment = $\$x$. $[7000(1+\frac{12\%}{12}) - x](1+\frac{12\%}{12}) - x = 0$,
$(7070 - x)(1.01) - x = 0$, $7140.7 - 1.01x - x = 0$, $2.01x = 7140.7$,
$\therefore x = 3552.6$

44. Let annual deposit = $\$x$. $x(1+6\%)^3 + x(1+6\%)^2 + x = 300000$,
$x[(1.06)^3 + (1.06)^2 + 1.06 + 1] = 300000$, $\therefore x = 68577$

45. Decrease in value $= 95000(1-12\%)^4 - 95000(1-12\%)^5$
$= 56971.06 - 50134.53 = \$6837$

46. Value $= 16000(1+5\%)^2(1-10\%)^5 = \$10416$

UNIT 6 PERCENTAGES (3)

1. B	2. D	3. A	4. A	5. C	6. B	7. C	8. D
9. A	10. D	11. C	12. B	13. A	14. B	15. A	16. B
17. D	18. A	19. A	20. A	21. D	22. B	23. B	24. C
25. B	26. A	27. B	28. C	29. C	30. C	31. B	32. C
33. B	34. A	35. C	36. D				

Explanatory Notes

5. Let the side of small cube = x.
total surface area of original cube $= 6(3x)^2 = 54x^2$;
total surface area of small cubes $= 6x^2 \times 27 = 162x^2$;
$\therefore$ percentage increase $= \dfrac{162x^2 - 54x^2}{54x^2} \times 100\% = 200\%$

8. Let x kg be the weight before joining the slimming programme.
The required percentage change $= \dfrac{x - (0.7x)(1.2x)}{(0.7x)(1.2x)} \times 100\% = +19\%$

9. Original volume $= \pi r^2 h$, new volume $= \pi[r(1+x\%)]^2[h(1-20\%)]$;
$\pi r^2 h = \pi[r(1+x\%)]^2[h(1-20\%)]$, $\pi r^2 h = \pi r^2 h(1+x\%)^2(0.8)$,
$0.8(1+x\%)^2 = 1$, $(1+x\%)^2 = 1.25$, $1 + x\% = 1.118$, $x = 11.8$.
$\therefore$ Percentage increase in radius = 11.8%

10. $A = C(1-15\%) = 0.85C$, $B = C(1+10\%) = 1.1C$,
$\therefore$ required percentage $= \dfrac{B}{A} \times 100\% = \dfrac{1.1C}{0.85C} \times 100\% = 129.4\%$

11. $Q = R \times 120\% = 1.2R$, $Q = S \times 75\% = 0.75S$;
I. $\dfrac{Q-R}{Q} \times 100\% = \dfrac{1.2R - R}{1.2R} \times 100\% = 16.6\%$
II. $\dfrac{S-Q}{S} \times 100\% = \dfrac{S - 0.75S}{S} \times 100\% = 25\%$
III. $\dfrac{S-R}{S} \times 100\% = \dfrac{\frac{1}{0.75}Q - \frac{1}{12}Q}{\frac{1}{0.75}Q} \times 100\% = 37.5\%$
$\therefore$ The answer is C.

12. Percentage change $= [(1+20\%)(1-15\%) - 1] \times 100\% = 2\%$

13. Percentage of failed students $=60\%(1-55\%)=27\%$,
percentage of students who passed $=1-27\%=73\%$

I. Percentage $=\dfrac{73}{27}\times100\%=270\%$

II. Percentage $=\dfrac{73-27}{27}\times100\%=170\%$

III. Percentage $=\dfrac{73-27}{73}\times100\%=63\%$

$\therefore$ The answer is A.

15. Percentage change
$=[0.55(1-10\%)+0.25(1+30\%)+0.2(1+5\%)-1]\times100\%=3\%$

17. Suppose x g of sugar should be added.
$\dfrac{400\times15\%+x}{400+x}\times100\%=20\%$, $\dfrac{60+x}{400+x}=\dfrac{1}{5}$, $300+5x=400+x$,
$4x=100$, $\therefore x=25$

22. Percy's allowance ≥ annual income = \$17 000×12 = \$204 000

23. Salaries tax at progressive rates
= \$[50 000×(2% + 6% + 10% + 14%) +
(4 302 000 – 302 000 – 50 000×4)×17%]
= \$662 000
Salaries tax at standard rate = 4 302 000×15% = \$645 300
$\because$ \$645 300 < \$662 000, $\therefore$ salaries tax payable = \$645 300

24. Let number of members in 2000 = x. $x(1+10\%)^3(1-5\%)^2\le900$,
$x\le749.2$. $\therefore$ Maximum number = 749

25. Let cost = \$$c$ and selling price = \$$s$, then $9c=6s$ or $s=1.5c$.
$\therefore$ Profit percentage $=\dfrac{s-c}{c}\times100\%=\dfrac{1.5c-c}{c}\times100\%=50\%$

26. $D=E(1+25\%)=1.25E$, $F=D(1-16\%)=0.84D$;
$\because \dfrac{F-E}{E}\times100\%=\dfrac{0.84D-\frac{D}{1.25}}{\frac{D}{1.25}}\times100\%=5\%$,
$\therefore$ A is true but C is false.
$\because \dfrac{F-E}{F}\times100\%=\dfrac{0.84D-\frac{D}{1.25}}{0.84D}\times100\%=4.76\%$,
$\therefore$ B and D are false.

27. Let number = N. $N(1+r\%)(1-r\%)=N(1-36\%)$,
$1-(r\%)^2=0.64$, $(r\%)^2=0.36$, $r\%=0.6$, $\therefore r=60$

28. Let original income = \$$x$, then original savings $=x\times20\%=0.2x$,
new savings $=x(1+15\%)-x(1-20\%)(1+10\%)=0.27x$.
$\therefore$ Percentage change $=\dfrac{0.27x-0.2x}{0.2x}\times100\%=35\%$

29. Original price per kg $= 60 \times \frac{3}{10} + 32 \times \frac{7}{10} = \$40.4$,

new price per kg $= 60(1-15\%) \times \frac{3}{10} + 32(1+25\%) \times \frac{7}{10} = \$43.3$,

$\therefore$ percentage change $= \frac{43.3 - 40.4}{40.4} \times 100\% = 7.2\%$

30. Let distance = D and speed = S,

then original time $= \frac{D}{S}$, new time $= \frac{D}{S(1-50\%)} = \frac{D}{0.5S}$.

$\therefore$ Percentage increase $= \frac{\frac{D}{0.5S} - \frac{D}{S}}{\frac{D}{S}} \times 100\% = 100\%$

31. Let original price per kg = $\$x$. $\frac{480}{x} - \frac{480}{x(1+20\%)} = 10$,

$1.2(480) - 480 = 10(1.2x)$, $96 = 12x$, $\therefore x = 8$

32. Let number of articles = n. $\frac{600}{n}(1+15\%)(n-5) - 600 = 21$,

$600(1.15)(n-5) = 621n$, $690n - 3450 = 621n$, $\therefore n = 50$

33. Let cost of X = a, then cost of Y = $a(1 + 25\%) = 1.25a$,
total cost = $a + 1.25a = 2.25a$.

If profit percentage on Y = $r\%$,
then total selling price $= a(1+60\%) + 1.25a(1+r\%)$,

$\therefore$ $(2.25a)(1+50\%) = a(1+60\%) + (1.25a)(1+r\%)$,
$1.775a = 1.25a(1+r\%)$, $1 + r\% = 1.42$, $r\% = 0.42$, $r = 42$.

$\therefore$ Profit percentage on Y = 42%

34. Let profit percentage of remaining stock = $r\%$.

$[\frac{1}{2}(1+20\%) + \frac{1}{6}(1-16\%) + (1 - \frac{1}{2} - \frac{1}{6})(1+r\%) - 1] \times 100\% = 15\%$,

$0.6 + 0.14 + \frac{1}{3}(1+r\%) - 1 = 0.15$, $\frac{1}{3}(1+r\%) = 0.41$, $r\% = 0.23$,

$r = 23$. $\therefore$ Profit percentage = 23%

35. Salaries tax on the 1st \$200 000
= \$50 000(2% + 6% + 10% + 14%) = \$16 000
$\therefore$ annual income = \$(16 000 ÷ 17% + 200 000 + 132 000)
= \$426 118

36. Let $\$x$ be annual income.
50 000(2%+6%+10%+14%)+(x–352000–200000)(17%) = x(15%)
16 000 + 0.17x – 93 840 = 0.15x
0.02x = 77 840, x = 3 892 000, $\therefore$ Annual income = \$3 892 000

UNIT 7 QUADRILATERALS

1. D	2. B	3. B	4. B	5. D	6. B	7. C	8. A
9. A	10. C	11. A	12. D	13. B	14. C	15. B	16. D
17. C	18. D	19. A	20. B	21. B	22. D	23. C	24. C
25. A	26. B	27. D	28. C	29. C	30. D	31. A	32. B
33. D	34. D	35. B	36. A	37. D	38. B	39. B	40. B
41. A							

Explanatory Notes

13. $x + 8 = 3y$, $x - 3y + 8 = 0$ ……(1);
$4x - y = 9 - x$, $5x - y - 9 = 0$ ……(2);
Solving (1) and (2), we have $x = 2.5$, $y = 3.5$.

15. $\angle ECB = 60° + 90° = 150°$;
$\because EC = CD = CB$, $\therefore \angle CBE = (180° - 150°) \div 2 = 15°$,
$\therefore \angle AFE = \angle CFB = 180° - \angle ACB - \angle CBE = 180° - 45° - 15°$
$= 120°$

16. I. $\because \Delta SKU \cong \Delta SKT$ (RHS/AAS),
$\therefore \angle KSU = \angle KST = 60° \div 2 = 30°$,
but $\angle RST = 90° - 60° = 30°$, $\therefore \angle KST = \angle RST$,
also $\angle SKT = \angle R = 90°$ and ST is common,
$\therefore \Delta RST \cong \Delta KST$ (AAS)
II. $\angle SKU = \angle QKT$, $KU = KT$, $\angle SUK = \angle QTK$,
$\therefore \Delta SKU \cong \Delta QKT$ (ASA), $\therefore$ SK = QK
III. $\because PS = QR$ and $SU = QT$, $\therefore PU = RT$
$\therefore$ The answer is D.

17. $QR = PS = 13$, $ST = \frac{1}{2}QS = \frac{1}{2} \times \sqrt{13^2 - 5^2} = \frac{1}{2} \times 12 = 6$,
$\therefore PR = 2RT = 2\sqrt{5^2 + 6^2} = 2\sqrt{61} = 15.6$ cm

22. Area of ΔAPQ : area of $PQCB = 1 : 3$

23. $\because$ AC = CE and BC // DE, $\therefore$ AB = BD (intercept thm.)
$\because$ AC = CE and CD // EF, $\therefore$ AD = DF (intercept thm.)
$\therefore y = 3 + 3 = 6$

24. $BG = \frac{1}{2}CE$, $BF = 2CE$,
$\therefore BG : BF = \frac{1}{2}CE : 2CE = 1 : 4$

28. $3y + 2 = x + y$, $x - 2y = 2$ ……(1);
$2x - 4 = x + y$, $x - y = 4$ ……(2);
Solving (1) and (2), we have $x = 6$, $y = 2$.
$\therefore$ Area = $\frac{1}{2}(6+2)^2 \times 4$ = 128 sq. units

29. Let $WZ = YZ = a$. $\because \Delta WZK \sim \Delta HYK$ (AAA),
$\therefore \frac{WZ}{HY} = \frac{ZK}{YK}$, $\frac{a}{9} = \frac{a-6}{6}$,
$6a = 9a - 54$, $3a = 54$, $\therefore a = 18$

30. $\angle EBA = \angle EAB = 55°$, $\angle DEA = 55° + 55° = 110°$,
$\angle AEF = 110° - 60° = 50°$, but $AE = DE = EF$,
$\therefore \angle AFE = (180° - 50°) \div 2 = 65°$

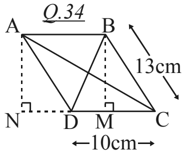
31. I. $\angle DGH = \angle EGH = 90°$,
$\angle HDG = \angle DEG = 90° - \angle EDG$,
$\therefore \Delta DHG \sim \Delta EDG$ (AAA)
II. $BC = DC$, $\angle BCH = \angle DCF = 90°$,
$\angle CBH = \angle DEG = \angle FDC$,
$\therefore \Delta BHC \cong \Delta DCF$ (ASA)
III. $\because \Delta GEF \cong \Delta GBF$ and $\Delta HBC \cong \Delta CDF$,
but ΔHBC is not congruent to ΔGBF,
$\therefore \Delta CDF$ is not congruent to ΔGEF.

32. $BD = DE = BF = \sqrt{12^2 + 12^2} = 12\sqrt{2}$, $CH = FC = 12\sqrt{2} - 12$,
$DH = 12 - (12\sqrt{2} - 12) = 24 - 12\sqrt{2}$,
$DG = \frac{1}{2}DF = \frac{1}{2}\sqrt{12^2 + (12\sqrt{2} - 12)^2} = 6.494$,
$\therefore GH = \sqrt{(24 - 12\sqrt{2})^2 - 6.494^2} = 2.69$ cm

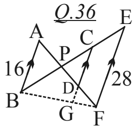
33. I. Size of each $\angle$ of pentagon = $\frac{(5-2)\times 180°}{5} = 108°$,
$\angle EAD = (180° - 108°) \div 2 = 36°$, $\angle FAB = 108° - 36° = 72°$,
but $\angle ABF = 108° \div 2 = 54°$,
$\therefore \angle AFB = 180° - 72° - 54° = 54°$,
$\therefore AB = AF$ and ΔABF is isosceles.
II. $\because \Delta ABF \cong \Delta CBF$ (SAS), $\therefore CF = AF = AB = CD$,
$\therefore \Delta CDF$ is isosceles.
III. $\because AB = AF = CB = CF$, $\therefore ABCD$ is a rhombus.

34. $CM = DM = 10 \div 2 = 5$,
$\therefore AN = BM = \sqrt{13^2 - 5^2} = 12$
but $CN = 10 + 5 = 15$,
$\therefore AC = \sqrt{12^2 + 15^2} = 19.2$ cm

Q.34



Q.36



36. $\because AB \,//\, CD \,//\, EF$ and $BC = CE$,
$\therefore AD = DF$ and $BG = GF$,
$\therefore CG = 28 \div 2 = 14$ and $DG = 16 \div 2 = 8$,
$\therefore CD = 14 - 8 = 6$

37. A. $\because AF = FC$ and $CE \,//\, FG$, $\therefore AG = GE$, $\therefore CE = 2FG$,
$\therefore CD = 2CE = 4FG$

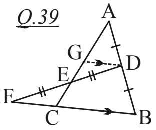
B. $\because \Delta DEH \sim \Delta FGH$ (AAA), $\therefore \dfrac{FH}{DH} = \dfrac{FG}{DE} = \dfrac{1}{2}$,
$\therefore DH = 2FH$,
$\therefore BD = 2DF = 2(DH + FH) = 2(2FH + FH) = 6FH$

C. $\dfrac{GH}{EH} = \dfrac{FG}{DE} = \dfrac{1}{2}$, $\therefore EH = 2GH$,
$\therefore AE = 2EG = 2(EH + GH) = 2(2GH + GH) = 6GH$

D. $\tan \angle AED = \dfrac{AD}{DE} = 2$, $\angle AED = 63.4°$,
$\therefore \angle DHE = 180° - 45° - 63.4° = 71.6°$,

$\therefore \Delta DEH$ is not isosceles.

Q.39



39. Draw $GD \,//\, FB$. $\because \Delta CEF \cong \Delta GED$ (ASA),
$\therefore CE = GE$. $\because AD = DB$ and $GD \,//\, CB$,
$\therefore AG = GC$. $\therefore AE : EC = 3 : 1$

40. Draw EG $\perp$ CD. $\because \Delta DEG \cong \Delta CEG$ (R.H.S.), $\therefore$ DG = CG.
$\because$ AD // EG // BC and DG = CG, $\therefore$ BE = EF (intercept thm.).

41. Let $\angle GAS = \angle DAS = a$ and $\angle FDS = \angle ADS = b$.
$\angle GAS + \angle DAS + \angle FDS + \angle ADS = 180°$,
$2a + 2b = 180°$, $a + b = 90°$,
$\therefore \angle DSA = 180° - (\angle DAS + \angle ADS) = 180° - (a + b) = 90°$,
$\therefore \angle PSR = \angle DSA = 90°$. Similarly, $\angle PQR = 90°$.
$\angle DEA = 180° - \angle EDA - \angle DAE = 180° - a - 2b$
$= 180° - (a + b) - b = 90° - b = a$,
but $\angle DCG = \dfrac{1}{2}\angle DCB = \dfrac{1}{2}\angle DAB = a$, $\therefore AE \,//\, GC$,
$\therefore \angle SRQ = \angle SPQ = 90°$.
$\because PS \neq SR$, $\therefore PQRS$ is a rectangle.

UNIT 8 CENTRES OF TRIANGLES

1. A	2. D	3. C	4. B	5. A	6. C	7. C	8. D
9. C	10. B	11. D	12. C	13. D	14. A	15. B	16. B
17. B	18. B	19. A	20. B	21. B	22. D	23. D	24. D
25. C	26. D	27. B	28. C	29. B	30. C	31. C	32. B

33. A 34. A 35. D 36. A

Explanatory Notes

4. Let N be a point on AC such that $BN \perp AC$.
$AD = DC$ (median of Δ)
$$\text{Area of } \Delta ABC = \frac{(AC)(BN)}{2} = \frac{(2AD)(BN)}{2}$$
$$= 2 \times \text{area of } \Delta ABD = 8 \text{ cm}^2$$

7. $BD = DC$ and $ED \perp BC$ (def. of ⊥ bisector)
$\therefore AE = EC$ (intercept thm)
Note that $\Delta CDE \sim \Delta CAB$ (AAA).
$$\therefore ED = \frac{AB}{2} = \frac{8}{2} \text{ cm} = 4 \text{ cm.}$$
$$\text{Area of } \Delta CDE = \frac{(ED)(CD)}{2} = 12 \text{ cm}^2,\ CD = 6 \text{ cm.}$$
$$AE = EC = \sqrt{ED^2 + CD^2}$$
$$= \sqrt{4^2 + 6^2} \text{ cm} = \sqrt{52} \text{ cm} = 7 \text{ cm (cor. to the nearest cm)}$$

11. I. $\because BC = CD$, $AB \perp BC$ and $AD \perp CD$ (given)
$\therefore AC$ is the angle bisector of $\angle BAD$. (converse of prop. of ∠ bisector)
II. $\because P$ is a point on the angle bisector of $\angle BAD$.
$\therefore BP = DP$ (prop. of ∠ bisector)
III. Note that $\Delta ABC \cong \Delta ADC$ (RHS/ AAS).
$\therefore AB = AD$ (corr. sides, ≅ Δs)
Then, $\Delta ABP \cong \Delta ADP$ (SSS).
$\therefore \angle ABP = \angle ADP$ (corr. ∠s, ≅ Δs)
$\therefore$ The answer is D.

12. $BE = AE$ and $BG = CG$ (prop. of ⊥ bisector)
$\therefore$ perimeter of $\Delta BEG = BE + EG + BG$
$= AE + EG + CG = AC = 40$ cm

13. I. $\because PR \perp QS$ and $PQ = PS$
$\therefore QT = ST$ (converse of property of ⊥ bisector)
II. $\because PR \perp QS$ and $QT = ST$
$\therefore QR = RS$ (property of ⊥ bisector)
III. $\because PQ = PS$, $QR = SR$ and $PR = PR$ (common side)
$\therefore \Delta PQR \cong \Delta PSR$ (SSS)
$\therefore \angle PRQ = \angle PRS$ (corr. ∠s, ≅ Δs)

15. I. $\because SR = PS$ (median of Δ)

$\therefore SR = QS$

$\therefore \angle RQS = \angle QRS$ (base $\angle$s, isos. Δ)

Also, $\angle QPS = \angle PQS$ (base $\angle$s, isos. Δ)

$\angle RQS + \angle QRS + \angle QPS + \angle PQS = 180°$ ($\angle$ sum of Δ)

$\angle RQS + \angle PQS = 90°$, i.e. $\angle PQR = 90°$

$\therefore \Delta PQR$ is a right-angled triangle, not obtuse

II. $\angle QSR = \angle PQS + \angle QPS$ (ext. $\angle$ of Δ) $= 2\angle QPS = 2\angle QPR$

III. $\because QS = SR$, $\therefore$ the perpendicular bisector of QR passes through S. (converse of prop. of $\perp$ bisector)

$\therefore$ The answer is B.

18. $OP = OQ = OR = 5$ (radii of circumcircle),

$\therefore OP^2 + OQ^2 + OR^2 = 5^2 + 5^2 + 5^2 = 75$

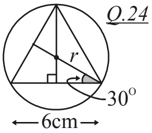
21. $x + y + z = 180°$ ($\angle$ sum of Δ)

$\angle IYZ = \dfrac{y}{2}$ and $\angle IZY = \dfrac{z}{2}$ (in-centre of Δ)

$\angle YIZ = 180° - \dfrac{y}{2} - \dfrac{z}{2} = (x + y + z) - \dfrac{y}{2} - \dfrac{z}{2} = x + \dfrac{y}{2} + \dfrac{z}{2}$

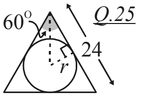
24. $\dfrac{3}{r} = \cos 30° = \dfrac{\sqrt{3}}{2}$, $r = \dfrac{6}{\sqrt{3}} = 2\sqrt{3}$,

$\therefore$ area $= \pi(2\sqrt{3})^2 = 12\pi \text{ cm}^2$



25. $\tan(\dfrac{60°}{2}) = \dfrac{r}{12}$,

$\therefore r = 12\tan 30° = 12 \times \dfrac{\sqrt{3}}{3} = 4\sqrt{3}$ cm



26. $\because \Delta DNF \sim \Delta EMF$ (AAA), $\therefore \dfrac{MF}{NF} = \dfrac{EF}{DF}$, $\dfrac{MF}{6} = \dfrac{12}{10}$,

$\therefore MF = \dfrac{6}{5} \times 6 = 7.2$ cm

27. $ME = \sqrt{12^2 - 7.2^2} = 9.6$. $\because \Delta EHN \sim \Delta EFM$ (AAA),

$\therefore \dfrac{HN}{FM} = \dfrac{EN}{EM}$, $\dfrac{HN}{7.2} = \dfrac{6}{9.6}$, $\therefore HN = \dfrac{6}{9.6} \times 7.2 = 4.5$ cm

28. PI, QI, RI are angle bisectors,

$\therefore \Delta API \cong \Delta CPI$, $\Delta QAI \cong \Delta QBI$, $\Delta RBI \cong \Delta RCI$ (AAS)

$\therefore AP = CP$, $AQ = BQ$ and $BR = CR$. (corr. sides, $\cong \Delta$s)

$\therefore$ Perimeter of $\Delta PQR = AP + CP + AQ + BQ + BR + CR$

$= 2(AP + AQ) + 2(BR) = 2(PQ) + 2(BR) = [2(7) + 2(10)]$ cm $= 34$ cm

29. Let $AI = r$ cm. $BI = CI = AI = r$ cm (in-centre of Δ)
area of ΔPQR = area of ΔPQI + area of ΔQRI + area of ΔPRI

$$= \frac{(PQ)(r\text{ cm})}{2} + \frac{(QR)(r\text{ cm})}{2} + \frac{(PR)(r\text{ cm})}{2}$$

$$= \frac{(r\text{ cm})(PQ + QR + PR)}{2} = \frac{(r\text{ cm})(34\text{ cm})}{2}$$

$\therefore \frac{34r}{2} = 68$, $r = 4$, $\therefore AI = 4$ cm

30. Produce AH to meet BC at N; produce BH to meet AC at K.
$\angle BKC = \angle ANB = 90°$ (orthocentre of Δ)
In ΔABK, $\angle KAH + a + b = 90°$ (ext. ∠ of Δ)
In ΔAHC, $\angle KAH + c = 90°$ (ext. ∠ of Δ)
$\therefore c = a + b$

31. $AP \perp BC$, $BQ \perp AC$, $CR \perp AB$ (orthocentre)
Let $\angle PAB = a$.
In ΔRCB, $\angle RCB = 180° - 90° - \angle CBR$ (∠ sum of Δ)
$= 180° - 90° - \angle PBR = \angle PAB = a$
Triangles with angles a and 90° included:
ΔABP, ΔAHR, ΔCBR, ΔCHP
∴ They are similar triangles.
ΔBCQ has angles: 90°, $a + \angle QCH$, $\angle CBQ$.
So ΔBCQ is similar to ΔABP only when $\angle CBQ = a$ which is not always true.
∴ The answer is C.

32. $CH = \sqrt{PH^2 + PC^2} = \sqrt{10^2 + 24^2}$ cm = 26 cm

$\because \Delta AHR \sim \Delta CHP$, $\therefore \frac{AR}{CP} = \frac{HR}{HP}$, $AR = \frac{25}{10} \times 24 = 60$ cm

Area of $\Delta AHC = \frac{(CH)(AR)}{2} = \frac{(26)(60)}{2} = 780\text{ cm}^2$

34. Area of $\Delta ABC = \frac{1}{2}(AB)(CP) = 30\text{ cm}^2$

$AP = \frac{1}{2}AB$ (median)

$PG = CP \times \frac{1}{2+1} = \frac{1}{3}CP$ (from hint)

area of $\Delta APG = \frac{1}{2}(AP)(PG) = \frac{1}{2}(\frac{1}{2}AB)(\frac{1}{3}CP)$

$$= \frac{1}{6}(\text{area of } \Delta ABC) = \frac{1}{6}(30)\text{ cm}^2 = 5\text{ cm}^2$$

35. I. $\because$ $\angle ADC = 90°$ (orthocentre of Δ) = $\angle CBF$ (given)
$\therefore$ $AD \,//\, FB$ (corr $\angle$s equal), i.e. $AH \,//\, FB$
Produce BH to meet AC at N.
$\therefore$ $\angle BNC = 90°$ (orthocentre of Δ) = $\angle CAF$ (given)
$\therefore$ $AF \,//\, BN$ (corr $\angle$s equal), i.e. $AF \,//\, BH$
$\therefore$ $AFBH$ is a parallelogram. (by definition)
II. $\because$ $\angle OEC = 90°$ (given) = $\angle CBF$,
$\therefore$ $OE \,//\, FB$ (corr $\angle$s equal).
$\because$ $CE = BE$ (circumcentre of Δ)
$\therefore$ $OC = OF$ (intercept thm)
$\therefore$ $AH = 2OE$ (mid-pt thm)
III. $\because$ $OA = OB = OC$ (circumcentre of Δ) = OF
$\therefore$ OF is a radius of the circumscribed circle of ΔABC.
$\therefore$ The answer is D.

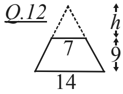
36. I. Produce CH to meet AB at N.
$CN \perp AB$ and $BE \perp AC$ (H is orthocentre)
In ΔACN: $\angle ACH = 180° - \angle A - 90°$
In ΔABD: $\angle ABE = 180° - \angle A - 90°$
$\angle ACH = \angle ABE$, $\therefore$ I is always true.
II. Note that $\Delta CDH \cong \Delta CDE$ (ASA).
$\therefore$ $DH = DE$ (corr. sides, $\cong$ Δs), $\therefore$ II is always true.
III. Note that $\Delta BDA \sim \Delta CDH$ (AAA).
$\therefore$ $\dfrac{DA}{DH} = \dfrac{DB}{DC}$ (corr. sides, $\sim\Delta$s)
$DA \times DC = DB \times DH$, $\therefore$ III is not always true.
$\therefore$ Answer is A

Unit 9 Areas and Volumes (3)

1. D	2. A	3. A	4. B	5. A	6. B	7. B	8. B
9. C	10. A	11. C	12. D	13. B	14. C	15. D	16. C
17. A	18. D	19. C	20. B	21. B	22. D	23. D	24. D
25. C	26. B	27. C	28. B	29. D	30. C	31. C	32. D
33. B	34. A	35. A	36. B	37. C	38. D	39. C	40. C
41. A	42. C	43. D	44. A	45. C	46. D	47. B	48. B
49. C	50. C	51. A	52. C	53. A	54. D	55. A	56. A
57. D	58. C	59. D	60. C	61. B	62. C	63. A	64. D
65. B	66. A	67. D	68. B	69. D	70. C	71. D	72. A
73. B							

Explanatory Notes

12. $\dfrac{h}{h+9}=\dfrac{7}{14}=\dfrac{1}{2}$, $2h = h + 9$, $h = 9$;

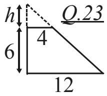


$\therefore$ Volume = $\dfrac{1}{3}(14)^2(9+9)-\dfrac{1}{3}(7)^2(9)=1029\text{ cm}^3$

22. $\pi(3)^2+\pi(3)(\ell)=33\pi$, $9+3\ell=33$, $\ell=8$;

$\sin\dfrac{\theta}{2}=\dfrac{3}{8}$, v $\dfrac{\theta}{2}=22.0°$, $\therefore \theta=44.0°$

23. $\dfrac{h}{h+6}=\dfrac{4}{12}=\dfrac{1}{3}$, $3h = h + 6$, $h = 3$;



$\therefore$ Volume = $\dfrac{1}{3}\pi(12)^2(3+6)-\dfrac{1}{3}\pi(4)^2(3)=416\pi\text{ cm}^3$

24. Curved surface area = $\pi(12)\sqrt{12^2+9^2}-\pi(4)\sqrt{4^2+3^2}$

$= 180\pi - 20\pi = 160\pi\text{ cm}^2$

28. Radius of largest sphere = $8\div 2=4$ cm;

$\therefore$ Volume = $\dfrac{4}{3}\pi(4)^3=268.1\text{ cm}^3$

29. $\dfrac{4}{3}\pi r^3\times 2=\dfrac{4}{3}\pi(10)^3$, $r^3=500$, $\therefore\ r=\sqrt[3]{500}=7.94$ cm

30. Percentage change = $\dfrac{4\pi(7.94)^2(2)-4\pi(10)^2}{4\pi(10)^2}\times 100\%=26.1\%$

34. Let h be height of cylinder.

$\pi(\dfrac{r}{2})^2h=\dfrac{4}{3}\pi r^3$, $(\dfrac{r^2}{4})(h)=\dfrac{4}{3}r^3$, $\therefore\ h=\dfrac{16}{3}r$

36. Let h cm be depth. $\pi(1)^2(h)+\dfrac{2}{3}\pi(1)^3=8\pi$, $h+\dfrac{2}{3}=8$, $\therefore\ h=\dfrac{22}{3}$

43. Let A cm^2 be curved surface area.

$\dfrac{y}{A}=\left[\dfrac{r}{r(1+200\%)}\right]^2=\dfrac{1}{9}$, $\therefore A=9y$

45. $V_A:(V_A+V_B):(V_A+V_B+V_C)=y^3:(y+2y)^3:(y+2y+y)^3$

$= 1 : 27 : 64$, $\therefore\ V_A:V_C=1:(64-27)=1:37$

46. $S_A:(S_A+S_B):(S_A+S_B+S_C)=y^2:(y+2y)^2(y+2y+y)^2$

$= 1 : 9 : 16$,

$\therefore\ S_B:S_C=(9-1):(16-9)=8:7$

47. $\dfrac{A_1}{A_2}=\left(\dfrac{1}{1-20\%}\right)^2=\dfrac{1}{0.64}$,

$\therefore$ percentage decrease = $(1-0.64)\times 100\%=36\%$

48. $\frac{r_1}{r_2} = \sqrt[3]{\frac{1}{1+72.8\%}} = \frac{1}{1.2}$,

$\therefore$ percentage change = $(1.2-1)\times 100\%$ = 20%

49. Suppose x cm^3 of water must be added.

$\frac{15}{x+15} = (\frac{1}{2})^3 = \frac{1}{8}$, $120 = x + 15$, $\therefore x = 105$

50. $\frac{A_1}{A_2} = (\frac{1}{2})^2 = \frac{1}{4}$, $\therefore$ percentage increase = $(4-1)\times 100\%$ = 300%

51. Original volume $V = \frac{1}{3}\pi r^2 h$,

new volume = $\frac{1}{3}[x(1-20\%)]^2[h(1+50\%)] = 0.96(\frac{1}{3}x^2 h) = 0.96V$,

$\therefore$ percentage change = $\frac{0.96V - V}{V}\times 100\% = -4\%$

52. Ratio = $\frac{1}{3}(\frac{ab}{2})(c) : [abc - \frac{1}{3}(\frac{ab}{2})(c)] = \frac{abc}{6} : \frac{5abc}{6} = 1 : 5$

53. $AB = FG = x$, $GH = y$, $BG = AF = z$;

Vol. of $AEFGH$: vol. of $ABCHG$ = $\frac{1}{3}(\frac{xy}{2})(z) : \frac{1}{3}(\frac{yz}{z})(x) = 1 : 1$

54. Original volume $V = \frac{1}{3}\pi r^2 h$,

new volume = $\frac{1}{3}\pi[r(1+40\%)]^2[h(1-25\%)] = 1.47(\frac{1}{3}\pi r^2 h) = 1.47V$,

$\therefore$ percentage change = $\frac{1.47V - V}{V}\times 100\% = 47\%$

55. Curved surface area = $\pi(\frac{r}{2})(2\ell) = \pi r\ell$ (unchanged)

56. Height = $12\cos 60° = 6$ cm, radius = $12\sin 60° \div 2 = 3\sqrt{3}$ cm,

$\therefore$ volume = $\frac{1}{3}\pi(3\sqrt{3})^2(6) = 54\pi$ cm^3

57. By cutting along the slant edge through P and flatting the cone to form a sector, the shortest distance is PP'.

$2\pi(15)\times\frac{\theta}{360°} = 2\pi(5)$, $\theta = 120°$;

$\therefore PP' = (15\sin\frac{120°}{2})\times 2 = 26$ cm

58. Increase in total surface area = $\pi r^2 \times 2 = 2\pi r^2$,

$\therefore$ percentage change = $\frac{2\pi r^2}{4\pi r^2}\times 100\% = 50\%$

59. I. $= \dfrac{4\pi r^2}{2\pi r(2r)} = \dfrac{4\pi r^2}{4\pi r^2} = 1$

II. $= \dfrac{4\pi r^2}{2\pi r(2r) + 2\pi r^2} = \dfrac{4\pi r^2}{6\pi r^2} = \dfrac{2}{3}$

III. $= \dfrac{\frac{4}{3}\pi r^3}{\pi r^2(2r)} = \dfrac{\frac{4}{3}\pi r^3}{2\pi r^3} = \dfrac{2}{3}$

$\therefore$ The answer is D.

60. $PQ : SR = 3 : 15 = 1 : 5$. Note that $\Delta RST \sim \Delta PQT$.

$\therefore$ Area of ΔPQT : Area of $\Delta SRT = PQ^2 : SR^2 = 1^2 : 5^2 = 1 : 25$

$ST : QT = RT : PT = SR : PQ = 5 : 1$ (corr. sides, ~Δs)

Area of ΔPQT : Area of $\Delta PST = QT : ST = 1 : 5$

$\therefore$ area of ΔSRT : area of $PQRS = 25 : (1 + 5 + 5 + 25) = 25 : 36$

61. $AE : EB : CD = 1 : 4 : (1 + 4) = 1 : 4 : 5$. Note that $\Delta EBF \sim \Delta CDF$.

Area of ΔEBF : Area of $\Delta CDF = BE^2 : CD^2 = 4^2 : 5^2 = 16 : 25$

$\therefore$ Area of $\Delta EBF = \dfrac{100}{25}(16)$ cm^2 = 64 cm^2

BF : FD = BE : CD (corr. sides, ~Δs)

= 4 : 5

$\therefore$ Area of ΔBFC : Area of ΔCDF = 4 : 5

Area of ΔDFC = $\dfrac{4}{5}(100)$ = 80 cm^2

Note that ΔDAB ≅ ΔBCD.

$\therefore$ Area of ADFE + 64 = 100 + 80

Area of ADFE = 180 – 64 = 116 cm^2

62. $\dfrac{V_1}{V_2} = \left(\sqrt{\dfrac{1}{1+125\%}}\right)^3 = (\dfrac{1}{1.5})^3 = \dfrac{1}{3.375}$,

$\therefore$ percentage change = $(3.375-1)\times 100\%$ = 237.5%

63. $\dfrac{r_A}{r_B} = \sqrt[3]{\dfrac{27}{125}} = \dfrac{3}{5}$, $\dfrac{r_B}{r_C} = \dfrac{1}{4\div 2} = \dfrac{1}{2}$, $\therefore$ $r_A : r_B : r_C$ = 3 : 5 :10,

$\therefore$ $(\dfrac{r_A}{r_B})^2 = (\dfrac{3}{10})^2 = \dfrac{9}{100}$. $\because$ C is a hemisphere, $\therefore$ $\dfrac{S_A}{S_C} = \dfrac{9}{50}$

64. Let V_W = volume of water, V_E = volume of empty part.

$\dfrac{V_E}{V_E + V_W} = (\dfrac{15-10}{15})^3 = (\dfrac{5}{15})^3 = \dfrac{1}{27}$, $\therefore V_E : V_W$ = 1 : (27 – 1) = 1 : 26.

Let d cm be the depth. $\dfrac{d}{15} = \sqrt[3]{\dfrac{26}{27}} = 0.987$, $\therefore$ d = 14.8

69. Let $ON = OM = r$. $\because$ $\Delta DNC \sim \Delta BMC$,

$\therefore \dfrac{ON}{BM} = \dfrac{OC}{BC}$, $\dfrac{r}{6} = \dfrac{8-r}{\sqrt{6^2+8^2}}$, $10r = 48 - 6r$, $16r = 48$, $\therefore r = 3$

71. Let d cm be the depth. $\dfrac{d}{8} = \sqrt[3]{\dfrac{3}{8}} = 0.721$, $\therefore d = 5.77$

72. $\dfrac{1}{3}\pi r^2 h = \dfrac{2}{3}\pi r^3(1-25\%)$, $h = 2r(0.75)$, $\dfrac{h}{r} = 1.5 = \dfrac{3}{2}$, $\therefore$ $h : r = 3 : 2$

73. $h = 30 \times \dfrac{3}{3+2} = 18$, $r = 30 - 18 = 12$,

$\therefore$ volume $= \dfrac{1}{3}\pi(12)^2(18) + \dfrac{2}{3}\pi(12)^3 = 2016\pi \text{ cm}^3$

UNIT 10 COORDINATE GEOMETRY

1. A	2. C	3. C	4. D	5. A	6. D	7. D	8. B
9. A	10. B	11. C	12. C	13. C	14. A	15. B	16. A
17. D	18. B	19. D	20. A	21. C	22. C	23. B	24. A
25. D	26. D	27. B	28. C	29. A	30. D	31. B	32. A
33. A	34. B	35. C	36. B	37. A	38. D	39. C	40. D
41. A	42. D	43. B	44. C	45. A	46. C	47. C	48. B
49. D	50. D	51. C	52. B	53. C	54. D	55. B	56. B
57. B	58. D	59. A	60. A	61. A	62. B	63. A	64. C
65. D	66. A						

Explanatory Notes

6. I. $AB = \sqrt{10}$, $BC = 2\sqrt{5}$, $AC = \sqrt{10}$, $\therefore$ ΔABC is isosceles.

II. $AB^2 + AC^2 = (\sqrt{10})^2 + (\sqrt{10})^2 = 20 = BC^2$,

$\therefore$ ΔABC is rt. $\angle$ed.

III. Area $= \dfrac{\sqrt{10} \times \sqrt{10}}{2} = 5$ sq. units

$\therefore$ The answer is D.

34. Let y-intercept $= a$. $\dfrac{a-0}{0-(-10)} \times 1.25 = -1$, $\dfrac{a}{10} \times \dfrac{5}{4} = -1$, $\therefore$ $a = -8$

36. $m_{PQ} = \frac{3-1}{3+1} = \frac{1}{2}$, $m_{QR} = \frac{3-1}{3-4} = -2$, $m_{RS} = \frac{1+1}{4-0} = \frac{1}{2}$,
$m_{PS} = \frac{1+1}{-1-0} = -2$.
$\because$ $m_{PQ} = m_{RS}$ and $m_{QR} = m_{PS}$, $\therefore$ PQ // RS and QR // PS;
$\because$ $m_{PQ} \times m_{QR} = m_{RS} \times m_{PS} = (\frac{1}{2})(-2) = -1$, $\therefore$ $PQ \perp QR$ and $RS \perp PS$;
But $PQ = \sqrt{(3+1)^2 + (3-1)^2} = 2\sqrt{5}$, $QR = \sqrt{(4-3)^2 + (1-3)^2} = \sqrt{5}$,
$\because$ $PQ \neq QR$, $\therefore$ $PQRS$ is a rectangle.

44. $3QR = PQ = PR + QR$, $2QR = PR$, $\therefore$ $PR : QR = 2 : 1$,
$\therefore$ $R = \left(\frac{1(-10)+2(2)}{1+2}, \frac{1-(-1)+2(5)}{1+2}\right) = (-2, 3)$

45. AP : PB = 1 : (4 – 1) = 1 : 3
Let $B = (x, y)$. $\frac{9(3)+x(1)}{1+3} = 3$, $27 + x = 12$, $x = -15$;
$\frac{-4(3)+y(1)}{1+3} = 2$, $-12 + y = 8$, $y = 20$. $\therefore$ $B = (-15, 20)$

46. $AC : BC = [(-3) - (-6)] : [4.5 - (-3)] = 3 : 7.5 = 2 : 5$

47. $\because$ x-coordinate of $P = 0$,
$\therefore$ $AP : PB = (6 - 0) : [0 - (-10)] = 6 : 10 = 3 : 5$

48. $PR : QR = [(k + 4) - k] : [k - (k - 1)] = 4 : 1$

49. $\sqrt{(-5+3)^2 + (k-7)^2} = 2\sqrt{5}$, $4 + (k-7)^2 = 20$, $(k-7)^2 = 16$,
$k - 7 = -4$ or 4, $\therefore$ $k = 3$ or 11

51. Suppose the line cuts the x-axis and the y-axis at $A(a, 0)$ and $B(0, b)$ respectively.
Slope of $AB = \frac{b-0}{0-a} = -\frac{b}{a} = \frac{3}{5}$
$OA = -a$, $OB = b$.
$\therefore$ $\tan\theta = \frac{OA}{OB} = \frac{-a}{b} = \frac{5}{3}$, $\theta = 59°$

53. Let $B = (0, y)$. $\because$ $L_1 \perp L_2$,
$\therefore$ $\frac{y-1}{0-8} \times \frac{5-1}{0-8} = -1$, $\frac{y-1}{-8} \times \frac{1}{-2} = -1$, $y - 1 = -16$, $y = -15$.
$\therefore$ Area = $\frac{1}{2}(5+15)(8)$ = 80 sq. units

54. Let y-intercept of $L_1 = k$, then x-intercept of $L_1 = 2k$.
$\because L_1 \perp L_2$, $\therefore$ $\frac{k-0}{0-2k} \times \frac{b-0}{a-0} = -1$,
$\frac{k}{-2k} \times \frac{b}{a} = -1$, $\frac{b}{a} = 2$, $\therefore$ $b = 2a$

55. Mid-point (M) of $PR = \left(\frac{3+1}{2}, \frac{6-4}{2}\right) = (2, 1)$. Let $S = (x, y)$.

$\because$ M is also the mid-pt. of QS (prop. of // gram),

$\therefore \frac{x-2}{2} = 2$, $x = 6$; $\frac{y+2}{2} = 1$, $y = 0$. $\therefore S = (6, 0)$

56. Let $A = (x, 0)$, $B = (0, y)$.

$\frac{x(2)+0(1)}{1+2} = 3$, $2x = 9$, $x = 4.5$; $\frac{y(1)+0(2)}{1+2} = 5$, $y = 15$.

$\therefore A = (4.5, 0)$, $B = (0, 15)$

57. Let D = $(x, 0)$. Mid-point of AB = $\left(\frac{-6+0}{2}, \frac{0+12}{2}\right) = (-3, 6)$.

$\because$ AB$\perp$CD, $\therefore \frac{12-0}{0+6} \times \frac{0-6}{x+3} = -1$, $\frac{-12}{x+3} = -1$, $x + 3 = 12$, $x = 9$.

$\therefore$ D = (9, 0)

58. Let B = $(x, 0)$. $\because$ A, B, D are collinear,

$\therefore \frac{0-6}{x-16} = \frac{6+9}{16+4} = \frac{3}{4}$, $-24 = 3x - 48$, $x = 8$;

$\because$ A, C, D are collinear,

$\therefore \frac{y-6}{0-16} = \frac{3}{4}$, $4y - 24 = -48$, $y = -6$.

$\therefore$ Area = $\frac{1}{2}(8)(6) = 24$ sq. units

60. $\because$ $AM = MB$ and $AN = NC$, $\therefore MN = \frac{1}{2}BC$ (mid-pt. thm.),

$\therefore MN = \frac{1}{2}\sqrt{(-6-10)^2 + (7+5)^2} = \frac{1}{2}(20) = 10$

61. $\because$ ΔAOC and ΔBOC have the same height,

$\therefore$ $AC : CB$ = area of ΔAOC : area of ΔBOC = 2 : 3,

$\therefore C = \left(\frac{3(-8)+2(0)}{2+3}, \frac{3(0)+2(-5)}{2+3}\right) = (-4.8, -2)$

62. $M = \left(\frac{3+20}{2}, \frac{0+2}{2}\right) = (11.5, 1)$,

$\therefore G = \left(\frac{1(10)+2(11.5)}{1+2}, \frac{1(10)+2(1)}{1+2}\right) = (11, 4)$

63. Circumcentre = $\left(\frac{18+0}{2}, \frac{0+24}{2}\right) = (9, 12)$

64. Radius = $\frac{\sqrt{(0-18)^2 + (24+0)^2}}{2} = 15$,

$\therefore$ area = $\pi(15)^2 = 225\pi$ sq. units

65. $P = \left(\frac{-2+8}{2}, \frac{3+5}{2}\right) = (3, 4)$, $Q = \left(\frac{-2+0}{2}, \frac{3-3}{2}\right) = (-1, 0)$.

Let $C = (x, y)$. $\because PC \perp XY$, $\therefore \frac{y-4}{x-3} \times \frac{5-3}{8+2} = -1$, $\frac{y-4}{x-3} = -5$,

$y - 4 = -5x + 15$, $5x + y = 19$ ……(1);

$\because CQ \perp XZ$, $\therefore \frac{y-0}{x+1} \times \frac{3+3}{-2-0} = -1$,

$\frac{y}{x+1} = \frac{1}{3}$, $3y = x + 1$, $x - 3y = -1$ ……(2);

Solving (1) and (2), we have $x = 3.5$, $y = 1.5$.

$\therefore C = (3.5, 1.5)$

66. Let $H = (x, y)$. $\because PH \perp RQ$, $\therefore \frac{y-5}{x-5} \times \frac{0+1}{-3-5} = -1$,

$\frac{y-5}{x-2} = 8$, $y - 5 = 8x - 16$, $8x - y = 11$ ……(1);

$\because QH \perp PR$, $\therefore \frac{y+1}{x-5} \times \frac{5-0}{2+3} = -1$,

$\frac{y+1}{x-5} = -1$, $y + 1 = -x + 5$, $x + y = 4$ ……(2);

Solving (1) and (2), we have $x = \frac{5}{3}, y = \frac{7}{3}$.

$\therefore H = \left(\frac{5}{3}, \frac{7}{3}\right)$

UNIT 11 TRIGONOMETRIC RELATIONS

1. A	2. D	3. D	4. C	5. B	6. A	7. C	8. A
9. D	10. D	11. B	12. C	13. B	14. A	15. B	16. C
17. A	18. C	19. A	20. A	21. C	22. D	23. C	24. B
25. D	26. A	27. D	28. B	29. A	30. C	31. B	32. A
33. B	34. B	35. D	36. A	37. C	38. C	39. D	40. C
41. D	42. D	43. B	44. A	45. B	46. B	47. B	48. C
49. A	50. D	51. A	52. C	53. C	54. D	55. A	56. C
57. A	58. B	59. B	60. A	61. D	62. C	63. C	64. D
65. B	66. A	67. C	68. A				

Explanatory Notes

17. $AB = 4 \div \tan 45° = 4$, $BD = 4 \div \sin 45 = 4\sqrt{2}$,
$CD = 4\sqrt{2}\sin 30° = 2\sqrt{2}$, $BC = 4\sqrt{2}\cos 30° = 2\sqrt{6}$,
$\therefore$ area $= \dfrac{4\times 4}{2} + \dfrac{2\sqrt{2}\times 2\sqrt{6}}{2} = (4\sqrt{3}+8)\ \text{cm}^2$

22. $\because AS : AP : PS = 1 : \sqrt{3} : 2$, $\therefore AB : PS = (1+\sqrt{3}) : 2$,
$\therefore$ area of $ABCD$: area of $PQRS$ $= (1+\sqrt{3})^2 : 2^2 = (4+2\sqrt{3}) : 4$
$= (2+\sqrt{3}) : 2$

23. $\because$ X, Y and Z are similar, $\therefore X : Y : Z = 1^2 : (\sqrt{3})^2 : 2^2 = 1 : 3 : 4$

24. $\sqrt{12} - \sqrt{6}\cos(x+5°) = \sqrt{3}$, $2\sqrt{3} - \sqrt{3} = \sqrt{6}\cos(x+5°)$,
$\cos(x+5°) = \dfrac{\sqrt{3}}{\sqrt{6}} = \dfrac{1}{\sqrt{2}}$, $x + 5° = 45$, $\therefore x = 40°$

25. $1 + \tan x = \dfrac{2}{\sqrt{3}-1} = \dfrac{2(\sqrt{3}+1)}{3-1} = \sqrt{3}+1$, $\tan x = \sqrt{3}$, $\therefore x = 60°$

30. $= \dfrac{(1-\cos x)-(1+\cos x)}{1^2 - \cos^2 x} = -\dfrac{2\cos x}{\sin^2 x}$

31. $= \dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

32. $= \dfrac{\sin^4 x + \sin^2 x\cos^2 x}{\cos^2 x} = \dfrac{\sin^2 x(\sin^2 x + \cos^2 x)}{\cos^2 x} = \tan^2 x$

33. $= (\dfrac{\sin^2\theta - 1}{\sin\theta})(\dfrac{\cos^2\theta - 1}{\cos\theta}) = (\dfrac{-\cos^2\theta}{\sin\theta})(\dfrac{-\sin^2\theta}{\cos\theta}) = \sin\theta\cos\theta$

35. $5\sin^2\theta + 4\cos^2\theta = 5$, $5\sin^2\theta + 4(1-\sin^2\theta) = 5$, $\sin^2\theta = 1$,
$\therefore \sin\theta = 1$

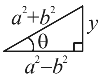
38. $= \tan\theta + \dfrac{1}{\tan\theta} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$

39. $= \dfrac{\sin^2 x}{\cos^2 x}\cdot\cos^2 x + \sin^2 x = 2\sin^2 x$

40. $= \cos\theta + \sin\theta\cdot\dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \dfrac{1}{\cos\theta}$

43. $\because y^2 = (a^2+b^2)^2 - (a^2-b^2)^2$
$= (a^2+b^2+a^2-b^2)(a^2+b^2-a^2+b^2)$
$= 4a^2b^2$, $\therefore y = 2ab$, $\therefore \tan\theta = \dfrac{2ab}{a^2-b^2}$

Q.43



44. $\begin{aligned} 10x &= 4.444\ldots\ldots \\ -\quad x &= 0.444\ldots\ldots \\ \hline 9x &= 4 \end{aligned}$ $\qquad \therefore x = \tan\theta = \dfrac{4}{9}$,

$\therefore \sin\theta - \cos\theta = \dfrac{4}{\sqrt{4^2+9^2}} - \dfrac{9}{\sqrt{4^2+9^2}} = \dfrac{-5}{\sqrt{97}} = \dfrac{-5\sqrt{97}}{97}$

45. $\sqrt{3}\sin 2\theta = \dfrac{3}{2}$, $\sin 2\theta = \dfrac{\sqrt{3}}{2}$, $2\theta = 60°$, $\therefore \theta = 30°$

46. $\cos\theta - \sqrt{3}\sin\theta = 0$, $\cos\theta = \sqrt{3}\sin\theta$, $\tan\theta = \dfrac{1}{\sqrt{3}}$, $\therefore \theta = 30°$

48. $2\sin(x+y) = \sqrt{3}$, $\sin(x+y) = \dfrac{\sqrt{3}}{2}$, $x+y = 60°$ …… (1);

$3\tan(x-y) = \sqrt{3}$, $\tan(x-y) = \dfrac{\sqrt{3}}{3}$, $x-y = 30°$ …… (2);

Solving (1) and (2), we have $x = 45°$, $y = 15°$.

49. $x\tan 60° - \sin 30° \le x\tan 45° + \cos 30°$, $x(\sqrt{3}) - \dfrac{1}{2} \le x + \dfrac{\sqrt{3}}{2}$,

$x(\sqrt{3}-1) \le \dfrac{\sqrt{3}+1}{2}$, $x \le \dfrac{\sqrt{3}+1}{2(\sqrt{3}-1)}$, $x \le \dfrac{(\sqrt{3}+1)^2}{2(3-1)}$, $x \le \dfrac{4+2\sqrt{3}}{4}$,

$\therefore x \le \dfrac{2+\sqrt{3}}{2}$

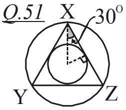
50. $\because AB = PR =$ diameter of circle,

$\therefore AB : PQ = PR : PQ = \sqrt{2} : 1$

51. $\because$ Radius of C_1 : radius of C_2 = 2 : 1,

$\therefore$ area of C_1 : area of C_2 = $2^2 : 1^2$ = 4 : 1

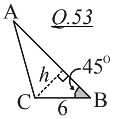
Q.51



53. $h = 6\sin 45° = 6(\dfrac{1}{\sqrt{2}}) = 3\sqrt{2}$;

$\dfrac{AB \times 3\sqrt{2}}{2} = 27$, $\therefore AB = \dfrac{54}{3\sqrt{2}} = 9\sqrt{2}$

Q.53



54. Let $AB = BC = a$.

$CD = \dfrac{2a}{\tan 60°} = \dfrac{2a}{\sqrt{3}} = \dfrac{2\sqrt{3}a}{3}$, $CE = \dfrac{a}{\tan 30°} = \sqrt{3}a$,

$\therefore CD : DE = \dfrac{2\sqrt{3}a}{3} : (\sqrt{3}a - \dfrac{2\sqrt{3}a}{3}) = \dfrac{2\sqrt{3}a}{3} : \dfrac{\sqrt{3}a}{3} = 2 : 1$

55. Let $AD = DC = a$.

$BC = \dfrac{2a}{\cos 30°} = 2a \times \dfrac{2}{\sqrt{3}} = \dfrac{4\sqrt{3}a}{3}$, $EC = a\cos 30° = \dfrac{\sqrt{3}a}{2}$,

$\therefore BE : EC = (\dfrac{4\sqrt{3}a}{3} - \dfrac{\sqrt{3}a}{2}) : \dfrac{\sqrt{3}a}{2} = \dfrac{5\sqrt{3}a}{6} : \dfrac{\sqrt{3}a}{2} = 5 : 3$

56. Let $AD = BD = a$.
$\angle ABD = 30°$ (base $\angle$s, isos. Δ), $\angle BDC = 60°$ (ext. $\angle$ of Δ),
$\therefore\ CD = BD\cos 60° = \frac{a}{2}$, $\therefore\ CD : AD = \frac{a}{2} : a = 1 : 2$

57. $= (\sin^2 x + \cos^2 x)^2 = 1$

58. $= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = (1)(1 - \sin^2 x - \sin^2 x)$
$= 1 - 2\sin^2 x$

59. $= \sin^2\theta + \cos^2\theta\,(\sin^2\theta + \cos^2\theta) = \sin^2\theta + \cos^2\theta(1) = 1$

60. $= \dfrac{\cos\theta\,(1+\cos\theta)}{1+\cos\theta} + \dfrac{\sin^2\theta}{1+\cos\theta} = \dfrac{\cos\theta + \cos^2\theta + \sin^2\theta}{1+\cos\theta}$
$= \dfrac{\cos\theta + 1}{1+\cos\theta} = 1$

61. $= \cos^2\theta\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right) = \cos^2\theta\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right) = \sin\theta\cos\theta$

62. $= \dfrac{\cos\theta}{1-\sin\theta} + \dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta + (1-\sin\theta)^2}{\cos\theta(1-\sin\theta)}$
$= \dfrac{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta}{\cos\theta\,(1-\sin\theta)} = \dfrac{2-2\sin\theta}{\cos\theta\,(1-\sin\theta)} = \dfrac{2}{\cos\theta}$

65. $\sin\theta + \cos\theta = \frac{3}{2}$, $(\sin\theta + \cos\theta)^2 = (\frac{3}{2})^2$,
$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = \frac{9}{4}$, $2\sin\theta\cos\theta = \frac{9}{4} - 1$,
$\therefore\ \sin\theta\cos\theta = \frac{5}{4} \times \frac{1}{2} = \frac{5}{8}$

66. $= \sin^2 1° + \sin^2 2° + \ldots\ldots + \sin^2 44° + \sin^2 45° + \cos^2 44° + \ldots\ldots$
$+ \cos^2 2° + \cos^2 1°$
$= (\sin^2 1° + \cos^2 1°) + \ldots\ldots + (\sin^2 44° + \cos^2 44°) + \sin^2 45°$
$= 44 \times 1 + (\frac{1}{\sqrt{2}})^2 = 44.5$

67. $= \tan 2° \times \tan 4° \times \ldots \times \tan 44° \times \dfrac{1}{\tan 44°} \times \ldots\ldots \times \dfrac{1}{\tan 4°} \times \dfrac{1}{\tan 2°} = 1$

68. $\tan\theta\tan(\theta + 20°) = 1$, $\dfrac{1}{\tan(90° - \theta)} \times \tan(\theta + 20°) = 1$,
$\tan(\theta + 20°) = \tan(90° - \theta)$, $\theta + 20° = 90° - \theta$, $2\theta = 70°$,
$\therefore\ \theta = 35°$

UNIT 12 APPLICATION OF TRIGONOMETRY

1. B	2. B	3. D	4. C	5. A	6. C	7. B	8. B
9. B	10. D	11. D	12. C	13. C	14. A	15. C	16. A
17. B	18. A	19. D	20. A	21. B	22. B	23. A	24. A
25. D	26. C	27. D	28.C	29. D	30. A	31. B	32. C
33. B	34. D	35. D	36. C	37. A	38. B	39. A	40. C
41. D	42. A	43. B	44. A	45. B	46. C	47. B	48. C
49. D	50. C	51. B	52. B	53. A	54. A	55. C	56. D
57. D	58. C	59. B	60. B	61. D	62. C	63. B	64. B
65. D	66. C	67. B	68. D	69. A	70. A	71. D	72. A
73. C	74. A	75. C	76. B				

Explanatory Notes

14. Let θ be the inclination of second slope.

$\because \tan\theta = \frac{1}{3}$, $\therefore \sin\theta = \frac{1}{\sqrt{1^2+3^2}} = \frac{1}{\sqrt{10}}$, $\cos\theta = \frac{3}{\sqrt{10}}$.

Total vertical distance = $220\sin 14° + 160 \times \frac{1}{\sqrt{10}}$ = 103.82,

total horizontal distance = $220\cos 14° + 160 \times \frac{3}{\sqrt{10}}$ = 365.25,

$\therefore$ angle of depression = $\tan^{-1}(\frac{103.82}{365.25})$ = 15.9°

16. Angle of elevation = $\tan^{-1}(\frac{4}{12})$ = 18.4°

20. Let x m be the height of the flagstaff.

$\frac{x}{\tan 46°} + \frac{x}{\tan 25°} = 15$, $x(\frac{1}{\tan 46°} + \frac{1}{\tan 25°}) = 15$,

$\therefore x = 15 \div (\frac{1}{\tan 46°} + \frac{1}{\tan 25°}) = 4.82$

21. $\frac{OR}{\tan 20°} - \frac{OR}{\tan 65°} = 10 \times 15$, $OR(\frac{1}{\tan 20°} - \frac{1}{\tan 65°}) = 150$,

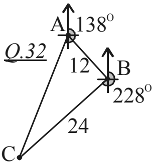
$\therefore OR = 150 \div (\frac{1}{\tan 20°} - \frac{1}{\tan 65°})$ = 65.8 m

22. Let h m be the height.

$\frac{h}{\tan 72°} = \frac{h-55}{\tan 39°}$, $h\tan 39° = h\tan 72° - 55\tan 72°$,

$h(\tan 72° - \tan 39°) = 55\tan 72°$, $\therefore h = 74.6$

23. $h + \frac{h}{\tan 24°} \times \tan 35° = 120$, $h(1 + \frac{\tan 35°}{\tan 24°}) = 120$, $\therefore h = 46.6$



32. $\angle ABC = 360° - 228° - (180° - 138°) = 90°$,

$\therefore\ AC = \sqrt{12^2 + 24^2} = \sqrt{720} = 12\sqrt{5}$ km

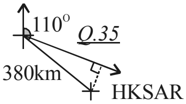
34. $\angle PAB = 180° - 156° = 24°$,

$\angle PBA = 270° - 225° = 45°$.

Let x m be the shortest distance.

$\dfrac{x}{\tan 24°} + \dfrac{x}{\tan 45°} = 460$, $x(\dfrac{1}{\tan 24°} + 1) = 460$,

$\therefore\ x = 460 \div (\dfrac{1}{\tan 24°} + 1) = 142$



35. Shortest distance = 380sin(180° – 110° – 45°)

= 380sin25° = 160.6 km

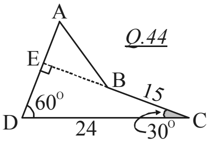
36. Time taken = $380\cos 25° \div 100$ = 3.4 h

43. The pentagon is formed by five identical isosceles triangles.

Each base angle = $(5-2)\times 180° \div 5 \div 2$ = 54°,

base = $15\cos 54° \times 2$ = 30cos54°, height = 15sin54°,

$\therefore$ area = $\pi(15)^2 - \dfrac{30\cos 54° \times 15\sin 54°}{2} \times 5$ = 172 cm^2



44. DE = 24cos60° = 12, CE = 24sin60° = $12\sqrt{3}$,

AE = 16 – 12 = 4, BE = $12\sqrt{3} - 15$,

$\therefore$ area = $\dfrac{12 \times 12\sqrt{3}}{2} + \dfrac{4(12\sqrt{3} - 15)}{2}$

= 136.3 cm^2

45. $\sin\theta = \dfrac{2\sin 45°}{7} = \dfrac{\sqrt{2}}{7}$, $\therefore\ \theta = 11.7°$

46. Height = 4sin(180° – 90° – 11.7°) = 4sin78.3° = 3.9 cm

47. Height = 3.9 + 7sin11.7° = 5.3 cm

48. Let $AB = AD = DE = BE = x$ cm. $\dfrac{x}{\tan 40°} + x + \dfrac{x}{\tan 60°} = 9$,

$x(\dfrac{1}{\tan 40°} + 1 + \dfrac{1}{\tan 60°}) = 9$, $x = 9 \div (\dfrac{1}{\tan 40°} + 1 + \dfrac{1}{\tan 60°}) = 3.25$.

$\therefore$ Area = $\dfrac{(3.25+9)(3.25)}{2}$ = 19.9 cm^2

49. Let r cm be the radius. $\dfrac{r}{\sin 30°} + r = 18$, $2r + r$ =18, $\therefore\ r = 6$

52. Let a be vertical distance between A and B.

Slope of $AB = \dfrac{a}{4}$, slope of $CD = \dfrac{2a}{5}$, slope of $EF = \dfrac{3a}{8}$,

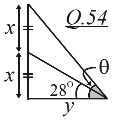
$\because\ \dfrac{2a}{5} > \dfrac{3a}{8} > \dfrac{a}{4}$, $\therefore$ CD has the greatest gradient.

53. Let a be vertical distance between A and B.

$\tan 10° = \frac{a}{4}$, $\therefore$ $a = 4 \tan 10°$.

Let θ be inclination of PQ. $\tan\theta = \frac{2a}{6} = \frac{4\tan 10°}{3}$, $\therefore$ $\theta = 13.2°$

54. $\tan 28° = \frac{x}{y}$. Let θ be the angle of depression.



$\tan\theta = \frac{2x}{y} = 2\tan 28°$, $\therefore$ $\theta = 46.8°$

59. $\tan \angle OPQ = \frac{30}{60}$, $\angle OPQ = 26.57°$;

$\sin \angle OPG = \frac{20}{\sqrt{30^2 + 60^2}}$, $\angle OPG = 17.35°$;

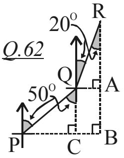
$\therefore$ Angle of elevation = 26.57° + 17.35° = 43.9°

60. $\because$ $\angle SBC = 20° + 40° = 60°$ and $\frac{BC}{SB} = \frac{100}{50} = 2$, $\therefore$ $\angle CSB = 90°$,

$\therefore$ $SC = 100\sin 60° = 50\sqrt{3}$ m

61. $\angle SCB = 180° - 90° - 60° = 30°$,

$\therefore$ the bearing is S(30° + 40°)W or S70°W.



62. $RB = RA + QC = 8\cos 20° + 12\cos 50° = 15.23$,

$PB = PC + QA = 12\sin 50° + 8\sin 20° = 11.93$,

$\therefore$ $PR = \sqrt{15.23^2 + 11.93^2} = 19.3$ km

64. $AQ = RC - PA = 190\sin 70° - 140\cos 60°$

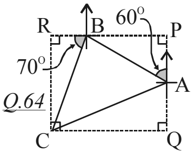
$= 108.54$,

$CQ = RB + BP = 190\cos 70° + 140\sin 60°$

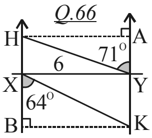
$= 186.23$,

$\therefore$ distance = $AC = \sqrt{108.54^2 + 186.23^2}$

$= 216$ m



66. $AK = AY + BX = \frac{6}{\tan 71°} + \frac{6}{\tan 64°} = 4.99$;



$\tan \angle AKH = \frac{6}{4.99}$, $\angle AKH = 50.2°$,

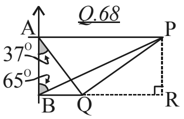
$\therefore$ bearing of H from K is N50.2°W.

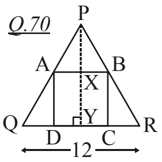
68. $QR = AP - BQ = 80\tan 65° - 80\tan 37°$

$= 111.28$;

$\tan \angle QPR = \frac{111.28}{80}$, $\angle QPR = 54°$,

$\therefore$ bearing of Q from P is 180° + 54° or 234°.





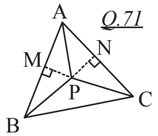
70. $PY = \frac{12}{2} \times \tan 60° = 6\sqrt{3}$.

Let a cm be the side of square $ABCD$.

$\because \Delta PAB \sim \Delta PQR$, $\therefore \frac{AB}{QR} = \frac{PX}{PY}$,

$\frac{a}{12} = \frac{6\sqrt{3} - a}{6\sqrt{3}}$, $6\sqrt{3}a = 72\sqrt{3} - 12a$,

$(6\sqrt{3} + 12)a = 72\sqrt{3}$, $\therefore a = 5.57$



71. $\angle CAP = \angle BAP = 46° \div 2 = 23°$,

$\angle CBP = \angle ABP = 62° \div 2 = 31°$,

$\angle ACP = \angle BCP = (180° - 62° - 46°) \div 2 = 36°$.

$PM = PN = 4\sin 23°$,

$\therefore BP = \frac{PM}{\sin 31°} = \frac{4\sin 23°}{\sin 31°} = 3.03$ cm,

$CP = \frac{PN}{\sin 36°} = \frac{4\sin 23°}{\sin 36°} = 2.66$ cm

75. $\because \Delta BMX \cong \Delta AMX$, $\therefore \angle MAX = \angle MBX = 50° \div 2 = 25°$,

but $\angle BAC = (180 - 50°) \div 2 = 65°$, $\therefore \angle MAN = 65° - 25° = 40°$,

$\therefore MN = AN \tan 40° = \frac{16}{2} \times \tan 40° = 6.71$ cm

76. $AM = \frac{AN}{\cos 40°} = \frac{8}{\cos 40°} = 10.44$,

$\therefore$ area $= \pi(10.44)^2 = 342.6\ \text{cm}^2$

UNIT 13 MEASURES OF CENTRAL TENDENCY

1. C	2. B	3. B	4. A	5. D	6. D	7. C	8. C
9. D	10. A	11. B	12. D	13. C	14. D	15. B	16. D
17. A	18. A	19. D	20. C	21.C	22. D	23. B	24. B
25. D	26. D	27. D	28. B	29. A	30. D	31. D	32. D
33. A	34. B	35. A	36. C	37. D	38. B	39. D	40. D
41. D	42. B	43. C	44. B	45. A	46. B	47. B	48. C
49. C	50. D	51. A	52. B	53. B	54. A	55. D	56. A
57. C	58. A	59. D	60. B	61. C	62. C		

Explanatory Notes

6. $8 \times 15 + 12n = 9.5(15 + n)$, $120 + 12n = 142.5 + 9.5n$, $\therefore n = 9$

7. $a+b+c+d=18\times 4=72$,

$\therefore$ mean $=\dfrac{(2a+1)+(b-4)+(c-5)+(9-a)+(d+7)}{5}$

$=\dfrac{(a+b+c+d+e)+8}{5}=\dfrac{72+8}{5}=16$

9. Present mean age $=\dfrac{(18+6)\times 16-27}{15}=23.8$

10. Let x = no. of men, y = no. of woman. $178x+158y=165.5(x+y)$,

$12.5x=7.5y$, $\dfrac{y}{x}=\dfrac{12.5}{7.5}=\dfrac{5}{3}$, $\therefore x:y=5:3$

13. Rearrange the data: $\dfrac{3k}{5}$, $\dfrac{2k}{3}$, $\dfrac{5k}{7}$, $\dfrac{3k}{4}$, $\dfrac{5k}{6}$;

$\therefore \dfrac{5k}{7}=15$, $k=21$

14. $\because$ The magnitude and sign of x are not known,

$\therefore$ the median cannot be determined.

15. x can be 5, 6, 7, 8, but different integers, $\therefore x=6$ or 7.

17. $\because \dfrac{8+10}{2}=9$, $\therefore a$ should be arranged after 10,

$\therefore a\geq 10$, that means, $a>9$

19. $\because$ 6 and 7 are smaller than 8 which is the median,

$\therefore$ there are two cases:

(1) $p-7$ and $p-2$ are the middle 2 numbers, then

$\dfrac{(p-7)+(p-2)}{2}=8$, $2p-9$, $p=12.5$

(2) 7 and $p-2$ are the middle 2 numbers, then

$\dfrac{7+(p-2)}{2}=8$, $p+5=16$, $p=11$

$\because p$ is an integer, $\therefore p=11$

26. For example, original set of numbers can be –1, –1, 1, 1, x, x, x.

When squared, the set becomes 1, 1, 1, 1, x^2, x^2, x^2.

$\because$ The mode is changed, $\therefore$ the new mode cannot be determined.

32. If the mean, mode and median are negative, they will become larger when multiplied by –3.

36. Mean $=\dfrac{3^{4x+1}+9^{2x+1}+81^{x+1}}{3}=\dfrac{3^{4x+1}+3^{4x+2}+3^{4x+4}}{3}$

$=\dfrac{3^{4x+1}(1+3+3^3)}{3}=3^{4x}\cdot 31$

41. $\because$ Mode $=15$, $\therefore a=15$. $13+15+15+b+19+22=17\times 6$,

$\therefore b=102-84=18$

42. $\because$ Median = 10, $\therefore c = 10$. $\because$ Mode = 8, $\therefore a = b = 8$.
$8+8+10+d+e=10\times5$, $d+e=24$,
but d and e should be different integers which are greater than 10,
$\therefore d = 11, e = 13$

43. $a = 18\times4\times\dfrac{2}{2+5+2+3} = 72\times\dfrac{1}{6} = 12$

44. Let $a = 2k$, $b = 5k$, $c = 2k$, $d = 3k$.
$\dfrac{2k+3k}{2} = 35$, $5k = 70$, $k = 14$, $\therefore d = 3(14) = 42$

45. $2+x+y+17=9\times4$, $x+y=17$ ……(1);
$2\times5+3x+6y+17\times6=9.8(5+3+6+6)$, $3x+6y=84$ ……(2);
Solving (1) and (2), we have $x = 6$, $y = 11$

46. $6\times18+7\times24+8k+9\times20+10\times13=7.86(18+24+k+20+13)$,
$8k+586=7.86(k+75)$, $8k-7.86k=589.5-586$, $\therefore k = 25$

47. Least possible value of $k = (18+24-20-13)+1 = 10$

59. Sets I and III are evenly distributed, while "20" is an extreme datum in set II.

62. $\because x+y=2a$, $y+z=2b$, $x+z=2c$,
$\therefore (x+y)+(y+z)+(x+z)=2a+2b+2c$,
$2(x+y+z)=2(a+b+c)$, $x+y+z=a+b+c$,
$\therefore$ mean $= \dfrac{x+y+z}{3} = \dfrac{a+b+c}{3}$

UNIT 14 INTRODUCTION TO PROBABILITY

1. D	2. B	3. A	4. C	5. C	6. B	7. C	8. B
9. D	10. C	11. D	12. C	13. A	14. A	15. C	16. C
17. C	18. A	19. B	20. D	21. C	22. C	23. B	24. C
25. B	26. C	27. D	28. D	29. B	30. B	31. C	32. A
33. A	34. B	35. C	36. C	37. A	38. C	39. C	40. D
41. A	42. A	43. D	44. B	45. A	46. D	47. A	48. B

Explanatory Notes

6. 2, 3 and 5 are prime numbers.

16. Let x = total no. of balls. $\dfrac{x-20}{x} = \dfrac{4}{9}$, $9x-180=4x$, $\therefore x = 36$

38. no. of prime = 12
no. of composite = 6
"1" is neither prime nor composite.
$\therefore$ Expected no. of tokens

$$= 5\times\frac{12}{20}+6\times\frac{6}{20}+0\times\frac{2}{20} = 4.8$$

	0	1	2	3	4
0	/	1	P	P	**C**
1	1	/	P	**C**	P
2	P	P	/	P	**C**
3	P	**C**	P	/	P
4	**C**	P	**C**	P	/

39. Expected value $= 1\times\frac{1}{6}+2\times\frac{1}{6}+3\times\frac{1}{6}+4\times\frac{1}{6}+5\times\frac{1}{6}+6\times\frac{1}{6} = 3.5$

45. Total no. of balls $= 18\div(1-\frac{1}{10}-\frac{3}{5}) = 18\div\frac{3}{10} = 60$,

$\therefore$ difference $= 60\times(\frac{3}{10}-\frac{1}{10}) = 12$

46. Total no. of coins $= 12\div(1-\frac{3}{4}) = 48$