# Answers

## Unit 1: Directed numbers

1. (a) $<$ (b) $>$ (c) $>$ (d) $>$

2. (a) $-8 < -5 < -0.87 < 9.99 < 13$ (b) $-2 < -\frac{13}{7} < 0 < 1 < \frac{8}{7}$

3. (a) $\frac{25}{30} > \frac{24}{30} > \frac{15}{30} > -\frac{15}{30} > -\frac{24}{30} > -\frac{25}{30}$,

   $\therefore \frac{5}{6} > \frac{4}{5} > \frac{1}{2} > -\frac{1}{2} > -\frac{4}{5} > -\frac{5}{6}$

   (b) $3 > 2 > -3\frac{3}{4} > -4 > -4\frac{1}{3} > -14$, $\therefore 3 > 2 > -\frac{15}{4} > -4 > -\frac{13}{3} > -14$

4. (a) $= 7+2 = 9$ (b) $= -61-39+21 = -100+21 = -79$

   (c) $= -17+14+17 = -17+17+14 = 14$ (d) $= +33-15-(-4+6) = 18-2 = 16$

   (e) $= -11-\frac{3}{4}+12+\frac{1}{3}+5 = -11+12+5+\frac{1}{3}-\frac{3}{4} = 6-\frac{5}{12} = 5\frac{7}{12}$

   (f) $= 9.34+2.53-15.17-4.66 = 11.87-15.17-4.66 = -3.3-4.66 = -7.96$

5. (a) $= 9\times6\div18 = 3$ (b) $= -\frac{4\times3}{6\times10} = -\frac{1}{5}$ (c) $= \frac{2}{3}\times\frac{5}{8}\times\frac{6}{1} = \frac{5}{2} = 2\frac{1}{2}$

   (d) $= -(1.6 \times 0.25 \div 1.2 \times 0.3) = -0.1$

6. (a) $= 7-(-19) = 26$ (b) $= -8+(-28) = -8-28 = -36$

   (c) $= (-15)\times(-4) = 60$ (d) $= (-10)\div6 = -\frac{5}{3} = -1\frac{2}{3}$

7. (a) $= -36\div(3\times3) = -4$ (b) $= (-12)\div(4)\times(-6) = 18$

   (c) $= -6-(-11+1) = -6+10 = 4$ (d) $= (-2)\times(15+3) = -36$

8. (a) $= +(\frac{8}{3}\times\frac{5}{2}\times\frac{1}{9})+1 = 1\frac{20}{27}$

   (b) $= -\frac{7}{6}+(-\frac{1}{9}) = -\frac{7}{6}-\frac{1}{9} = -\frac{21}{18}-\frac{2}{18} = -\frac{23}{18} = -1\frac{5}{18}$

   (c) $= \frac{(-5)\times(-6)}{(-10+8)\times10} = \frac{30}{(-2)\times10} = -\frac{3}{2} = -1\frac{1}{2}$

9. (a) $= (-3)^2 - 4(-5) = 9 + 20 = 29$ (b) $= (-2)(-6) + (-2)^2 = 12 + 4 = 16$

   (c) $= 3(-\frac{1}{3})^2 + 2(-\frac{1}{3}) - 4 = 3(\frac{1}{9}) - \frac{2}{3} - 4 = \frac{1}{3} - \frac{2}{3} - 4 = -4\frac{1}{3}$

   (d) $= (-8)(\frac{2}{5}) + (\frac{2}{5})(-3) + (-3)(-8) = -\frac{16}{5} - \frac{6}{5} + 24 = 24 - \frac{22}{5} = 19\frac{3}{5}$

10. Let $x$ be the number, $x+(-26) = -53$, $x = (-53) - (-26)$, $x = -27$.
    *Ans. The number is –27.*

11. $27-(-7) = 34$. *Ans. The temperature drop is 34ºC.*

12. Let $x$ºC be the temperature of Beijing, $8-12 = x-5$, $x = 8-12+5$, $x = 1$.
    *Ans. The temperature of Beijing is 1ºC.*

13. $+250 - (-150) = 400$. *Ans. The rocket is 400m above the submarine.*

14. $+80 + (-150) = -70$. *Ans. The shopkeeper lost $70.*

15. $32(+2) + 10(-2) + 8(-1) = +64 - 20 - 8 = 36$. *Ans. Andrew's test score was 36.*

16. (a) $p = (-7) - (6) = -13$, $q = (-7)+(5) = -2$, $\therefore p-q = (-13) - (-2) = -11$

    (b) $p+q = (-13) + (-2) = -15$

17. (a) $= (-8)\times 4 + 3 = -32 + 3 = -29$ (b) $= (-8 + 6)^4 = (-2)^4 = 16$

    (c) $= \dfrac{4\times[-(5\times 5\times 5)]}{15\times 15} = -\dfrac{20}{9} = -2\dfrac{2}{9}$ (d) $= 36 - (-32) = 68$ (e) $= 4 \div \dfrac{1}{9} = 36$

18. (a) $= (3 + \dfrac{1}{2} - 7 - \dfrac{1}{6}) \div [-4 - (\dfrac{4}{9})] = (-4 + \dfrac{1}{3}) \div (-4\dfrac{4}{9}) = (-3\dfrac{2}{3}) \div (-\dfrac{40}{9})$

    $= \dfrac{11}{3} \times \dfrac{9}{40} = \dfrac{33}{40}$

    (b) $= -(5 - 3) \times 36 - (-4) = -2 \times 36 + 4 = -68$

    (c) $= \dfrac{36 - 10 - 8}{-9} = \dfrac{18}{-9} = -2$

19. $(3 + n)^2 = 0$, $3 + n = 0$, $\therefore n = -3$ 20. $= -1 - (+1) = -1 - 1 = -2$

21. (a) $= (-\dfrac{9}{4}) \div [(-\dfrac{1}{8}) + (-1)] = (-\dfrac{9}{4}) \div (-\dfrac{9}{8}) = +(\dfrac{9}{4} \times \dfrac{8}{9}) = 2$

    (b) $= -\dfrac{1}{3} - \{\dfrac{-3}{4} - [-\dfrac{1}{3} - (\dfrac{1}{2})^2]\} = -\dfrac{1}{3} - [-\dfrac{3}{4} - (\dfrac{-7}{12})] = -\dfrac{1}{3} - (\dfrac{-1}{6}) = -\dfrac{1}{6}$

22. (a) $= (-81 + 1) \times (\dfrac{7}{2} \times \dfrac{5}{14} - \dfrac{4}{5}) = (-80) \times (\dfrac{5}{4} - \dfrac{4}{5}) = -80 \times \dfrac{9}{20} = -36$

    (b) $= (2\dfrac{1}{6} - 2\dfrac{1}{4}) \div (-\dfrac{1}{3}) - [15 \times (-\dfrac{3}{20})] = (-\dfrac{1}{12}) \times (-\dfrac{3}{1}) - (-\dfrac{9}{4})$

    $= \dfrac{1}{4} + \dfrac{9}{4} = 2\dfrac{1}{2}$

23. $= 1 - 1 \div (1 - \dfrac{1}{1 - \frac{1}{9}})^3 = 1 - 1 \div (1 - \dfrac{1}{\frac{8}{9}})^3 = 1 - 1 \div (1 - \dfrac{9}{8})^3 = 1 - 1 \div (-\dfrac{1}{8})^3 =$

    $= 1 - 1 \times (-512) = 513$

24. (a) $= \dfrac{(2)(-1)}{-[0.3 + (-5)(-\frac{1}{4})]} = \dfrac{-2}{-(0.3 + \frac{5}{4})} = 2 \div (\dfrac{6}{20} + \dfrac{25}{20}) = 2 \div \dfrac{31}{20} = \dfrac{40}{31} = 1\dfrac{9}{31}$

    (b) $= \dfrac{-[(\frac{-1}{4})(0.3) - (-2)(1)]}{-(-5)^2} = (\dfrac{-3}{40} + 2) \cdot \dfrac{1}{25} = \dfrac{77}{40} \cdot \dfrac{1}{25} = \dfrac{77}{1000} = 0.077$

25. (a) $\dfrac{x}{-25} - \dfrac{9}{100} = -\dfrac{1}{4}$, $-4x - 9 = -25$, $-4x = -16$, $\therefore x = 4$

    (b) $(\dfrac{3}{4})(-\dfrac{2}{3}) - (\dfrac{4}{5})(-\dfrac{5}{8}x) = (\dfrac{1}{2})(-\dfrac{2}{3})$, $-\dfrac{1}{2} + \dfrac{1}{2}x = -\dfrac{1}{3}$,

$\frac{1}{2}x = -\frac{1}{3}+\frac{1}{2}, \quad x = \frac{1}{6} \times 2, \ \therefore \ x = \frac{1}{3}$

26. (a) $= (99) \times ... \times 1 \times 0 \times (-1) \times ... \times (-99) = 0$

    (b) $= (-1 + 2) + (-3 + 4) + (-5 + 6) + ... + (-49 + 50) = 1 \times 25 = 25$

27. There are 79 terms in which 40 terms are negative numbers.

    $\therefore$ The product is a positive number. $= +(\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times ... \times \frac{80}{79} \times \frac{81}{80}) = \frac{81}{2} = 40\frac{1}{2}$

28. (a) $y = 1^2 + 1^3 + ... + 1^{100} = 1 + 1 + 1 + ... + 1 = 99$ ($\because$ there are 99 terms)

    (b) $y = (-1)^2 + (-1)^3 + ... + (-1)^{100} = +1 - 1 + ... + 1 - 1 + 1 = 1$

29. (a) $\because$ $60 = 2 \times 2 \times 3 \times 5$, $\therefore$the smallest value of $a + b + c = 3 + 4 + 5 = 12 > 10$,

    $\therefore$ $a$, $b$, $c$ can't be all positive.

    $\because$ $abc = 60$, but $a$, $b$, $c$ can't be all positive,

    $\therefore$ two of them must be negative numbers.

    (b) The positive factors of 60 include : 1, 2, 3, 4, 5, 6, 10, 12,15, 20, 30, 60

    By inspection, $(-1) + (-4) + (+15) = 10$,

    $(-1)(-4)(+15) = 60$. $\therefore$ $a = -4$ (the smallest), $b = 15$ ( the largest), $c = -1$

30. (a) $(50 - 5) \times 4 + 5 \times (-2) = 180 - 10 = 170$. *Ans. He might get 170 marks.*

    (b) Let $x$ be the number of correct answers.

    $4x + (-3)(50 - x) = 123$, $4x - 150 + 3x = 123$, $7x = 273$, $x = 39$.

    *Ans. His percentage of correct answers* $= \frac{39}{50} \times 100\% = 78\%$.

31. (a) False. $-(-5) = +5$ which is not a negative number.

    (b) False. $-0 = +0 = 0$ (c) True (d) True (e) False. $-3 + 7 = 4$

32. (a) $(+2) - (-12) = 14$. *Ans. The time in Sydney is 14h ahead that of New York.*

    (b) The time difference $= a - (-1) = (a + 1)$ h.

    (c) Sydney – 19:00 on Thursday, London – 10:00 on Thursday

    The time difference between Sydney and London $= 2 - a$

    $19 - 10 = 2 - a$, $9 = 2 - a$, $\therefore$ $a = -7$

    (d) Vancouver – 19:00 on Monday,

    Hong Kong – 10:00 on Tuesday = 34:00 on Monday

    $34 + b = 19$, $\therefore b = 19 - 34 = -15$.

    *Ans. The time in Vancouver is 15 h behind that of H.K.*

    (e) (i) The time difference between Toyko and Vancouver

    $= +1 - b = 1 - (-15) = 16$h

    (ii) When Toyko is 09:00 on Friday, Vancouver is $(9 - 16 + 24) = 17:00$ on Thursday. When the person arrives at Vancouver, the time is 17:00 + 14 = 31:00 on Thursday = 7:00 am on Friday.

## Unit 2: Introduction to algebra

1. (a) $12 - 10x$ (b) $2a + 5b$ (c) $6x^2$ (d) $2a^4$ (e) $5p - 2q$

(f) $= 4n + 4n = 8n$ (g) $5xy - 3x^2$

2. (a) $\frac{1}{6}$ (b) 12 (c) $= \frac{p^2 \cdot p}{2} = \frac{p^3}{2}$ (d) $\frac{4ab}{c} - 2$

3. (a) $= 5(3)^2 - 3(4) = 45 - 12 = 33$ (b) $= (6)^2 - 3(6) + 8 = 36 - 18 + 8 = 26$

(c) $= 8(5) - (4)(16) = 40 - 64 = -24$ (d) $= \frac{5(3)^2 - 1}{2(3) + 5} = \frac{44}{11} = 4$

4. (a) $C = \frac{5}{9}(104 - 32) = \frac{5}{9} \times 72 = 40$ (b) $A = \frac{7}{2}(6 + 9) = \frac{105}{2} = 52\frac{1}{2}$

(c) $P = 10(10 - 2)(10 - 6)(10 - 5) = 10 \times 8 \times 4 \times 5 = 1600$

(d) $\frac{1}{f} = \frac{1}{\frac{2}{7}} - \frac{1}{1.5} = \frac{7}{2} - \frac{2}{3} = \frac{17}{6}$, $\therefore f = \frac{6}{17}$

5. (a) sum (b) difference (c) product (d) quotient

6. (a) $\frac{k}{3}$ (b) $x^2 - 7$ (c) $x(\frac{y}{2}) = \frac{xy}{2}$ (d) $(p + q) \div 2 = \frac{p + q}{2}$

7. (a) $\frac{t}{60}$ hours (b) $100n$ cents (c) $\frac{y}{1000}$ km (d) $(1000x + y)$ grams

8. $y - 2$ 9. $5 - x$ 10. $12 + k$ 11. $x - 3$ 12. $bh$ cm$^2$

13. $\$\frac{800}{n}$ 14. $\$(2x + 3y)$ 15. $\$\frac{k}{21}$ 16. $\frac{x}{y}$ h 17. $\frac{s}{x}$ km/h

18. $y \times \frac{15}{60} = \frac{y}{4}$ km

19. (a) $= 14mn - 17mn = -3mn$ (b) $= -4k^3 - 2$ (c) $= -7x^2 + 6x + 4$

(d) $= 7xy^2 - 6x^2y + x^2y = 7xy^2 - 5x^2y$ (e) $= p - q - 5p - 9q = -4p - 10q$

(f) $= 2x - 7y + 6x - y - 2x - 3y = 6x - 11y$

20. (a) $(a+b)^2$ (b) $a^2 + b^2$ (c) $\frac{2}{5} \times n = \frac{2n}{5}$ kg (d) $\frac{p}{4}$ cups

(e) $\frac{x}{3} \div x \times y = \frac{y}{3}$ g

21. $p \times q - \frac{k}{2} \div \frac{1}{3} = pq - \frac{3k}{2}$ 22. $(x - 7kn)$ kg

23. $10y + x$ 24. $\frac{1000x}{96 \times 60} = \frac{25x}{144}$ minutes

25. (a) The total distance travelled = $(mx + ny)$ km

(b) The total time taken = $(m + n)$ hours, $\therefore$ the average speed = $\frac{mx + ny}{m + n}$ km/h

26. (a) 6 years ago Paul was $(x - 6)$ years old.

(b) 6 years ago Helen's age = $\frac{34 - (x - 6)}{2} = \frac{40 - x}{2} = (20 - \frac{x}{2})$ years old

(c) Helen's age now = $20 - \frac{x}{2} + 6 = (26 - \frac{x}{2})$ years old

$\therefore$ Their total ages now = $x + (26 - \frac{x}{2}) = (\frac{x}{2} + 26)$ years old

27. (a) The value of letter G is 7.

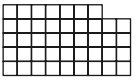
(b) T = 8 (7) + 7 (4) + 6 (6) + 5 (7) + 4 (5) + 3 (5) + 2 (6)
= 56 + 28 + 36 + 35 + 20 + 15 + 12 = 202

(c) 202 ÷ 11 = 18 .... 4, $\therefore$ R = 4, $x = 11 - 4 = 7$

28. (a) $= \frac{r-s}{r-s} = 1$ (b) $= \frac{a-b}{-a+b} = \frac{(a-b)}{-(a-b)} = -1$ (c) No.

(d) $= \frac{(p+q)^2}{(p+q)^2} = 1$ (e) $= \frac{(x-y)^2}{[-(x-y)]^2} = \frac{(x-y)^2}{(x-y)^2} = 1$ (f) No.

## Unit 3: Patterns and sequences

1. (a) 

(b) The no. of squares = 6 × 11 – 2 = 64

2. (a) 22, 25 (b) $\frac{1}{4}, \frac{1}{8}$ (c) 16, 27

3. 1, 3, 6, 10, 15, 21 4. 64, 81, 100, 121, 144

5. (a) 1 – 5 = –4, –3 – 1 = –4, –7 – (–3) = –4, –11 – (–7) = –4. *Ans. There is a common difference, so the numbers form an arithmetic sequence.*

(b) 19 – 13 = 6, but 26 – 19 = 7.
*Ans. The numbers don't form an arithmetic sequence.*

6. (a) $\frac{-8}{4} = -2, \frac{16}{-8} = -2, \frac{-32}{16} = -2, \frac{64}{-32} = -2.$
*Ans. There is a common ratio, so the numbers form a geometric sequence.*

(b) $\frac{5}{25} = \frac{1}{5}, \frac{1}{5} = \frac{1}{5}, \frac{0.2}{1} = \frac{1}{5}, \frac{0.04}{0.2} = \frac{1}{5}, \frac{0.008}{0.04} = \frac{1}{5}.$
*Ans. There is a common ratio, so the numbers form a geometric sequence.*

7.

| 5th | 6th | 7th | 8th | 9th | 10th | 11th | 12th |
|---|---|---|---|---|---|---|---|
| 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 |

*Ans. The eighth term is 21 and the twelfth term is 144.*

8. (a) $a_n = 4 - 3n$, $\therefore a_1 = 4 - 3(1) = 1, a_2 = 4 - 3(2) = -2, a_3 = 4 - 3(3) = -5,$
$a_4 = 4 - 3(4) = -8$. *Ans. The first four terms are* 1, –2, –5, –8.

(b) $a_n = \frac{n+1}{n+2}$, $\therefore a_1 = \frac{1+1}{1+2} = \frac{2}{3}, a_2 = \frac{2+1}{2+2} = \frac{3}{4}, a_3 = \frac{3+1}{3+2} = \frac{4}{5}, a_4 = \frac{4+1}{4+2} = \frac{5}{6}.$

*Ans. The first four terms are* $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}.$

(c) $a_n = \frac{n(n+1)}{2}$, $\therefore a_1 = \frac{1(1+1)}{2} = 1, a_2 = \frac{2(2+1)}{2} = 3, a_3 = \frac{3(3+1)}{2} = 6,$

$a_4 = \frac{4(4+1)}{2} = 10$. *Ans. The first four terms are* 1, 3, 6, 10.

(d) $a_n = n^2 - 1$, $\therefore a_1 = 1^2 - 1 = 0, a_2 = 2^2 - 1 = 3, a_3 = 3^2 - 1 = 8,$

$a_4 = 4^2 - 1 = 15$. *Ans. The first four terms are* 0, 3, 8, 15.

(e) $a_n = 2^n + 3$, $\therefore\ a_1 = 2^1 + 3 = 5$, $a_2 = 2^2 + 3 = 7$, $a_3 = 2^3 + 3 = 11$,
$a_4 = 2^4 + 3 = 19$. *Ans. The first four terms are 5, 7, 11, 19.*

9. $a_1 = 2 \times 3 - 1 = (\mathbf{1}+1)(\mathbf{1}+2) - \mathbf{1}$ $\quad\therefore\ a_n = (n+1)(n+2) - n$
$a_2 = 3 \times 4 - 2 = (\mathbf{2}+1)(\mathbf{2}+2) - \mathbf{2}$ $\quad$[or: $a_n = n^2 + n + 2n + 2 - n = n^2 + 2n + 2$]
$a_3 = 4 \times 5 - 3 = (\mathbf{3}+1)(\mathbf{3}+2) - \mathbf{3}$

10. (a) $a_1 = 0 = 8 \times (\mathbf{1} - 1)$
$a_2 = 8 = 8 \times (\mathbf{2} - 1)$
$a_3 = 16 = 8 \times (\mathbf{3} - 1)$
$a_4 = 24 = 8 \times (\mathbf{4} - 1)$
.....
$\therefore\ a_n = 8(n - 1)$

(b) $a_1 = 3 = 3^{\mathbf{1}}$
$a_2 = 9 = 3^{\mathbf{2}}$
$a_3 = 27 = 3^{\mathbf{3}}$
.....
$\therefore\ a_n = 3^n$

11. (a) $a_n = 14 + (-3)^n$
$a_1 = 14 + (-3)^1 = 14 - 3 = 11$
$a_2 = 14 + (-3)^2 = 14 + 9 = 23$
$a_3 = 14 + (-3)^3 = 14 - 27 = -13$
$a_4 = 14 + (-3)^4 = 14 + 81 = 95$
$\therefore$ The first four terms are 11, 23, –13, 95.

(b) $a_n = -2^n - 11$
$a_1 = -2^1 - 11 = -2 - 11 = -13$
$a_2 = -2^2 - 11 = -4 - 11 = -15$
$a_3 = -2^3 - 11 = -8 - 11 = -19$
$a_4 = -2^4 - 11 = -16 - 11 = -27$
$\therefore$ The first four terms are –13, –15, –19, –27.

(c) $a_n = 3^{n+1} + (-1)^{n+1}$
$a_1 = 3^2 + (-1)^2 = 9 + 1 = 10$
$a_2 = 3^3 + (-1)^3 = 27 - 1 = 26$
$a_3 = 3^4 + (-1)^4 = 81 + 1 = 82$
$a_4 = 3^5 + (-1)^5 = 243 - 1 = 242$
$\therefore$ The first four terms are 10, 26, 82, 242.

12. Let $a_n$ be the *n*th term.

(a) The common difference = 2
$a_1 = 2(1) + 7$, $a_2 = 2(2) + 7$, $a_3 = 2(3) + 7$, .... $\quad\therefore\ a_n = 2n + 7$

(b) The common difference = 7
$a_1 = 7(1) + 6$, $a_2 = 7(2) + 6$, $a_3 = 7(3) + 6$, .... $\quad\therefore\ a_n = 7n + 6$

(c) The common difference = –3
$a_1 = -3(1) + 38$, $a_2 = -3(2) + 38$, $a_3 = -3(3) + 38$, .... $\quad\therefore\ a_n = -3n + 38$

(d) The common difference = 4
$a_1 = 4(1) - 59$, $a_2 = 4(2) - 59$, $a_3 = 4(3) - 59$, .... $\quad\therefore\ a_n = 4n - 59$

13. Let $a_n$ be the *n*th term.

(a) The common ratio = 2 , $a_1 = 7(2^1)$, $a_2 = 7(2^2)$, $a_3 = 7(2^3)$, .... $\quad\therefore\ a_n = 7(2^n)$

(b) The common ratio = 0.1 , $a_1 = (0.3)(0.1)^1$, $a_2 = (0.3)(0.1)^2$, $a_3 = (0.3)(0.1)^3$, ....
$\therefore\ a_n = (0.3)(0.1)^n$ $\quad$[or: $3(0.1)^{n+1}$]

(c) The common ratio = –3

$a_1 = (-\frac{4}{3})(-3)^1$, $a_2 = (-\frac{4}{3})(-3)^2$, $a_3 = (-\frac{4}{3})(-3)^3$, ....

$\therefore\ a_n = (-\frac{4}{3})(-3)^n$ $\quad$[or: $4(-3)^{n-1}$]

14. (a) $a_1 = 4 = 1 + 3 = 1^2 + 3$
$a_2 = 7 = 4 + 3 = 2^2 + 3$
$a_3 = 12 = 9 + 3 = 3^2 + 3$
$a_4 = 19 = 16 + 3 = 4^2 + 3$
.....
$\therefore\ a_n = n^2 + 3$

(b) $a_1 = 3 = 1 \times 3 = 1 \times (1 + 2)$
$a_2 = 8 = 2 \times 4 = 2 \times (2 + 2)$
$a_3 = 15 = 3 \times 5 = 3 \times (3 + 2)$
$a_4 = 24 = 4 \times 6 = 4 \times (4 + 2)$
.....
$\therefore\ a_n = n\,(n + 2)$

15. (a) $a_1 = 9,\ a_2 = 13,\ a_3 = 17$
$a_3 - a_2 = 17 - 13 = 4$
$a_2 - a_1 = 13 - 9 = 4$
$\therefore$ They form an arithmetic sequence.

(b) $a_1 = 9 = 1 \times 4 + 5$
$a_2 = 13 = 2 \times 4 + 5$
$a_3 = 17 = 3 \times 4 + 5$
.....
$\therefore\ a_n = 4n + 5$

16. (a) 15 (b) $8 - 3 = 5,\ 7 - 8 = -1,\ 12 - 7 = 5,\ 11 - 12 = -1$.
$\therefore$ A term is formed by either adding + 5 or –1 to the previous term.

(c) For the odd numbers in the sequence, the next term is formed by adding +5 to the preceding terms. $\therefore$ The term after 107 = 107 + 5 = 112.

17. $10 - 4n = -58,\ 10 + 58 = 4n,\ n = \dfrac{68}{4} = 17$. $\therefore$ The 17th term is –58.

18. (a) $a_3 = (-3)^3 + 10$
$= -27 + 10$
$= -17$

(b) $(-3)^n + 10 = -233$
$(-3)^n = -243$
$(-3)^n = (-3)^5$
$n = 5$. $\therefore$ The 5th term is –233.

19. $29 - 4n > 0,\ 29 > 4n,\ n < \dfrac{29}{4},\ n < 7\dfrac{1}{4}$.

*Ans. 7 terms of this sequence are positive numbers.*

20. In each row, the first and the last number are 1.
Observing the pattern: $1 + 3 = 4,\ 3 + 3 = 6,\ 3 + 1 = 4$
$\therefore$ The 6th layer is: 1 5 10 10 5 1
The 7th layer is: 1 6 15 20 15 6 1

1 3 3 1
1 4 6 4 1

21. (a) $\boxed{1}\ 5^2 = \boxed{2}\ 25,\quad \boxed{2}\ 5^2 = \boxed{6}\ 25,\quad \boxed{3}\ 5^2 = \boxed{12}\ 25$,
$1 \times 2$ $2 \times 3$ $3 \times 4$
$\therefore$ The last 2 digits of the answer are always 25. And if the tens digit of the given number is n, the front digit(s) of the answer will be $n(n+1)$.

(b) The tens digit is 9, and 9(9+1) = 90. The last 2 digits are 25. $\therefore\ 95^2 = 9025$

22. (a) The first 8 terms are: –1, 1, –1, 1, –1, 1, –1, 1

(b) The first 8 terms are: 1, 4, 9, 16, 25, 36, 49, 64

(c) Comparing each term of the sequence in (b) and (c), the difference is either +1 or –1.
$\therefore\ a_n = n^2 + (-1)^n$

23. (a) In each column, the sum of the first number and the third number is always twice that of the middle number, e.g. 1+3 = 2×2, 4+12=8×2, 2+4=3×2, .....

(b) $10 + 4 = 2x,\ \therefore\ x = 7$ ; $11 + y = 2 \times 9,\ \therefore\ y = 7$

24. (a) In each column, the sum of the squares of the first and the middle number is always half of the last number, e.g. $1^2+2^2 = \dfrac{10}{2},\ 2^2+3^2 = \dfrac{26}{2},\ 3^2+1^2 = \dfrac{20}{2}$, ...

(b) $6^2+1^2 = \frac{h}{2}$, $\therefore\ h = 74$ ; $7^2+k^2 = \frac{116}{2}$, $k^2 = 58-49$, $k^2 = 9$, $\therefore k = 3$

25. (a)

| Number of participants | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Number of handshakes | 1 | 3 | 6 | 10 | 15 | 21 | 28 |

(b) Triangular Number Pattern.

(c) The new sequence is: 2, 6, 12, 20, 30, 42, 56

$2 = 1\times2$, $6 = 2\times3$, $12 = 3\times4$, $20 = 4\times5$, $30 = 5\times6$, $42 = 6\times7$, $56 = 7\times8$

(d) From (c), the following pattern is observed:

| No. of participants | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| No. of handshakes | $1 = \frac{1\times2}{2}$ | $2 = \frac{2\times3}{2}$ | $6 = \frac{3\times4}{2}$ | $10 = \frac{4\times5}{2}$ |

$\therefore$ When there are $n$ participants, the no. of handshakes $= \frac{n(n-1)}{2}$.

(e) When there are 50 participants, the no. of handshakes $= \frac{50\times49}{2} = 1225$.

## Unit 4: Operations of algebraic expressions

1. (a) $\frac{5}{4}a$ (b) $-\frac{4}{15}y^2$ (c) $-\frac{5}{7}b^3$ (d) $= -\frac{4}{3}xy^2 + \frac{1}{3}xy^2 = -xy^2$

2. (a) $= 14mn - 17mn = -3mn$ (b) $-4k^3 - 2$ (c) $-7x^2 + 6x + 4$
   (d) $= xy^2 - 6x^2y + x^2y = xy^2 - 5x^2y$ (e) $= p - q - 5p - 9q = -4p - 10q$
   (f) $= 2x - 7y + 6x - y - 2x - 3y = 6x - 11y$

3. (a) $4, 1, -\frac{1}{2}, 3, -2$ (b) $3, -1, 0, -5, 4$ (c) $3, 3, -6, \frac{1}{2}, 0$

4. (a) $2x^2 + 6x - 10$ (b) $x^3 - x^2 - 6x - 5$
   (c) $= 5x^3y - 9x^3y - 6x^3y + x^3y = -9x^3y$

5. (a) $= 12x^2 + 3y^2 - 7x^2 - 8y^2 = -5y^2 + 5x^2$
   (b) $= -3x^2 + 2 - 11x + 10 + 8x^2 - 6x = 12 - 17x + 5x^2$
   (c) $= x^2 - 8x + 1 - 3x^2 - 5x - 2 + 4x^2 - 3x + 6 = 5 - 16x + 2x^2$
   (d) $= -7y^2 - 9xy + 4x^2 + 5xy - 6y^2 + 11x^2 - 3y^2 + 8xy + 5x^2$
   $= 4x^2 + 11x^2 + 5x^2 - 9xy + 5xy + 8xy - 7y^2 - 6y^2 - 3y^2 = -16y^2 + 4xy + 20x^2$

6. (a) $12y^2 - 30y + 6$ (b) $-6a^2 + 10a$ (c) $6w^4 + 16w^3 + 36w^2$
   (d) $-10x^3 - 25x^2 + 10x$

7. (a) $= 8x - 6 - 12x + 6 = -4x$ (b) $= 24a^2 - 15a - 8a^2 - 12a = 16a^2 - 27a$
   (c) $= 16b - 4(7 + 5 - 10b) = 16b - 48 + 40b = 56b - 48$

8. (a) $= x(2x + 5) + 3(2x + 5) = 2x^2 + 5x + 6x + 15 = 2x^2 + 11x + 15$
   (b) $= 4y(y + 3) - 1(y + 3) = 4y^2 + 12y - y - 3 = 4y^2 + 11y - 3$
   (c) $= 6m(2m - 7n) - n(2m - 7n) = 12m^2 - 42mn - 2mn + 7n^2$
   $= 12m^2 - 44mn + 7n^2$

9. (a) $6x$ (b) $x^6$ (c) $= x^3 + x^3 = 2x^3$ (d) $x^4 + x^2$ (e) $x^5 + x$
   (f) $= x^2 + x^2 + x^2 = 3x^2$ (g) $= x + x^4 + x = 2x + x^4$

(h) $= x^2 + 2x + x^2 = 2x^2 + 2x$ (i) $= x + x^2 + x^2 + x = 2x + 2x^2$

(j) $= 2x + x^2 + 2x = 4x + x^2$

10. (a) $= \dfrac{6k \times (-18k)}{4} = -27k^2$ (b) $= -\dfrac{6}{7}mn - mn = -\dfrac{13}{7}mn$

11. (a) $= -6\,[3y\,(2y + 3) - 2\,(2y + 3)] = -6\,(6y^2 + 9y - 4y - 6) = -6\,(6y^2 + 5y - 6)$
$= -36y^2 - 30y + 36$

(b) $= x\,(9x^2 + xy - 4y^2) + 2y\,(9x^2 + xy - 4y^2)$
$= 9x^3 + x^2y - 4xy^2 + 18x^2y + 2xy^2 - 8y^3 = 9x^3 + 19x^2y - 2xy^2 - 8y^3$

12. (a) $(5 + 3)^2 = 8^2 = 64$ ; $5^2 + 3^2 = 25 + 9 = 34$. They are not equal.

(b) $= (x + y)\,(x + y) = x\,(x + y) + y\,(x + y) = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$

(c) $= (3a - 2)\,(3a - 2) = 3a\,(3a - 2) - 2\,(3a - 2) = 9a^2 - 6a - 6a + 4$
$= 9a^2 - 12a + 4$

13. $2\,(a + 5) + 5\,(2a^2 + 3a - 4) = 2a + 10 + 10a^2 + 15a - 20 = 10a^2 + 17a - 10$

*Ans. The total value of the coins is* $\$(10a^2 + 17a - 10)$.

14. $(x + 3y)\,(y - 2x) \div 2 = [x\,(y - 2x) + 3y\,(y - 2x)] \div 2 = [xy - 2x^2 + 3y^2 - 6xy] \div 2$

$= (-2x^2 - 5xy + 3y^2) \div 2 = -x^2 - \dfrac{5}{2}xy + \dfrac{3}{2}y^2$

*Ans. The area of the triangle is* $(-x^2 - \dfrac{5}{2}xy + \dfrac{3}{2}y^2)$ *sq. units.*

15. (a) $(12x - 2) \div 2 - 3x = 6x - 1 - 3x = 3x - 1$

*Ans. The width of the rectangle is* $(3x - 1)$ *cm.*

(b) $= 3x\,(3x - 1) - (3x - x)\,(3x - 1 - 1) = 9x^2 - 3x - 2x\,(3x - 2)$
$= 9x^2 - 3x - 6x^2 + 4x = 3x^2 + x$ *Ans. The decrease in area is* $(3x^3 + x)$ *cm*$^2$.

16. (a) $(x - 1) \times (y + 2) = x\,(y + 2) - 1\,(y + 2) = xy + 2x - y - 2$

*Ans. The total number of marbles in the remaining jars is* $(xy + 2x - y - 2)$.

(b) $xy - (xy + 2x - y - 2) = xy - xy - 2x + y + 2 = -2x + y + 2$

*Ans. The number of the marbles left is* $(-2x + y + 2)$.

## Unit 5: Simple indices

1. (a) $= 1 - 27 = -26$ (b) 81 (c) $= -16 + 49 = 33$

(d) $= \dfrac{19^7 \cdot 19^7}{19^{15}} = \dfrac{19^{14}}{19^{15}} = \dfrac{1}{19}$ (e) $= \dfrac{64 + 27}{16 - 9} = \dfrac{91}{7} = 13$

(f) $= \dfrac{3^{15}}{4^{15}} \times (-\dfrac{4^{17}}{3^{17}}) = -\dfrac{4^2}{3^2} = -\dfrac{16}{9}$

2. (a) $54x^6$ (b) $25a^3b^3$ (c) $8x^3y^6z^3$ (d) $= 3y^2 \cdot 9y^2 = 27y^4$

(e) $36a^{12}$ (f) $= (4a^5b^4)\,(27a^3b^9) = 108a^8b^{13}$ (g) $= \dfrac{4m^6n}{8m^6} = \dfrac{n}{2}$

(h) $= \dfrac{15ab^3}{4a^2b^4} \times 3ab = \dfrac{45}{4} = 11\dfrac{1}{4}$

3. (a) $k^{10}$ (b) $= (-x^3)\,(9x^2) = -9x^5$ (c) $= (2y^3)\,(16y^{16}) = 32y^{19}$

(d) $= \dfrac{5^8 a^4}{(-a^3)\cdot(5^6 a^6)} = -\dfrac{25}{a^5}$ (e) $= \dfrac{16a^4b^6}{-6a^2b} \times a^{12}b^8 = -\dfrac{8a^{14}b^{13}}{3}$

(f) $= \dfrac{-35x^6y^2}{14x^4y^{12}\cdot 2x^2} = \dfrac{-5}{4y^{10}}$

4. (a) $= \dfrac{(-12ab^3)\times(8a^3b^{12})}{36a^4b^2} = -\dfrac{8b^{13}}{3}$ (b) $= (-14x^3y^6)\times(-\dfrac{7}{xy^9})\times(-2x^2y^3) = -196x^4$

(c) $= \dfrac{(16x^{12})(3x^5)}{6x^4} = 8x^{13}$ (d) $\dfrac{a^{12}\cdot b^6}{a^5b^5} = a^7b$

5. (a) $45xy^2z^6 = 3\cdot 15\cdot x\cdot y^2\cdot z^6$ ; $105x^2yz^3 = 7\cdot 15\cdot x^2\cdot y\cdot z^3$ $\therefore$ HCF $= 15xyz^3$

(b) $6a^2b^3 = 6\cdot a^2\cdot b^3$ ; $18ab^4 = 6\cdot 3\cdot a\cdot b^4$ ; $24b^2 = 6\cdot 4\cdot b^2$ $\therefore$ HCF $= 6b^2$

6. (a) $33a^2bc^8 = 11\cdot 3\cdot a^2\cdot b\cdot c^8$ ; $44ab^5c^6 = 11\cdot 4\cdot a\cdot b^5\cdot c^6$

$\therefore$ LCM $= 11\cdot 3\cdot 4\cdot a^2\cdot b^5\cdot c^8 = 132a^2b^5c^8$

(b) $15p^5q^3 = 3\cdot 5\cdot p^5\cdot q^3$ ; $6pq^4 = 2\cdot 3\cdot p\cdot q^4$ ; $10p^2q = 2\cdot 5\cdot p^2\cdot q$

$\therefore$ LCM $= 2\cdot 3\cdot 5\cdot p^5\cdot q^4 = 30p^5q^4$

7. $48a^2b^3 \times 2 \div 6ab = \dfrac{96a^2b^3}{6ab} = 16\text{ab}^2$

*Ans. The corresponding height of the triangle is $16ab^2$ units.*

8. $3x^3 + 3 = 15$, $3x^3 = 12$, $x^3 = 4$ $\therefore$ $\dfrac{x^6}{3} = \dfrac{(x^3)^2}{3} = \dfrac{4^2}{3} = \dfrac{16}{3} = 5\dfrac{1}{3}$

9. $4^2 \times 4^6 = 16^n$, $4^{6+2} = (4^2)^n$, $4^8 = 4^{2n}$ $\therefore$ $8 = 2n$, $n = 4$

10. $2^3 = x^3$ $\therefore$ $x = 2$. $9^3 = 3^y$, $(3^2)^3 = 3^y$, $3^6 = 3^y$ $\therefore$ $y = 6$. $\therefore$ $y^x = 6^2 = 36$

11. $= \dfrac{k^k}{k\cdot k} = \dfrac{k^k}{k^2} = k^{k-2}$

12. (a) $5^{3x-1} = 25$, $5^{3x-1} = 5^2$ $\therefore$ $3^x - 1 = 2$, $3x = 3$, $x = 1$

(b) $\dfrac{1}{2^m} = 0.125$, $\dfrac{1}{2^m} = \dfrac{1}{8}$, $2^m = 8$, $2^m = 2^3$ $\therefore$ $m = 3$

13. (a) $= 3^n \times 3^n = 3^{n+n} = 3^{2n}$ [ or $= (3\times 3)^n = 9^n$] (b) $2^x + 2^x = 2\cdot(2^x) = 2^{x+1}$

14. (a) $= \dfrac{2^{n+1}\cdot(2^2)^n}{(2^3)^n} = \dfrac{2^{n+1}\cdot 2^{2n}}{2^{3n}} = 2^{n+1+2n-3n} = 2$

(b) $= \dfrac{12n}{3^{n+1}\cdot 2^{2n}} = \dfrac{(2^2\cdot 3)^n}{3^{n+1}\cdot 2^{2n}} = \dfrac{2^{2n}\cdot 3^n}{3^{n+1}\cdot 2^{2n}} = \dfrac{3^n}{3^{n+1}} = \dfrac{1}{3}$

15. $1000000 = 10^6 = (2\times 5)^6 = 2^6 \times 5^6$ $\therefore$ The smaller integer $= 2^6 = 64$

16. $\dfrac{3^{21}}{2^{31}} = \dfrac{3^{20}\cdot 3}{2^{30}\cdot 2} = \dfrac{3^{2\times 10}\cdot 3}{2^{3\times 10}\cdot 2} = \dfrac{9^{10}}{8^{10}}\times\dfrac{3}{2} = (\dfrac{9}{8})^{10}\times\dfrac{3}{2} > 1$ $\therefore$ $3^{21} > 2^{31}$

*Ans. John's answer is greater than Peter's answer.*

## Unit 6: Linear equations (1)

1. (a) $y = \dfrac{3}{5}\times 6$, $y = 3\dfrac{3}{5}$ (b) $3 - \dfrac{1}{4} = y$, $y = 2\dfrac{3}{4}$ (c) $m = 0$

(d) $4n=-2$, $n=-\frac{1}{2}$ (e) $-2a=-16$, $a=8$

(f) $-9x=-18$, $x=2$ (g) $-5x+3+x=0$, $-4x=-3$, $x=\frac{3}{4}$

(h) $7x-(2x+3)=15-6x$, $11x=18$, $x=1\frac{7}{11}$

2. (a) $12x+10x-60=25x+60$, $-3x=120$, $x=-40$

(b) $2x+1-15=30$, $2x=44$, $x=22$

(c) $30+20x=42$, $20x=12$, $x=\frac{3}{5}$

(d) $6+(x+1)=2x$, $-x=-7$, $x=7$

(e) $4x+12=3x+15$, $x=3$

(f) $4y-(y-3)=36$, $3y=33$, $y=11$

3. (a) $0.2p=2.2$, $p=11$

(b) $0.6y+1.2=1.8y+0.6+1$, $-1.2y=0.4$, $y=-\frac{1}{3}$

(c) $0.8x-4=0.08$, $0.8x=4.08$, $x=5.1$

4. (a) $30x+40=12x$, $18x=-40$, $x=-2\frac{2}{9}$

(b) $15-18x=-8x$, $15=10x$, $x=1\frac{1}{2}$

(c) $x-2+6x+15=4$, $7x=-9$, $x=-1\frac{2}{7}$

(d) $2x-16-3x-6=4x+7$, $-5x=29$, $x=-5\frac{4}{5}$

(e) $3x-2x-6=4-x$, $2x=10$, $x=5$

(f) $3a-5a-5=2a+10$, $-4a=15$, $a=-3\frac{3}{4}$

5. (a) $3k-2(2k+2)=0$, $3k-4k-4=0$, $k=-4$

(b) $-6(7a-2)+2-2a=9a-2$, $-44a+14=9a-2$, $-53a=-16$, $a=\frac{16}{53}$

6. (a) $6+3(-x+7)=2(2x-14)$, $-3x+27=4x-28$, $-7x=-55$, $x=7\frac{6}{7}$

(b) $15(x-3)+8(x+1)=2(7x-2)$, $23x-37=14x-4$, $9x=33$, $x=3\frac{2}{3}$

(c) $2(x-2)-7(3x+6)=4x-1$, $-19x-46=4x-1$, $-23x=45$, $x=-1\frac{22}{23}$

(d) $4(6x+7)-5(x-8)=120$, $19x+68=120$, $19x=52$, $x=2\frac{14}{19}$

7. (a) $0.48-5.85x=0.25(4.6x+0.8)$, $0.48-5.85x=1.15x+0.2$,

$0.28 = 7x$, $x = 0.04$

(b) $\frac{15}{2} + \frac{10y}{3} + \frac{1-6y}{5} = \frac{1}{3}(-\frac{13}{20} + \frac{9y}{4} - \frac{1}{4})$, $\frac{15}{2} + \frac{10y}{3} + \frac{1-6y}{5} = -\frac{3}{10} + \frac{3y}{4}$,

$450 + 200y + 12(1-6y) = -18 + 45y$, $83y = -480$, $y = -5\frac{65}{83}$

8. (a) $7a - 1 = 3a$, $4a = 1$, $a = \frac{1}{4}$ (b) $3m - 15 = 5m$, $-2m = 15$, $m = -7\frac{1}{2}$

(c) $36 = 3x - 21$, $57 = 3x$, $x = 19$ (d) $9 - 6 + 4 = 84x$, $84x = 7$, $x = \frac{1}{12}$

(e) $x - (4x - 1) = -(x + 1)$, $-2x = -2$, $x = 1$

9. (a) $\frac{7x}{3} = 5x - 3x + 2$, $7x = 15x - 9x + 6$, $x = 6$

(b) $\frac{5y}{12} + \frac{4y-3}{60} + y = \frac{4y-3}{10} - y$, $25y + (4y - 3) + 60y = 6(4y - 3) - 60y$,

$125y = -15$, $y = -\frac{3}{25}$

10. (a) $35 - (2x^2 + 4x + 7x + 14) = 9 - 2x^2$, $-11x = -12$, $x = 1\frac{1}{11}$

(b) $(y - 3)(y + 1) = (y - 4)(y - 5)$, $y^2 + y - 3y - 3 = y^2 - 5y - 4y + 20$, $7y = 23$,

$y = 3\frac{2}{7}$

11. $2x + 9 = 2$, $2x = -7$, $x = -3.5$, $\therefore (x + 3)(x + 6) = (-3.5 + 3)(-3.5 + 6) = -1.25$

12. Let $y = 4x$, $\frac{y-1}{y+1} = 6$, $y - 1 = 6(y + 1)$, $-5y = 7$, $y = -1.4$,

$\therefore$ $(4x - 1)(4x + 1) = (y - 1)(y + 1) = (-1.4 - 1)(-1.4 + 1) = 0.96$

13. When $x + 3 = 0$, L.H.S = 0 = R.H.S., $\therefore$ $x = -3$

When $x + 3 \neq 0$, $5 - 2x = 7 + x$, $-3x = 2$, $\therefore$ $x = -\frac{2}{3}$ *Ans.* $x = -3$ *or* $-\frac{2}{3}$

14. (a) $a = 3$ or $a = -3$

(b) $3x + 2 = 3$ or $3x + 2 = -3$, $3x = 1$ or $3x = -5$, $x = \frac{1}{3}$ or $x = -1\frac{2}{3}$

## Unit 7: Linear equations (2)

1. $4x + 8 = 44$, $4x = 36$, $x = 9$

2. Let $x$ be the number, $23 + \frac{x}{7} = 44$, $\frac{x}{7} = 21$, $x = 147$. *Ans. The number is 147.*

3. Let $x$ be the number, $x - 7 = \frac{2}{3}x$, $\frac{x}{3} = 7$, $x = 21$. *Ans. The number is 21.*

4. Let $x$ be the number, $\frac{x-17}{7} = 5$, $x - 17 = 35$, $x = 52$. *Ans. The number is 52.*

5. Let $x$ be the smallest number, then other two numbers are $(x + 1)$ and $(x + 2)$,
   $x + (x + 1) + (x + 2) = 114$, $3x = 111$, $x = 37$. *Ans. The numbers are 37, 38, 39.*
6. Let $x$ be the smaller number, then the larger number is $(124 - x)$,
   $(124 - x) - x = 22$, $-2x = -102$, $x = 51$. *Ans. The smaller number is 51.*
7. Let $x$ be the greater number, then the smaller number is $(x - 15)$,
   $2x - (x - 15) = 47$, $x = 32$. *Ans. The numbers are 17 and 32.*
8. Let $x$ cm be the width, then the length is $(x + 3)$ cm, $2[x + (x + 3)] = 78$,
   $2x + 3 = 39$, $2x = 36$, $x = 18$. *Ans. The width of the rectangle is 18 cm.*
9. Let $x$ cm be the width, then the length is $(\frac{x}{2}+12)$ cm, $2[x + (\frac{x}{2}+12)] = 32$,

   $\frac{3x}{2}+12=16$, $3x = 8$, $x=2\frac{2}{3}$. $\therefore$ The length is $2\frac{2}{3}\times\frac{1}{2}+12=13\frac{1}{3}$ cm.

   *Ans. The width is* $2\frac{2}{3}$ *cm and the area is* $2\frac{2}{3}\times13\frac{1}{3}=35\frac{5}{9}\text{cm}^2$ .
10. Let $\$x$ be the amount Maggie has, then Kathy has $\$3x$, $x + 3x = 640$, $4x = 640$,
    $x = 160$. *Ans. Maggie has $160 and Kathy has $480.*
11. Let $x$ kg be the weight of Benson's son, then Benson's weight is $(2x + 27)$ kg,
    $2x + 27 = 73$, $2x = 46$, $x = 23$. *Ans. The weight of Benson's son is 23 kg.*
12. Let $x$ be the present age of Paul, then Paul's mother is $3x$ years old now,
    $3x - 4 = 35$, $3x = 39$, $x = 13$. *Ans. Paul is 13 years old now.*
13. Let $x$ be the present age of the son, then the woman is $(54 - x)$ years old now,
    $(54 - x) - 3 = 5(x - 3)$, $66 = 6x$, $x = 11$. *Ans. The son is 11 years old now.*
14. Let $x$ be the number of years, $69 - x = 7(15 - x)$, $6x = 36$, $x = 6$.
    *Ans. The number of years is 6.*
15. Let $x$ be the present age of the son, then the father is $4x$ years old now,
    $4x + 15 = 2(x + 15) + 5$, $2x = 20$, $x = 10$.
    *Ans. The present ages of the father and the son are 40 and 10 respectively.*
16. Let $\$x$ be the amount A originally has, then B originally has $\$(x - 80)$,
    $x - 10 = 6[(x - 80) + 10]$, $x - 10 = 6(x - 70)$, $410 = 5x$, $x = 82$.
    *Ans. A originally has $82.*
17. Let $\$x$ be the amount Betty originally had, $x-43-\frac{1}{3}(x-43)=48$,

    $3x - 129 - (x - 43) = 144$, $2x = 230$, $x = 115$. *Ans. Betty originally had $115.*
18. Let $\$x$ be the amount Adam originally had, $\frac{x}{6}\times4=\frac{3}{4}(x-\frac{x}{6})+14$,

    $\frac{2x}{3}=\frac{5x}{8}+14$, $16x = 15x + 336$, $x = 336$. *Ans. Adam originally had $336.*
19. Let $\$x$ be the original price of an orange, then the new price is $\$(x + 0.4)$,
    $35x + 3 = 30(x + 0.4)$, $5x = 9$, $x = 1.8$. *Ans. Original price of an orange is $1.8.*
20. Let $x$ be the number of $2 coins, then the number of $5 coins is $(x + 10)$,

$2x + 5(x + 10) = 134$, $7x = 84$, $x = 12$. $\therefore$ Number of \$5 coins = 12 + 10 = 22.
*Ans. The total number of coins is 12 + 22 = 34.*

21. Let \$$x$ be the amount received by the younger son, then the amount received by the elder son is \$$(6000 - x)$, $\frac{1}{3}(6000 - x) = x + 500$, $6000 - x = 3x + 1500$, $4500 = 4x$, $x = 1125$. *Ans. The amount received by the younger son is \$1125.*

22. Let $x$ be the number of biscuits in the smaller portion, then the number of biscuits in the bigger portion is $(50 - x)$, $4x = 3(50 - x) + 4$, $7x = 154$, $x = 22$.
*Ans. The number of biscuits in the smaller portion is 22.*

23. Let \$$x$ be the amount received by the elder brother, then the younger brother received \$$(800 - x)$, $800 - x = \frac{3}{4}x + 30$, $770 = \frac{7}{4}x$, $3080 = 7x$, $x = 440$.
*Ans. The elder and the younger brother received \$440 and \$360 respectively.*

24. Let $x$ h be the time taken he had ridden at the faster speed, then he had ridden $(10 - x)$ h at the slower speed, $7(10 - x) + 12x = 95$, $5x = 25$, $x = 5$.
*Ans. He had ridden 5 h at the faster speed.*

25. Let $x$ km be the distance between A and B, $\frac{x}{12} + \frac{x}{10} = 3\frac{2}{3}$, $\frac{x}{12} + \frac{x}{10} = \frac{11}{3}$, $5x + 6x = 220$, $11x = 220$, $x = 20$. *Ans. A and B is 20 km apart.*

26. Let $x$ km/h be the power speed of the boat, $x + 6 = 2(x - 6)$, $x = 18$.
*Ans. The power speed of the boat is 18 km/h.*

27. Let $x$ km/h be Car A's speed, then Car B's speed is $2x$ km/h, $4(x + 2x) = 540$, $3x = 135$, $x = 45$. *Ans. Car A's speed is 45 km/h and Car B's speed is 90 km/h.*

28. Let $x$ h be the time taken, $(45 + 75)x = 408$, $120x = 408$, $x = 3.4$.
*Ans. They will meet after 3.4 h.*

29. Let $x$ m/s be the speed of Jason, $(x - 7) \times 2 \times 60 = 240$, $x - 7 = 2$, $x = 9$.
*Ans. The speed of Jason is 9 m/s.*

30. Let $x$ be the smallest number, then the other two numbers are $(x + 2)$ and $(x + 4)$, $x + (x + 2) + (x + 4) = 27$, $3x = 21$, $x = 7$. $\therefore$ The other two numbers are 9 and 11.
*Ans. Their product is $7 \times 9 \times 11 = 693$.*

31. Let $x$ be the tens digit of the original number, then the units digit is $(11 - x)$, $10x + (11 - x) = [10(11 - x) + x] + 27$, $9x + 11 = -9x + 137$, $18x = 126$, $x = 7$.
$\therefore$ The units digit is $11 - 7 = 4$. *Ans. The original two-digit number is 74.*

32. Let $x$ be the present age of the son, then the father is $5x$ years old now, $5x + 13 = 2(x + 13) + 8$, $3x = 21$, $x = 7$.
*Ans. The age of the father when his son was born was $5(7) - 7 = 28$.*

33. Let \$$x$ be the cost of a scarf, then the cost of a brooch is \$$(243 - 2x)$, $x = 2(243 - 2x) + 9$, $5x = 495$, $x = 99$, $\therefore$ The cost of a brooch = 243 – 2(99) = \$45. *Ans. The difference in cost between a scarf and a brooch is 99 – 45 = \$54.*

34. Let $x$ be the number of \$1 coins Mandy has, then the number of 50-cent coins is

$90 - 16 - x = 74 - x$, $0.2(16) + 0.5(74 - x) + 1(x) = 53.2$, $0.5x = 13$, $x = 26$.

*Ans. Mandy has 26 $1 coins.*

35. Let $x$ be the number of 50-cent coins, then there are $(2x - 4)$ $1 coins and the number of 20-cent coins is $2x - 4 + 5 = 2x + 1$,

$0.2(2x + 1) + 0.5x + 1(2x - 4) = 54.2$, $2.9x = 58$, $x = 20$.

*Ans. The number of $1 coins is 2(20) – 4 = 36.*

36. Let $\$x$ be the price of 1 kg of Type A coffee, then the price of 1 kg of Type B coffee is $\$(x + 30)$, $7x + 8(x + 30) = 33.5 \times 60$, $15x = 1770$, $x = 118$.

*Ans. The price of 1 kg of Type A coffee is $118.*

37. Let $x$ km be the distance between A and B,

$\frac{x}{8} + \frac{x}{12} = \frac{2x}{11} + \frac{21}{60}$, $\frac{x}{8} + \frac{x}{12} - \frac{2x}{11} = \frac{7}{20}$, $\frac{7x}{264} = \frac{7}{20}$, $x = \frac{7}{20} \times \frac{264}{7}$, $x = 13.2$.

*Ans. The distance between A and B is 13.2 km.*

38. Let $x$ km/h be the speed for the return journey and $d$ km be the distance of the journey,

$\frac{d}{40} + \frac{d}{x} = \frac{2d}{60}$, $\frac{1}{40} + \frac{1}{x} = \frac{1}{30}$, $\frac{1}{x} = \frac{4}{120} - \frac{3}{120}$, $\frac{1}{x} = \frac{1}{120}$, $x = 120$.

*Ans. The speed for the return journey should be 120 km/h.*

39. Let $x$ h be the time taken for the later train to catch up the earlier train,

$250x = 100(x + 3)$, $150x = 300$, $x = 2$.

*Ans. It will catch up the earlier train at 9pm.*

40. Let $x$ min be the time taken and $y$ m/min be the original speed, $\frac{3}{4} y \cdot x = 2(15y)$,

$3x = 120$, $x = 40$. *Ans. The required time taken is 40 min.*

41. Let $x$ h be the time Q takes to overtake P, $50x = 40(x + 0.5)$, $10x = 20$, $x = 2$,

$\therefore$ P has travelled 2.5 h before he is overtaken by Q.

Let $y$ h be the time R starts after P, $60(2.5 - y) = 40(2.5)$, $50 = 60y$, $y = \frac{5}{6}$.

*Ans. R must start* $\frac{5}{6}$ *h after P.*

42. $\frac{x}{4} \times 4 \times 16 = 4x + 60$, $16x = 4x + 60$, $12x = 60$, $x = 5$

43. Let $x$ be the number of $5 coins, $\frac{x}{12} = \frac{x}{15} + 15$, $5x = 4x + 900$, $x = 900$.

*Ans. The amount of money in the safe is 5(900) = $4500.*

44. Let $x$ be the total number of books, $\frac{x-2}{14} = \frac{x-6}{18} + 4$, $9(x - 2) = 7(x - 6) + 504$,

$2x = 480$, $x = 240$. *Ans. The total number of books is 240.*

45. Let $x$ be the number of cards John has, then David has $(240 - x)$ cards,

$x - 20 = 2[(240 - x) + 20]$, $x - 20 = 2(260 - x)$, $3x = 540$, $x = 180$.

$\therefore$ John has 180 cards and David has $240 - 180 = 60$ cards.

Let $y$ be the number of cards John should give to David more,
$3[180 – (20 + y)] = 60 + (20 + y)$, $3(160 – y) = 80 + y$, $400 = 4y$, $y = 100$.
*Ans. John should give David 100 more flashing cards.*

46. (a) The difference = $2 \times 7 = 14$.
    (b) Let $y$ be the number of marbles Mary has, then Paul has $(y + 14)$ marbles,
    $(y + 14) + 4 = 2(y – 4)$, $y = 26$. *Ans. $x = 26 + (26 + 14) = 66$.*
    (c) The number of marbles Mary has = 26.

47. (a) Let $x$ be the actual number of wrong answers, then the actual numbers of correct answers and unanswered questions are $(40 – 2x)$ and $x$ respectively,
    $4[(40 – 2x) + 2] + (–2)(x – 2) + (–1)(x) = 106$, $4(42 – 2x) – 2x + 4 – x = 106$,
    $–11x = –66$, $x = 6$.
    *Ans. She should have got $4(40 – 2 \times 6) + (–2)(6) + (–1)(6) = 94$ marks.*
    (b) The actual number of correct answers = $40 – 2 \times 6 = 28$.

48. (a) Let $x$ be the original number of passengers, $\frac{3}{4}(\frac{4}{5}x + 4) + 1 = \frac{2}{3}x$,
    $\frac{3x}{5} + 4 = \frac{2x}{3}$, $9x + 60 = 10x$, $x = 60$. *Ans. Original number of passengers is 60.*
    (b) The total fares collected = $20(60 + 4 + 1) = \$1300$.

## Unit 8: Percentages (1)

1.

| Fraction | $\frac{7}{8}$ | $\frac{27}{2500}$ | $\frac{19}{40}$ | $10\frac{1}{5}$ | $3\frac{1}{4}$ | $\frac{33}{400}$ |
|---|---|---|---|---|---|---|
| Decimal | 0.875 | 0.0108 | 0.475 | 10.2 | 3.25 | 0.0825 |
| Percentage | 87.5% | 1.08% | 47.5% | 1020% | 325% | $8\frac{1}{4}\%$ |

2. (a) Percentage = $\frac{3}{8} \times 100\% = 37.5\%$ (b) Percentage = $\frac{1}{4} \times 100\% = 25\%$
   (c) Percentage = $\frac{3}{8} \times 100\% = 37.5\%$

3. (a) $= 3 + 0.17 = 3.17$ (b) $= 3 – 0.59 = 2.41$ (c) $= 72 \times \frac{3}{4} = 54$
   (d) $= 0.7 \times 0.3 = 0.21$ (e) $= 210 \div 30\% = 210 \div 0.3 = 700$
   (f) $= 130 \times (1 + 15\%) = 130 \times 1.15 = 149.5$ (g) $= \frac{2}{7} \div \frac{13}{10} \times \frac{91}{100} = \frac{2}{7} \times \frac{10}{13} \times \frac{91}{100} = \frac{2}{10} = 0.2$

4. Percentage = $\frac{3}{2.5} \times 100\% = 120\%$

5. The value $= 12 \times 25\% = 12 \times \frac{1}{4} = 3$

6. Let $y$ be the number, $y \times 84\% = 1008$, $y = 1008 \times \frac{100}{84}$, $y = 1200$.
   *Ans. The number is 1200.*

7. Percentage of female members = $\frac{60-15}{60}\times 100\% = 75\%$
8. Volume of water needed = $800\times(1-42\%) = 800\times 0.58 = 464$ mL
9. Let \$$y$ be his monthly income, $y\times(1-20\%) = 7600$, $0.8y = 7600$,
   $y = 9500$. *Ans. His monthly income is \$9500.*
10. Number of girls = $640\times(1-60\%)\times(1-12.5\%) = 640\times\frac{40}{100}\times\frac{87.5}{100} = 224$
11. Let \$$y$ be the amount of money Susan's sister has, $y\times 62\% = 930$,
    $y = 930\div 0.62$, $y = 1500$.
    *Ans. The difference between the money they have is 1500 – 930 = \$570.*
12. Let $y$ be the number of story books Peter has, $y + y\times 40\% = 84$,
    $1.4y = 84$, $y = 60$. *Ans. Peter has 60 story books.*
13. Number of stamps Margaret will have = $80\times(1-45\%)\times(1-25\%) = 33$
14. Percentage change = $\frac{6-18}{18}\times 100\% = \frac{-12}{18}\times 100\% = -66\frac{2}{3}\%$
15. Let $y$ be the number, $y\times(1+67.5\%) = 4020$, $1.675y = 4020$,
    $y = 2400$. *Ans. The number is 2400.*
16. Let $y$ be the number, $y-4 = y\times(1-20\%)$, $y-4 = 0.8y$,
    $0.2y = 4$, $y = 20$. *Ans. The number is 20.*
17. Percentage change = $\frac{50-45}{45}\times 100\% = \frac{5}{45}\times 100\% = 11\frac{1}{9}\%$
18. Let $y$ m be the length of cloth Mrs. Ng should buy,
    $y\times(1-5\%) = 3.8$, $0.95y = 3.8$, $y = 4$. *Ans. Mrs. Ng should buy 4 m of cloth.*
19. Let $y$ be the number, the net percentage change
    $= \frac{y(1+20\%)(1-20\%)-y}{y}\times 100\% = \frac{-0.04y}{y}\times 100\% = -4\%$
20. New area = $8\,(1-10\%)\times 12\,(1-10\%) = 7.2\times 10.8 = 77.76\,\text{m}^2$
21. Population after 2 years = $50000\,(1+10\%)\,(1+10\%) = 60500$
22. Number of Chinese books = $6000\times 70\%\times(1-10\%) = 3780$
23. Let \$$y$ be the cost of the lamp, $y\times(1-15\%) = 714$, $0.85y = 714$,
    $y = 840$. *Ans. The cost of the lamp is \$840.*
24. Selling price of each skirt = $960\,(1+80\%)\div 12 = 960(1.8)\div 12 = \$144$
25. Selling price of all the eggs = $6\div 12\times 1200 = \$600$,
    $\therefore$ Profit percentage = $\frac{600-240}{240}\times 100\% = \frac{360}{240}\times 100\% = 150\%$
26. Amount C paid = $250\,(1+40\%)\,(1+10\%) = 250\,(1.4)\,(1.1) = \$385$
27. Selling price = $5\times 56 + 2.5\times 40 = \$380$
    Let \$$y$ be the total cost, $y\,(1-5\%) = 380$, $y = 380\times\frac{100}{95} = 400$
    *Ans. The total cost is \$400.*

28. Total selling price = 20000 (1 + 20%) + 20000 (1 – 30%)
= 24000 + 14000 = \$38000, total cost = 20000 × 2 = 40000
∴ Loss percentage = $\frac{40000-38000}{40000}\times 100\% = \frac{2000}{40000}\times 100\% = 5\%$

29. Let \$$y$ be the cost of each shirt, cost of 40 shirts = \$$40y$
selling price of all shirts = $30y\times(1 + 10\%) + 10y\times(1 – 20\%) = \$41y$,
∴ Profit percentage = $\frac{41y-40y}{40y}\times 100\% = \frac{y}{40y}\times 100\% = 2.5\%$

30. Selling price = 1000 (1 + 10%) (1 – 10%) = 1000 (1.1) (0.9) = \$990,
∴ The storekeeper lost by $\frac{1000-990}{1000}\times 100\% = 1\%$.

31. Amount the customer can save = 900 × 25% = \$225

32. Let \$$y$ be the marked price, $y \times 20\% = 560$, $y = 560 \div 0.2$, $y = 2800$.
*Ans. The marked price of the sofa is \$2800.*

33. Selling price = 180 (1 – 40%) = 180 (0.6) = \$108

34. Let \$$y$ be the fee for each person,
discount percentage = $\frac{4y-3y}{4y}\times 100\% = \frac{y}{4y}\times 100\% = 25\%$

35. (a) Let \$$y$ be the marked price, $y \times 15\% = 96$, $y = 96 \div 0.15$, $y = 640$.
*Ans. The marked price of the tent is \$640.*
(b) Selling price = 640 – 96 = \$544
(c) Let \$$y$ be the cost of the tent, $y\,(1 + 36\%) = 544$, $1.36y = 544$,
$y = 400$. *Ans. The cost of the tent is \$400.*

36. (a) Let $y$ be the total number of students, $y\times 45\% - y\times 22\% = 253$,
$0.23y = 253$, $y = 1100$. *Ans. The total number of students is 1100.*
(b) Number of students going home by other means = $1100\times(1 – 45\% – 22\ \%)$
= $1100\times 0.33 = 363$.

37. (a) Weight of water left = $4\times 70\%\times(1-30\%) = 4\times 0.7\times 0.7 = 1.96$ kg.
(b) Final weight of water-melon = $4 - 4\times 70\%\times 30\% = 4 - 4\times 0.7\times 0.3 = 3.16$ kg.

38. (a) Total mathematics score = $85\times 20\% + 35\times 80\% = 85\times 0.2 + 35\times 0.8 = 45$ marks.
(b) Let $y$ marks be the examination score, $30\times 20\% + y\times 80\% = 70$,
$6 + 0.8y = 70$, $0.8y = 64$, $y = 80$.
*Ans. The examination score should be 80 marks.*

39. Let $y$ be the number of female workers, ∴ no. of male workers = $y\times 60\% = 0.6y$
total number of workers = $y + 0.6y = 1.6y$,
∴ Percentage of male workers = $\frac{0.6y}{1.6y}\times 100\% = 37.5\%$

40. $\frac{x(1-60\%)}{x-10}\times 100\% = 50\%$, $\frac{0.4x}{x-10} = \frac{1}{2}$, $0.8x = x - 10$, $10 = 0.2x$, ∴ $x = 50$

41. Let $x$ be the number of people in the team, ∴ no. of female workers = $0.5x$

$\frac{0.5x+4}{x}\times 100\% = 52\%$, $(0.5x + 4) \times 100 = 52x$, $50x + 400 = 52x$,

$400 = 2x$, $x = 200$. *Ans. There are 200 people in the team.*

42. Number of students failed in either Mathematics or English

$= 200\times 32\% + 200\times 27\% - 16 = 64 + 54 - 16 = 102$,

$\therefore$ Percentage of students passing both subjects $= \frac{200-102}{200}\times 100\% = 49\%$

43. Let $y$ be the number of students in the school, $y(1-65\%)(1-60\%) = 168$,

$y\times 35\%\times 40\% = 168$, $y = 1200$, $\therefore$ There are 1200 students in the school.

*Ans. Number of boys wearing glasses in the school is* $1200\times 65\%\times 30\% = 234$.

44. Let $a$ cm be the original length of the square, and $y$% be the percentage decrease in the width,

$a(1+25\%)\times a(1-y\%) = a^2$, $125\%\times(1-y\%) = 1$,

$1-y\% = \frac{100}{125}$, $y\% = 0.2$, $y = 20$. *Ans. Percentage decrease in the width is 20%.*

45. Let $y$ be the number of eggs he originally had, $y(1 - 5\%)(1 - 80\%) = 266$,

$y(0.95)(0.2) = 266$, $y = 1400$. *Ans. He originally had 1400 eggs.*

46. Let \$$y$ be the cost of one egg, $\therefore$ the selling price of one egg = $\$\frac{15y}{12}$,

profit percent = $\frac{\frac{15y}{12}-y}{y}\times 100\% = \frac{0.25y}{y}\times 100\% = 25\%$

47. Let $y$% be the highest discount percentage, $400(1-y\%) = 240$, $1 - y\% = 0.6$,

$y\% = 0.4$, $y = 40$. *Ans. The highest discount percentage is 40%.*

48. Let \$$y$ be the cost price, $y(1+60\%) = x(1-20\%)$, $1.6y = 0.8x$, $x = 2y$,

$\therefore$ Profit percent = $\frac{x-y}{y}\times 100\% = \frac{2y-y}{y}\times 100\% = 100\%$

49. The cost = $x \div (1+40\%) = \frac{100x}{140} = \frac{5x}{7}$, the selling price = $x\,(1 - 25\%) = \frac{3x}{4}$,

the profit percentage = $[(\frac{3x}{4} - \frac{5x}{7}) \div \frac{5x}{7}] \times 100\% = \frac{x}{28}\times\frac{7}{5x}\times 100\% = \frac{1}{20}\times 100\% = 5\%$

50. Suppose the shopkeeper set \$$y$ as the marked price,

his cost price = $5000\times(1 - 10\%) = 4500$, $\therefore$ $y(1 - 20\%) = 4500\times(1 + 12\%)$,

$y = 4500\times\frac{112}{100}\times\frac{100}{80}$, y = 6300. *Ans. The marked price of the watch is \$6300.*

51. Let \$$x$ be the cost price and $y$% be the discount percentage, $x(1+25\%)(1-y\%) = x$,

$1.25(1 - y\%) = 1$, $1 - y\% = 0.8$, $y\% = 0.2$, $y = 20$.

*Ans. The discount percentage on the marked price should be 20%.*

52. (a) $x(1+15\%) = 2760$, $x = 2760\times\frac{100}{115}$, $x = 2400$

(b) The amount Peter originally had = 12000 – 2400 = 9600, the amount he gives Emily =

2760 – 2400 = 360

$\therefore$ Percentage decrease = $\frac{360}{9600}\times 100\% = 3.75\%$

53. Let \$$y$ be the amount she originally had,
$y(1 - 40\%)(1 - 20\%) - 342 = 1026$, $y(60\%)(80\%) = 1368$,
$y = 2850$. The total amount she had spent = 2850 – 1026 = 1824.
*Ans. Percentage of money spent altogether is* $\frac{1824}{2850}\times 100\% = 64\%$.

54. $P = (1 + 20\%)\,Q$, $P = (\frac{120}{100})\,Q$, $Q = P \times \frac{100}{120} = P \times (\frac{100}{120} \times 100\%) = P \times (83\frac{1}{3}\%)$

*Ans.* $83\frac{1}{3}\%$ *of P is equal to Q.*

55. Percentage = $\frac{B - A}{A}\times 100\% = \frac{B - B\times 40\%}{B\times 40\%}\times 100\% = \frac{0.6B}{0.4B}\times 100\% = 150\%$
*Ans. B is 150% greater than A.*

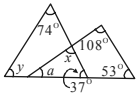
56. $p = q \times 36\% = 0.36q$, $r = q(1 - 60\%) = 0.4q$, $\therefore$ $p$ is smaller than $r$.
$\frac{r - p}{r}\times 100\% = \frac{0.4q - 0.36q}{0.4q}\times 100\% = \frac{0.04q}{0.4q}\times 100\% = 10\%$
*Ans. p is smaller than r by 10%.*

## Unit 9: Basic geometry

1. (a) points (b) straight (c) curve
2. (a) B (b) $b$, $\angle B$, $\angle ABC$, $\angle CBA$
3. (a) $116^\circ$ (b) $320^\circ$
4. (a) $360^\circ \times \frac{1}{5} = 72^\circ$ (b) $360^\circ \times 2\frac{2}{3} = 360^\circ \times 2 + 360^\circ \times \frac{2}{3} = 960^\circ$
5. (a) $360^\circ \times \frac{40}{60} = 240^\circ$ (b) $360^\circ \times \frac{20}{60} = 120^\circ$ (c) $360^\circ \times \frac{2.5}{12} = 360^\circ \times \frac{25}{120} = 75^\circ$
6. (a) obtuse (b) reflex (c) acute
7. (a) equilateral (b) isosceles
8. (a) rhombus (b) rectangle (c) parallelogram
   (d) trapezium (e) square
9. (a) $x + 30^\circ + 40^\circ = 180^\circ$ ($\angle$ sum of $\triangle$), $\therefore$ $x = 110^\circ$
   (b) $3y = 180^\circ$ ($\angle$ sum of $\triangle$), $\therefore$ $y = 60^\circ$
   (c) $a + b + 90^\circ = 180^\circ$ ($\angle$ sum of $\triangle$), $a + a + 90^\circ = 180^\circ$ ($a = b$), $2a = 90^\circ$,
   $\therefore$ $a = \underline{\underline{45^\circ}}$, and $b = a = \underline{\underline{45^\circ}}$
10. $\angle C + 38^\circ 31' + 75^\circ 39' = 180^\circ$ ($\angle$ sum of $\triangle$),
    $\therefore$ $\angle C = 180^\circ - 114^\circ 10' = 65^\circ 50'$

11. $(2x+22)^\circ+(x-15)^\circ+(x+17)^\circ=180^\circ$ ($\angle$ sum of $\triangle$), $4x^\circ+24^\circ=180^\circ$,
$4x^\circ=156^\circ$, $\therefore\ x=39$

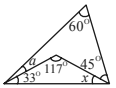
12. (a) $y+37^\circ+74^\circ=180^\circ$ ($\angle$ sum of $\triangle$), $\therefore\ y=\underline{\underline{69^\circ}}$
$a+53^\circ+108^\circ=180^\circ$ ($\angle$ sum of $\triangle$), $\therefore\ a=19^\circ$



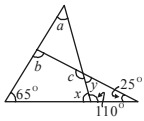

$x+a+37^\circ=180^\circ$ ($\angle$ sum of $\triangle$), $x+19^\circ+37^\circ=180^\circ$, $\therefore\ x=\underline{\underline{124^\circ}}$

(b) $3y=180^\circ$ ($\angle$ sum of $\triangle$), $\therefore\ y=\underline{\underline{60^\circ}}$
$x+x+y+y=180^\circ$ ($\angle$ sum of $\triangle$), $2x+60^\circ+60^\circ=180^\circ$,
$2x=60^\circ$, $\therefore\ x=\underline{\underline{30^\circ}}$


13. $x+33^\circ+117^\circ=180^\circ$ ($\angle$ sum of $\triangle$), $x=30^\circ$
$a+33^\circ+x+45^\circ+60^\circ=180^\circ$ ($\angle$ sum of $\triangle$),
$a+30^\circ+138^\circ=180^\circ$, $\therefore\ a=12^\circ$



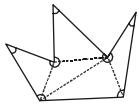

14. $b+65^\circ+25^\circ=180^\circ$ ($\angle$ sum of $\triangle$), $\therefore\ b=\underline{\underline{90^\circ}}$
$x=180^\circ-110^\circ=70^\circ$, $a+x+65^\circ=180^\circ$ ($\angle$ sum of $\triangle$), $\therefore\ a=\underline{\underline{45^\circ}}$
$y+25^\circ+110^\circ=180^\circ$ ($\angle$ sum of $\triangle$), $y=45^\circ$
$\therefore\ c=180^\circ-y=180^\circ-45^\circ=\underline{\underline{135^\circ}}$




15. (a) The statement is correct. If there were 2 interior right angles, the remaining angle would then be $0^\circ$.
(b) Incorrect. An angle between $180^\circ$ and $270^\circ$ is also called a reflex angle.
(c) Incorrect. The 2 pairs of opposite sides of a square are parallel to each other.

16. $\angle R+45^\circ+90^\circ=180^\circ$ ($\angle$ sum of $\triangle$), $\angle R=45^\circ$,
$\therefore\ \triangle PQR$ is isosceles, and PQ = QR = 6 cm
$\therefore$ Area of $\triangle PQR=\dfrac{1}{2}\times 6\times 6\ =18\,cm^2$

17. Divide the heptagon into 5 triangles.
Sum of interior angles of heptagon = sum of interior angles of the 5 triangles
$=\ 180^\circ\times 5\ =\ 900^\circ$



18. (a) $\angle A+\angle ABC+\angle C=180^\circ$ ($\angle$ sum of $\triangle$),
$2\angle A+46^\circ=180^\circ$ ($\angle A=\angle ABC$), $2\angle A=134^\circ$, $\therefore\ \angle A=67^\circ$
(b) $\angle A+\angle ADB+\angle ABD=180^\circ$ ($\angle$ sum of $\triangle$),
$2\angle A+\angle ABD=180^\circ$ ($\angle A=\angle ADB$),
$\angle ABD=180^\circ-2\times 67^\circ=46^\circ$ ; $\angle ABC=\angle A=67°$ ;
$\therefore\ \angle DBC=\angle ABC-\angle ABD=67^\circ-46^\circ=21^\circ$

19. $\angle ADE=90^\circ+60^\circ=150^\circ$,
$\triangle ADE$ is isosceles, $\therefore 2\angle DEA+150^\circ=180^\circ$
($\angle$ sum of $\triangle$, and $\angle DEA=\angle DAE$), $\angle DEA=15^\circ$.
Similarly, $\angle CEB=15^\circ$.
$\therefore\ \angle AEB=\angle DEC-\angle DEA-\angle CEB=60^\circ-15^\circ-15^\circ=30^\circ$

20. (a) $28^\circ+2\angle ABC=180^\circ$ ($\angle$ sum of $\triangle$, and $\angle ABC=\angle ACB$),
$\therefore\ \angle ABC=152^\circ\div 2=76^\circ$
(b) $\angle BCD+\angle DBC+\angle BDC=180^\circ$ ($\angle$ sum of $\triangle$),

$\angle BCD + 2\angle DBC = 180^\circ$ ($\angle DBC = \angle BDC$),

$\therefore \angle BCD = 180^\circ - 2\times 76^\circ = 28^\circ$

(c) $\angle ACB = \angle ABC = 76^\circ$, $\therefore$ $\angle DCE = \angle ACB - \angle BCD = 76^\circ - 28^\circ = 48^\circ$

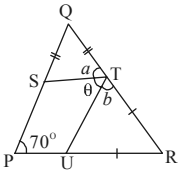
(d) $\angle DEC = \angle DCE = 48^\circ$; $\angle AED = 180^\circ - \angle DEC = 180^\circ - 48^\circ = 132^\circ$;

$\theta + \angle AED + \angle A = 180^\circ$ ($\angle$ sum of $\triangle$),

$\theta + 132^\circ + 28^\circ = 180^\circ$, $\therefore$ $\theta = 20^\circ$

21. $\angle DAB + \angle DBA + \angle DAC + \angle DCA = 180^\circ$ ($\angle$ sum of $\triangle$), $2\angle DAB + 2\angle DAC = 180^\circ$
($\angle DAB = \angle DBA$, $\angle DAC = \angle DCA$), $2(\angle DAB + \angle DAC) = 180^\circ$, $2\angle BAC = 180^\circ$,
$\therefore$ $\angle BAC = 90^\circ$

22. Let $\angle QTS = a$, $\angle RTU = b$.
$\angle Q = 180° - 2a$, $\angle R = 180° - 2b$.
$70° + \angle Q + \angle R = 180°$,
$70° + (180° - 2a) + (180° - 2b) = 180°$,
$250° - 2a - 2b = 0$, $a + b = 125°$.
$a + \theta + b = 180°$ (straight angle),
$\theta + 125° = 180°$, $\therefore$ $\theta = 55°$




23. (a) In an hour, the hour hand moves: $360° \times \frac{1}{12} = 30°$.

$\therefore$ From 3:00 to 4:00, it moves 30°, and from 4:00 to 4:15, it moves: $30° \times \frac{15}{60} = 7.5°$.

$\therefore$ The angle between the two hands = 30° + 7.5° = 37.5°.

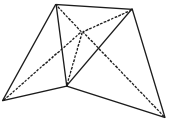
(b) From 6:00 to 7:50, there are 110 min. The hour hand moves: $30° \times \frac{110}{60} = 55°$.

From 7:30 to 7:50, there are 20 min. The minute hand moves: $360° \times \frac{20}{60} = 120°$.

$\therefore$ The angle between the two hands = 120° − 55° = 65°.

24. (a) Min. number of edges = 9.
The number of faces = 5.

(b) Max. number of edges = 12.
The number of faces = 8.
The polyhydron is formed by joining 3 tetrahedrons (四面體)

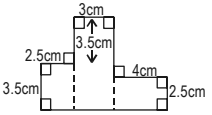


25. Join CD. $\because$ AC = AD = CD, $\therefore$ ACD is an equilateral $\triangle$.
$\therefore$ $\angle CAD = 180^\circ \div 3 = 60^\circ$

## Unit 10: Area and volume of simple figures

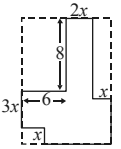
1. (a) 324.8 mm (b) 0.05 cm (c) 104000 cm (d) 1.62 m
(e) 12700 m (f) 0.096 km (g) 5000000 cm (h) 0.0082 km
2. (a) 1800 $mm^2$ (b) 0.65 $cm^2$ (c) 420000 $cm^2$ (d) 0.01 $m^2$
(e) 9060000 $m^2$ (f) 0.037 $km^2$
3. (a) 2660 $mm^3$ (b) 0.24 $cm^3$ (c) 42000000 $cm^3$ (d) 0.0001 $m^3$
(e) 1500 $cm^3$

4. Area $= \frac{6\times3}{2}+\frac{(6+4)\times5}{2}+\frac{4\times4}{2} = 9+25+8 = 42\,\text{cm}^2$




5. Area $= 2.5\times3.5+(3.5+3.5)\times3+4\times2.5$
$= 8.75+21+10 = 39.75\,\text{cm}^2$

6. $[(8+3x+x)+(6+2x+x)]\times2 = 56$, $7x+14 = 28$, $7x = 14$, $\therefore\ x = 2$




7. (a) $\frac{15\times h}{2} = \frac{12\times9}{2}$, $15h = 108$, $\therefore\ h = 7.2$

(b) $\frac{9\times h}{2} = \frac{3\times6}{2}$, $9h = 18$, $\therefore\ h = 2$

8. (a) Area $= 12\times8 = 96\,\text{cm}^2$
(b) $10\times h = 96$, $\therefore\ h = 9.6$

9. Area $= 3\times3-\frac{1\times1}{2}\times4 = 9-2 = 7\,\text{cm}^2$

10. (a) Cost of the carpet $= (12-2-2)\times(10-2-2)\times180 = 48\times180 = \$8640$

(b) Area of the uniform path $= 12\times10-48 = 120-48 = 72\,\text{m}^2$

11. Internal surface area $= (22\times3)\times2+(18\times3)\times2+(22\times18) = 636\ \text{cm}^2$

12. Volume of prism $= \frac{(3+8)\times5}{2}\times4 = 110\,\text{cm}^3$

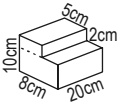
13. (a) Volume $= \frac{(30+9)\times28}{2}\times20 = 546\times20 = 10920\,\text{cm}^3$

Total surface area $= \frac{(30+9)\times28}{2}\times2+(30+28+9+35)\times20 = 3132\,\text{cm}^2$

(b) Volume $= 2\times7\times5+12\times6\times30 = 70+2160 = 2230\,\text{cm}^3$
Total surface area $= (12\times6+6\times30+12\times30)\times2+(5\times2+7\times5)\times2$
$= 1314\,\text{cm}^2$

14. Number of rectangular blocks $= (50\times40\times45)\div(2\times4\times3) = 25\times10\times15 = 3750$

15. Volume $= [5\times2+(10-2)\times8]\times20 = 74\times20 = 1480\,\text{cm}^3$
Total surface area $= [5\times2+(10-2)\times8]\times2+8\times20\times2+10\times20\times2$
$= 868\,\text{cm}^2$




or: Total surface area $= (10\times8+20\times10+8\times20)\times2-[(8-5)\times2]\times2$
$= 868\,\text{cm}^2$

16. Volume $= 15\times30\times20-\frac{(15-5-5)\times(30-5)}{2}\times20 = 9000-1250 = 7750\,\text{cm}^3$

17. Volume $= (7\times7-4\times4)\times8 = 33\times8 = 264\,\text{cm}^3$
Total surface area $= 33\times2+7\times8\times4+4\times8\times4 = 66+224+128 = 418\ \text{cm}^2$

18. (a) Internal volume $= (20-1-1)\times(15-1-1)\times(10-1) = 18\times13\times9 = 2106\ \text{cm}^3$

(b) Volume $= 20\times15\times10-2106 = 3000-2106 = 894\,\text{cm}^3$

19. Height of water level $= 6000\div(24\times25) = 6000\div600 = 10$ cm

20. The area $= 0.08\times100\times100 = 800\text{cm}^2$

Let $x$ cm be the height of the triangle, $\frac{80x}{2} = 800$, $40x = 800$, $x = 20$.

*Ans. The height of the triangle is 20 cm.*

21. Let $x$ cm be the upper base, $\frac{(22+x)\times5.5}{2} = 99$, $22+x = 36$, $x = 14$.

*Ans. The upper base is 14 cm.*

22. Let $x$ cm be the side of the new square, $x^2 = 500(1-20\%)$, $x^2 = 400$,
$\therefore\ x = 20$, $\therefore$ Perimeter of the new square $= 20 \times 4 = 80$ cm

23. Let $x$ cm be the side of each cube, $3x^3 = 4 \times 6 \times 8$, $x^3 = 64$, $x^3 = 4^3$, $\therefore\ x = 4$,
$\therefore$ Total surface area of the three cubes $= (4 \times 4 \times 6) \times 3 = 288\,\text{cm}^2$

24. 1m = 100cm, $\therefore$ Volume of water needed $= 80 \times 35 \times 100 = 280000\,\text{cm}^3$

25. 120cm = 1.2m, 100cm = 1m,
$\therefore$ Height of the water level $= 1.5 \div (1.2 \times 1) = 1.25$ m

26. Height of water $= (4 \times 10 \times 4 \times 6) \div 50 = 19.2$ cm

27. 12mm = 1.2cm, $\therefore$ Length $= (40 \times 1.2) \div 30 = 48 \div 30 = 1.6$ cm

28. Volume of the water $= \dfrac{(1+2.5) \times 25}{2} \times 12 = 525\,\text{m}^3$

29. 4km = 4000m, 14cm = 0.14m, $\therefore$ Time required $= (4000 \times 3 \times 0.14) \div 56 = 30$ h

30. Weight of the box $= [(60 \times 50 \times 40) - (60-5-5) \times (50-5-5) \times (40-5)] \times 0.8$
$= (120000 - 70000) \times 0.8 = 40000$ g $= 40$ kg

31. $\dfrac{4.8 \times 10}{2} = \dfrac{8 \times \text{BC}}{2}$, 8BC = 48, BC= 6, $\therefore$ The area of ABCD = 8×6 = 48 cm$^2$

32. Let $h$ cm be the height of $\triangle$ABC, $\therefore$ $h$ cm is also the height of $\Delta$ABD.
$\dfrac{12.5 \times 4}{2} = \dfrac{5 \times h}{2}$, $h = \dfrac{50}{5} = 10$ $\therefore$ Area of $\triangle$ABC $= \dfrac{(5+6) \times 10}{2} = 55\,\text{cm}^2$

33. BC = 60 – 16 – 24 = 20 cm, $\dfrac{20 \times \text{AD}}{2} = \dfrac{24 \times x}{2}$, 20AD = 24$x$, $\therefore$ AD = 1.2$x$ (or: $\dfrac{6}{5}x$)

34. Area of the parallelogram $= 4 \times 10 = 40$ sq. units

35. Shaded area is a trapezium. Area of shaded region $= \dfrac{(4+7) \times 4}{2} = 22\,\text{cm}^2$

36. (a) Area of $\triangle$BCG $= \dfrac{5 \times (8-5)}{2} = 7.5\,\text{cm}^2$

(b) Shaded area = Area of ABCD + Area of $\Delta$DGE + Area of $\triangle$BCG – Area of $\triangle$ABE
$= 5^2 + \dfrac{8 \times 8}{2} + 7.5 - \dfrac{5 \times (5+8)}{2}$
$= 25 + 32 + 7.5 - 32.5 = 32\,\text{cm}^2$

37. (a) Minimum number of tiles required $= (600 \times 450) \div (20 \times 20) = 675$

(b) $\because$ $600 \div 20 = 30$ and $450 \div 20 = 22.5$,
$\therefore$ Maximum number of whole tiles $= 30 \times 22 = 660$

38. (a) Area of square ABCD $= 6 \times 6 + 16 \times 4 = 36 + 64 = 100\,\text{cm}^2$

(b) The side of square ABCD $= \sqrt{100} = 10$ cm
$\therefore$ Height of each trapezium $= \dfrac{\text{DC} - \text{SR}}{2} = (10-6) \div 2 = 2$ cm

39. (a) No. of cubes = 8 + 3 + 1 = 12, $\therefore$ Volume = 12 × 1 = 12 cm$^3$
No. of squares: bottom = 8, back = 7, left side = 5, right side = 3, others = 19,

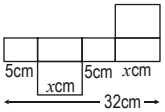
$\therefore$ Total surface area = 8 + 7 + 5 + 3 + 19 = 42 cm$^2$

(b) No. of cubes = 11, $\therefore$ Volume = 11 × 1 = 11 cm$^3$

No. of squares: bottom = 5, left back = 7, right back = 8, others = 20,

$\therefore$ Total surface area = 5 + 7 + 8 + 20 = 40 cm$^2$

40. (a) Volume of the triangular prism formed = $\frac{6\times 8}{2}\times 5 = 120\,\text{cm}^3$




(b) Let the unknown dimension of the cuboid be $x$ cm,

$5 + x + 5 + x = 32$, $2x = 22$, $x = 11$,

$\therefore$ Volume of the cuboid = 3 × 5 × 11 = 165 cm$^3$

41. Weight of the remaining solid = $[10\times 12-\frac{(4+8)\times 6}{2}]\times 32\times 0.8$

$= (120-36)\times 32\times 0.8 = 2150.4$ g

42. (a) Volume of the remaining solid = $80\times 80\times 80-20\times 20\times 20\times 6$

$= 512000 - 48000 = 464000\,\text{cm}^3$

(b) Total surface area = $80\times 80\times 6+20\times 20\times 4\times 6 = 38400 + 9600 = 48000\,\text{cm}^2$

43. Volume = $24\times 15\times 8-8\times 16\times 2 = 2880 - 256 = 2624\,\text{cm}^3$

Total surface area = $(24\times 8+15\times 8+24\times 15+8\times 2+16\times 2)\times 2 = 1440\,\text{cm}^2$

44. Volume = $8\times 4\times 3 = 96\,\text{cm}^3$

Total surface area = $(8\times 4+3\times 5+8\times 3)\times 2 = 71\times 2 = 142\,\text{cm}^2$

45. Volume = $(4\times 10+\frac{5\times 5}{2})\times 2 = 52.5\times 2 = 105\,\text{cm}^3$

Total surface area = $(4\times 10+\frac{5\times 5}{2}+10\times 2+1\times 2+5\times 2)\times 2+4\times 2 = 177\,\text{cm}^2$

46. Area of the water surface = $(12\times 12\times 12)\div 8 = 1728\div 8 = 216\,\text{cm}^2$

47. (a) Height of the triangular prism = $(5\times 5\times 5)\div 20 = 125\div 20 = 6.25$ cm

(b) Original area = $5^2\times 6 = 150$ cm$^2$, new area = 24 × 6.25 + 20 × 2 = 190 cm$^2$,

$\therefore$ % increase in total surface area = $\frac{190-150}{150}\times 100\% = \frac{40}{150}\times 100\% = 26\frac{2}{3}\%$

48. Capacity of the swimming pool = $\frac{(1.2+2.6)\times 50}{2}\times 12 = 95\times 12 = 1140\,\text{m}^3$

= 1140000 L, $\therefore$ Time required = $1140000\div 8000 = 142.5$ min

49. Let $x$ be the number of marbles, $8x > 14\times 18\times 15\times(1-\frac{1}{3})$, $8x > 2520$, $x > 315$.

*Ans. The minimum number of marbles required is 316.*

50. (a) $\frac{h\times 4}{2}\times 10 = \frac{1}{8}\times 4\times 10\times(12-h)$, $20h = 5(12 - h)$, $20h = 60 - 5h$, $25h = 60$,

$\therefore$ $h = 2.4$

(b) Minimum volume of wood = $(12\times 4\times 10)-[(12-2.4)\times 4+\frac{2.4\times 4}{2}]\times 10$

$= 480 - 432 = 48\,\text{cm}^3$

51. $(12\times 15-4\times 6)\times(10+h) = 12\times 15\times 10$, $156(10 + h) = 1800$, $1560 + 156h = 1800$,

$156h = 240, \quad \therefore \quad h = 1\frac{7}{13}$

## Unit 11: Rate and ratio

1. (a) $= \frac{1200}{25} = 48$ (b) $= \frac{128}{32} = 4$ (c) $= \frac{48}{30} = 1.6$

2. 90 km/day $= \frac{90 \times 1000\text{ m}}{24 \times 60\text{ min}} = 62.5$ m/min, $\therefore$ 70 m/min is the higher rate

3. Speed $= \frac{15}{6} = 2.5$ m/s, $\therefore$ Time taken $= \frac{100}{2.5} = 40$ s

4. (a) Time taken $= \frac{100}{80} = 1.25$ h

   (b) Distance travelled $= 80 \times \frac{3}{4} = 60$ km

5. (a) $= \frac{96}{24} : \frac{360}{24} = 4 : 15$

   (b) $= 9.8 \times 10 : 4.2 \times 10 = \frac{98}{14} : \frac{42}{14} = 7 : 3$

   (c) $= \frac{3}{10} : \frac{32}{15} = \frac{3}{10} \times 30 : \frac{32}{15} \times 30 = 9 : 64$

   (d) $= (3 \times 60 \times 60)$ seconds : 200 seconds $= \frac{10800}{200} : \frac{200}{200} = 54 : 1$

   (e) $= (2.5 \times 100)\text{¢} : 15\text{¢} = \frac{250}{5} : \frac{15}{5} = 50 : 3$

   (f) $= 240\text{ g} : (1.2 \times 1000)\text{ g} = \frac{240}{240} : \frac{1200}{240} = 1 : 5$

   (g) $= (3 \times 1000)\text{ m} : 800\text{ m} = \frac{3000}{200} : \frac{800}{200} = 15 : 4$

   (h) $= (0.4 \times 10000)\text{ cm}^2 : 500\text{ cm}^2 = \frac{4000}{500} : \frac{500}{500} = 8 : 1$

6. (a) $\frac{x}{3} = \frac{7}{2}, \quad 2x = 21, \quad \therefore \quad x = 10\frac{1}{2}$

   (b) $\frac{6}{y} = \frac{5}{4}, \quad 24 = 5y, \quad \therefore \quad y = 4\frac{4}{5}$

7. (a) $\frac{800g}{1000g} = \frac{\$a}{\$120}, \quad \frac{4}{5} = \frac{a}{120}, \quad 480 = 5a, \quad \therefore \quad a = 96$

   (b) $\frac{(7 \times 100000)\text{ cm}}{3\text{ cm}} = \frac{(2 \times 100000)\text{ cm}}{k\text{ cm}}, \quad 7k = 6, \quad \therefore \quad k = \frac{6}{7}$

8. Let $x$ be the original number, new number : original number

$= x(1+\frac{1}{8}) : x = \frac{9x}{8} : x = \frac{9}{8} \times 8 : 1 \times 8 = 9 : 8$

9. Paul's weight : his brother's weight = 48 : (48 – 16) = $\frac{48}{16} : \frac{32}{16} = 3 : 2$

10. (a) Speed of car = $33 \div \frac{1}{3} = 99$ km/h; speed of train = $\frac{270}{2} = 135$ km/h

    (b) Speed of car : speed of train = 99 : 135 = $\frac{99}{9} : \frac{135}{9} = 11 : 15$

11. Let $x$ be the number of boys in the group, $\frac{x}{x+10} = \frac{4}{5}$, $5x = 4x + 40$, $x = 40$.

    *Ans. Number of boys in the group is 40.*

12. Let \$$x$ be his monthly salary, $x(\frac{3}{5+3}) = 4800$, $3x = 4800 \times 8$, $x = 12800$.

    *Ans. His monthly salary is \$12800.*

13. Let $x$ litres be the volume of water added, $\frac{6}{4+x} = \frac{2}{3}$, $18 = 2(4 + x)$, $10 = 2x$, $x = 5$.

    *Ans. 5 litres of water must be added.*

14. New salary rate = $\$\frac{276}{10}$/h = \$ 27.6 / h,

    $\therefore$ New salary : original salary = 27.6 : 24 = $\frac{276}{12} : \frac{240}{12} = 23 : 20$

15. (a) $x : y = 5 \times 5 : 3 \times 5 = 25 : 15$, $y : z = 5 \times 3 : 2 \times 3 = 15 : 6$,
    $\therefore$ $x : y : z = 25 : 15 : 6$

    (b) $y : x = 4 : 3$, $x : z = 1 \times 3 : 2 \times 3 = 3 : 6$, $\therefore$ $x : y : z = 3 : 4 : 6$

16. (a) $4a = 3b$, $\frac{a}{b} = \frac{3}{4}$, $\therefore$ $a : b = 3 : 4$ (b) $\frac{4}{5} = \frac{a}{b}$, $\therefore$ $a : b = 4 : 5$

    (c) $7b = a$, $\frac{7}{1} = \frac{a}{b}$, $\therefore$ $a : b = 7 : 1$

    (d) $4a + 2b = 21a - 7b$, $9b = 17a$, $\frac{9}{17} = \frac{a}{b}$, $\therefore$ $a : b = 9 : 17$

17. $x : 12 = 4 : y$, $\frac{x}{12} = \frac{4}{y}$, $\therefore$ $xy = 48$

18. A : B = 1 : 3, B : C = 1 : 2 = $1 \times 3 : 2 \times 3 = 3 : 6$, $\therefore$ A : B : C = 1 : 3 : 6

    $\therefore$ The amount B gets = $500 \times \frac{3}{1+3+6} = 500 \times \frac{3}{10} = \$150$

19. The scale = 4 cm : 2 km = 4 cm : 200000 cm = 1 : 50000

20. Map distance of a runway = $2.5 \text{ km} \times \frac{1}{100000} = \frac{250000 \text{ cm}}{100000} = 2.5$ cm

21. Actual distance = 3 mm × 2500000 = 7500000 mm = 7.5 km

22. Length of the hall = 5 cm × 400 = 2000 cm = 20 m

Width of the hall = $4\text{ cm} \times 400 = 1600\text{ cm} = 16\text{ m}$

$\therefore$ Actual area of the hall = $20 \times 16 = 320\text{ m}^2$

23. Distance travelled = $60 \times \dfrac{90}{60} = 90\text{ km}$,

$\therefore$ Volume of petrol used = $\dfrac{90}{6} = 15\text{ litres}$

24. No. of \$1 coins = $240 \times \dfrac{8}{8+3+4} = 128$, no. of 50¢ coins

$= 240 \times \dfrac{3}{8+3+4} = 48$, no. of 20¢ coints = $240 \times \dfrac{4}{8+3+4} = 64$,

$\therefore$ Total amount = $128 \times 1 + 48 \times 0.5 + 64 \times 0.2 = \$164.8$

25. Average speed = $\dfrac{5}{1000}\text{km} \div \dfrac{3}{3600}\text{h} = \dfrac{5}{1000} \times \dfrac{3600}{3}\text{ km / h} = 6\text{ km/h}$

26. Let $2k$ be the present age of Mary, then the present age of Lily is $3k$,

$\dfrac{2k+4}{3k+4} = \dfrac{5}{7}$, $14k + 28 = 15k + 20$, $k = 8$, $\therefore$ $2k = 16$, $3k = 24$.

*Ans. The present ages of Mary and Lily are 16 and 24 respectively.*

27. Let $3k$ be the length of the smaller part, then the length of the larger part is $7k$,

$\therefore$ Ratio of the three parts = $3k : 7k \times \dfrac{2}{2+3} : 7k \times \dfrac{3}{2+3}$

$= 3k \times 5 : \dfrac{14k}{5} \times 5 : \dfrac{21k}{5} \times 5 = 15 : 14 : 21$

28. $\dfrac{4}{a+1} \div \dfrac{3}{a} = \dfrac{7}{6}$, $\dfrac{4a}{3(a+1)} = \dfrac{7}{6}$, $24a = 21a + 21$, $3a = 21$, $\therefore$ $a = 7$

29. $a : b = 3 \times 25 : 2 \times 25 = 75 : 50$, $a : c = \dfrac{5}{2} \times 30 : \dfrac{2}{5} \times 30 = 75 : 12$,

$\therefore$ $a : b : c = 75 : 50 : 12$, $b : c = 50 : 12 = 25 : 6$; but $b : d = 6 : 7$,

$\therefore$ $b : c : d = 25 \times 6 : 6 \times 6 : 7 \times 25 = 150 : 36 : 175$, $\therefore$ $c : d = 36 : 175$

30. $2p = 3q$, $p : q = 3 : 2 = 3 \times 9 : 2 \times 9 = 27 : 18$,

$8q = 9r$, $q : r = 9 : 8 = 9 \times 2 : 8 \times 2 = 18 : 16$, $\therefore$ $p : q : r = 27 : 18 : 16$

31. $\dfrac{1}{a} : \dfrac{1}{b} = 3 : 4$, $\dfrac{1}{a} \div \dfrac{1}{b} = \dfrac{3}{4}$, $\dfrac{b}{a} = \dfrac{3}{4}$, $\therefore$ $a : b = 4 : 3$, $\dfrac{1}{b} : \dfrac{1}{c} = 4 : 5$, $\dfrac{1}{b} \div \dfrac{1}{c} = \dfrac{4}{5}$,

$\dfrac{c}{b} = \dfrac{4}{5}$, $\therefore$ $b : c = 5 : 4$, $\therefore$ $a : b : c = 4 \times 5 : 3 \times 5 : 4 \times 3 = 20 : 15 : 12$

(OR: $\dfrac{1}{a} = 3k$, $\dfrac{1}{b} = 4k$, $\dfrac{1}{c} = 5k$, $\therefore$ $a : b : c = \dfrac{1}{3k} : \dfrac{1}{4k} : \dfrac{1}{5k} =$

$\dfrac{1}{3k} \times 60k : \dfrac{1}{4k} \times 60k : \dfrac{1}{5k} \times 60k = 20 : 15 : 12$)

32. Length scale of map = 1 mm : 0.25 m = 0.1 cm : 0.25 m = 1 cm : 2.5 m,

area scale of map = $(1 \times 1)\text{ cm}^2 : (2.5 \times 2.5)\text{ m}^2 = 1\text{ cm}^2 : 6.25\text{ m}^2$,

$\therefore$ Map area of the field = $400 \times \frac{1}{6.25} = 64\,\text{cm}^2$

33. Area scale of map = $(4 \times 4)\,\text{cm}^2 : (1 \times 1)\,\text{km}^2 = 16\,\text{cm}^2 : 1\,\text{km}^2$,

map area = $(2 \times 3)\,\text{cm}^2 = 6\,\text{cm}^2$, $\therefore$ Actual area = $6 \times \frac{1}{16} = \frac{3}{8}\,\text{km}^2$

34. $x = y(1 + 25\%)$, $\frac{x}{y} = 1.25 = \frac{5}{4}$, $\therefore$ $x : y = 5 : 4$,

$x = z(1 - 20\%)$, $\frac{x}{z} = 0.8 = \frac{4}{5}$, $\therefore$ $x : z = 4 : 5$,

$\therefore$ $x : y : z = 5 \times 4 : 4 \times 4 : 5 \times 5 = 20 : 16 : 25$

35. $2a + 3b = 3a + b$, $a = 2b$,

$\therefore$ $\sqrt{4a + b} : \sqrt{3a - 2b} = \sqrt{4(2b) + b} : \sqrt{3(2b) - 2b} = \sqrt{9b} : \sqrt{4b} = 3 : 2$

36. Let $2a = 3b = 7c = k$, then $a = \frac{k}{2}$, $b = \frac{k}{3}$, $c = \frac{k}{7}$,

$\therefore$ $(a - b + c) : (a + b - c) = (\frac{k}{2} - \frac{k}{3} + \frac{k}{7}) : (\frac{k}{2} + \frac{k}{3} - \frac{k}{7}) = \frac{13}{42} : \frac{29}{42} = 13 : 29$

37. Let $\frac{a}{b} = \frac{c}{d} = k$, then $a = bk$, $c = dk$, L.H.S. = $\frac{a + c}{b + d} = \frac{bk + dk}{b + d} = \frac{k(b + d)}{b + d} = k$,

R.H.S. = $\frac{a - c}{b - d} = \frac{bk - dk}{b - d} = \frac{k(b - d)}{b - d} = k$ = L.H.S., $\therefore$ $\frac{a + c}{b + d} = \frac{a - c}{b - d}$

38. $\because$ The largest angle = $180^\circ \times \frac{8}{3 + 8 + 1} = 180^\circ \times \frac{8}{12} = 120^\circ > 90^\circ$,

$\therefore$ It is an obtuse-angled triangle.

39. Let $3k$ cm and $4k$ cm be the width and the length of the rectangle respectively,
$(3k)(4k) = 432$, $12k^2 = 432$, $k^2 = 36$, $k = 6$, $\therefore$ $3k = 18$, $4k = 24$.
*Ans. Perimeter of the rectangle is 2(18+24) = 84 cm.*

40. Let $r$ : 1 be the ratio of the two types of tea,

$75(\frac{r}{1 + r}) + 50(\frac{1}{1 + r}) = 60$, $75r + 50 = 60(1 + r)$, $15r = 10$, $r = \frac{2}{3}$.

*Ans. The ratio of the mixture is* $\frac{2}{3} : 1 = 2 : 3$.

41. The cost of the mixture = 60 ÷ (1 + 25%) = \$48.
Let the cost price of coffee B be \$$y$.

$15 \times \frac{2}{2 + 3} + y \times \frac{3}{2 + 3} = 48$, $30 + 3y = 240$, $y = 70$.

*Ans. The cost price of coffee B is \$70/kg.*

42. $150 \times \frac{m}{m + n} + 200 \times \frac{n}{m + n} = 150\,(1 + 10\%) \times \frac{m}{m + n} + 200\,(1 - 30\%) \times \frac{n}{m + n}$,

$150m + 200n = 165m + 140n$, $60n = 15m$, $\frac{m}{n} = \frac{60}{15} = \frac{4}{1}$, $\therefore\ m : n = 4 : 1$

43. Let \$$2k$ and \$$k$ be the daily wages of a man and a woman respectively,

$\therefore$ Ratio of hourly wages of a man and a woman $= \frac{2k}{10} : \frac{k}{8} = \frac{1}{5} \times 40 : \frac{1}{8} \times 40 = 8 : 5$

44. Let $d$ m be the distance between home and school,

total time taken $= \frac{d}{x} + \frac{d}{y} = \frac{d(x+y)}{xy}$ seconds,

$\therefore$ His average speed $= 2d \div \frac{d(x+y)}{xy} = 2d \times \frac{xy}{d(x+y)} = \frac{2xy}{x+y}$ m/s

45. Let $x$ litres be the amount of water, amount of water 1 man needs per day $= \frac{x}{6 \times 8}$

$= \frac{x}{48}$ litres, amount of water 1 boy needs per day $= \frac{x}{8 \times 10} = \frac{x}{80}$ litres,

$\therefore$ Time lasting $= x \div (\frac{x}{48} \times 12 + \frac{x}{80} \times 4) = x \div \frac{3x}{10} = x \times \frac{10}{3x} = 3\frac{1}{3}$ days

46. Work done by A : work done by B : work done by C

$= \frac{1}{10} : \frac{1}{20} : \frac{1}{30} = \frac{60}{10} : \frac{60}{20} : \frac{60}{30} = 6 : 3 : 2$

$\therefore$ The amount A will receive $= 11000 \times \frac{6}{6+3+2} = \$6000$

47. If there are 60 chickens, the remaining food can last for 30 – 10 = 20 days,

$\therefore$ The number of days for 50 chickens $= \frac{60 \times 20}{50} = 24.$

*Ans. The remaining food can last for 24 – 20 = 4 days more.*

48. Let BQ = $x$, then QC = $36 - x$, $\frac{(18+x) \times \text{AB}}{2} : \frac{[18+(36-x)] \times \text{AB}}{2} = 3 : 2$,

$\frac{18+x}{54-x} = \frac{3}{2}$, $36 + 2x = 162 - 3x$, $5x = 126$, $x = 25.2$.

*Ans. The value of BQ is 25.2.*

49. Let the longer side and the shorter side of the original rectangle be $b$ cm and $a$ cm respectively.

The longer side of the new rectangle will be $a$ cm, and the shorter side will be $\frac{b}{2}$ cm,

$a : \frac{b}{2} = b : a$, $\frac{2a}{b} = \frac{b}{a}$, $2a^2 = b^2$, $\frac{a^2}{b^2} = \frac{1}{2}$, $\frac{a}{b} = \frac{1}{\sqrt{2}}$.

*Ans. The ratio of lengths of the original rectangle is* $1 : \sqrt{2}$. [OR: $\sqrt{2} : 1$]

50. Let AB = $x$, BC = $2x$; let PQ = $3y$, QR = $2y$, $2(x+2x) = 2(3y+2y)$, $3x = 5y$, $x = \frac{5y}{3}$,

$\therefore$ Area of ABCD : area of PQRS $= x(2x) : (3y)(2y) = x^2 : 3y^2$

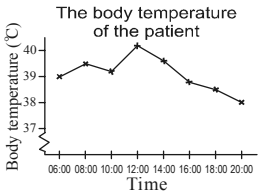
$= (\frac{5y}{3})^2 : 3y^2 = \frac{25}{9} : 3 = \frac{25}{9} \times 9 : 3 \times 9 = 25 : 27$

## Unit 12: Introduction to statistics

1. (a) 1 student
   (b) History
   (c) Total number of students
   $= 3 + 8 + 14 + 6 + 8 = 39$
   (d) Percentage $= \frac{14}{39} \times 100\% = 35.9\%$

2. (a) Total profit = 6 + 4 + 7 + 10 = 27 million dollars
   (b) Difference in profit = 10 – 4 = 6 million dollars
   (c) Percentage change $= \frac{7-4}{4} \times 100\% = \frac{3}{4} \times 100\% = 75\%$ (increase)

3. (a)




   (b) Before 12:00, the body temperature bounced between 39°C and 40.2°C. After that, it decreased gradually and reached 38°C at 20:00.

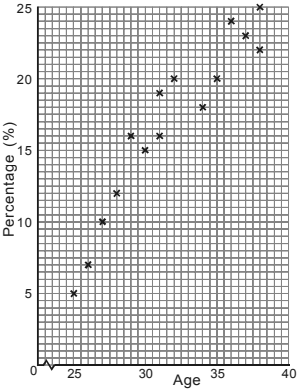
4. (a) Number of students = 3 + 4 + 5 + 6 + 7 + 4 + 6 = 35
   (b) Highest score = 96, lowest score = 35
   (c) Percentage of students passed $= \frac{35-3-4}{35} \times 100\% = \frac{28}{35} \times 100\% = 80\%$

5. (a) Percentage $= \frac{40^\circ}{360^\circ} \times 100\% = \frac{1}{9} \times 100\% = 11\frac{1}{9}\%$
   (b) Total number of students $= 200 \div \frac{1}{9} = 200 \times 9 = 1800$
   (c) Number of students $= 1800 \times \frac{100^\circ}{360^\circ} = 1800 \times \frac{5}{18} = 500$

6. (a) Monthly income $= 7500 \div \frac{150^\circ}{360^\circ} = 7500 \times \frac{12}{5} = \$18000$
   (b) Amount spent on food $= 18000 \times \frac{84^\circ}{360^\circ} = 18000 \times \frac{7}{30} = \$4200$
   (c) $\frac{x^\circ}{360^\circ} = \frac{3000}{18000}$, $\frac{x}{360} = \frac{1}{6}$, $\therefore\ x = 60$
   (d) Angle of sector of transportation $= 360^\circ - 150^\circ - 84^\circ - 42^\circ - 60^\circ = 24^\circ$,
   $\therefore$ Amount spent on transportation $= 18000 \times \frac{24^\circ}{360^\circ} = 18000 \times \frac{1}{15} = \$1200$

7. (a) No obvious relation
   (b) Negative relation
   (c) Positive relation
8. (b) The greater the ages are, the higher are the percentages of income spent on skin care products. They have a positive relation.
9. (a) Back-to-back stem-and-leaf diagram
   (b) Broken-line graph
   (c) Scatter diagram
   (d) Pie chart
10. (a) The unit of the vertical axis is not shown.
    (b) All bars should be equally spaced.
    (c) The value represented by one complete graphical symbol is not shown.
    (d) 1. The numbers of each 'leaf' should be listed in ascending order.
        2. The tens digits in the numbers of each 'leaf' should not be shown.

8. (a)




11. (a) Percentage = $\frac{64+9+4}{160}\times 100\% = \frac{77}{160}\times 100\% = 48\frac{1}{8}\%$

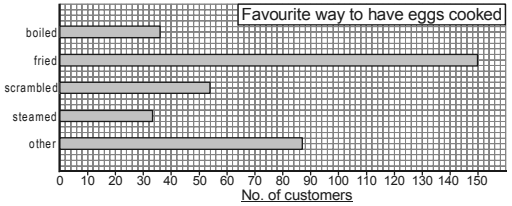
    (b) Angle of sector = $360^\circ \times \frac{50}{160} = 112.5^\circ$

    (c) Min. total number of children = 0(33) + 1(50) + 2(64) + 3(9) + 4(4) = 221

12. (a) Required ratio = $36^\circ : 150^\circ = 6 : 25$

    (b) Total number of customers = $54 \div \frac{54^\circ}{360^\circ} = 54 \times \frac{360}{54} = 360$

    (c)




    others
    = 36 – 33 –54 – 150 – 36
    = 87

13. Let $y$ be the angle representing the sale of sausage A, then the angle representing the sale of sausage C is $y - 40^\circ$, $\frac{y}{y-40^\circ} = \frac{3}{2}$, $2y = 3y - 120^\circ$, $y = 120^\circ$,

    $\therefore$ Angle representing the sale of sausage B = $360^\circ - 120^\circ - (120^\circ - 40^\circ) = 160^\circ$

    Their ratio of sales = 120° : 160° : 80° = 3 : 4 : 2 $\quad \therefore\ k = 2$

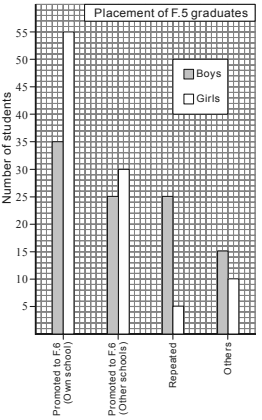
14. (a) Percentage $= \dfrac{35+25+55+30}{100+100}\times 100\%$

$= \dfrac{145}{200}\times 100\% = 72.5\%$

(b) Percentage $= \dfrac{55}{55+30}\times 100\%$

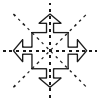
$= \dfrac{55}{85}\times 100\% = 64\dfrac{12}{17}\%$

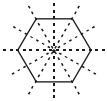
(c)




## Unit 13: Symmetry and transformation

1. (a) 4 axes (b) 5 axes (c) 6 axes (d) 1 axis

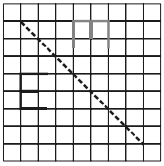




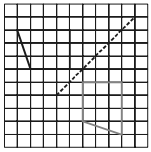

2. (a) order 2 (b) order 2 (c) order 3

3. (i) C (ii) D (iii) A, B, E (iv) C (v) A, B, D, E

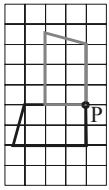
4. (a)



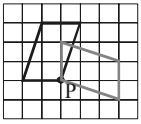
(b)



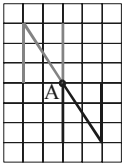
5. (a)



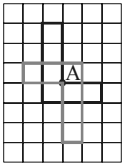

(b)



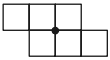

6. (a)




(b)




7. (a)

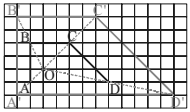


(b)

8. Infinite

9. (a)


(b)




10. The scale factor $= \dfrac{OC'}{OC} = \dfrac{6}{6+3} = \dfrac{6}{9} = \dfrac{2}{3}$

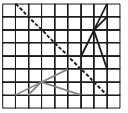
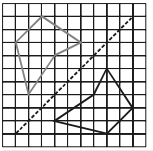
11. (a)

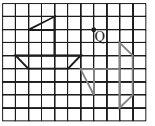
(b)

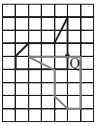
12. (a)

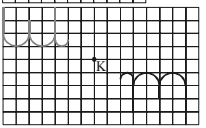
(b)

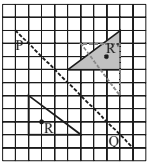
(c)

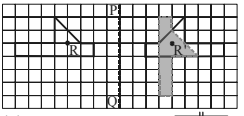
13. (a) 
(b) 

14. (a) 

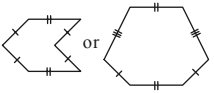
(b) 

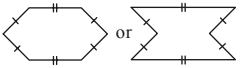

15. 


16. (a) 

(b) 


17. (a) translation (b) reflection or rotation
(c) rotation and enlargement

18. (a) 

(b) 

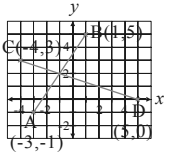
(c) 

19. (a) 3 (b) 1 (c) 1 (∵ 70°, 55°, 55°) (d) 0

20. (a) rotate 120° clockwise about E
(b) rotate 180° about E, then enlarge with scale factor 2 with F as the projection point
(c) rotate 60° anti-clockwise about I, then rotate 180° about E

## Unit 14: Introduction to coordinates

1. (a) D (b) A (c) F (d) B (e) E (f) C, G

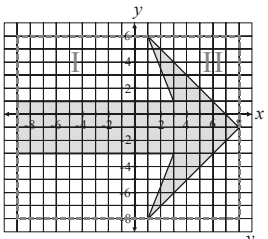
2. (a) 

(b) (–1, 2)

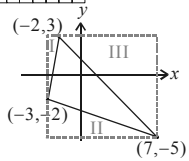
3. (a) Distance = 6 – (– 2) = 8 units
(b) Distance = 15 – 8 = 7 units
(c) Distance = – 2 – (– 7) = 5 units
(d) Distance = 9 – (– 4) = 13 units
(e) Distance = $-3\frac{3}{4}-(-5\frac{1}{3}) = 1\frac{7}{12}$ units

4. Radius = $[2-(-10)]\div 2 = 6$ units

5. Area = Area of rectangle – 2 × (Area I + Area II)
$= 14\times 17-2\times[\frac{(10+12)\times 5}{2}+\frac{7\times 7}{2}] = 238-159$
= 79 sq. units



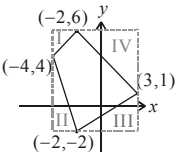

6. (a) Area = $\frac{8\times 7}{2}$ = 28 sq. units
(b) Area = Area of rectangle – Area I – Area II – Area III
$= 10\times 8-\frac{1\times 5}{2}-\frac{3\times 10}{2}-\frac{9\times 8}{2} = 80-2.5-15-36$
= 26.5 sq. units



(c) Area = Area of rectangle – Area I – Area II – Area III – Area IV

$= 7\times 8 - \frac{2\times 2}{2} - \frac{2\times 6}{2} - \frac{5\times 3}{2} - \frac{5\times 5}{2} = 56 - 2 - 6 - 7.5 - 12.5$

$= 28$ sq. units

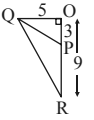



7. (a) L (3, 0); M (7, 0); N (11, 0)

(b) Area of $\Delta ABC$ = Area of ALNC – Area of ALMB – Area of BMNC

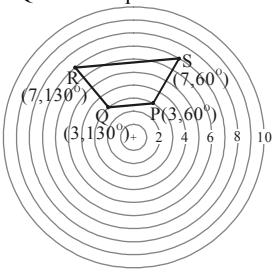
$= \frac{(4+9)\times 8}{2} - \frac{(4+5)\times 4}{2} - \frac{(5+9)\times 4}{2} = 52 - 18 - 28 = 6$ sq. units

8. QPR is an obtuse-angled triangle. Let $O(0,0^\circ)$ be the pole,




$\therefore$ Area $= \frac{OQ\times PR}{2} = \frac{5\times(9-3)}{2} = \frac{5\times 6}{2} = 15$ sq. units

9. PQRS is a trapezium.




10. (a) AB and CD are vertical.

(b) Coordinates of H = (3, 2)

(c) AB = 2 – (–1) = 3, CD = 7 – (–2) = 9,

BH = 3 – (–5) = 8,

$\therefore$ Area of trapezium $= \frac{(AB+CD)\times BH}{2}$

$= \frac{(3+9)\times 8}{2} = 48$ sq. units

11. (a) Coordinates of B = (–4, –1); Coordinates of D = (3, 5)

(b) AB = 3 – (–4) = 7, BC = 5 – (–1) = 6,

$\therefore$ Area of rectangle = $AB\times BC = 7\times 6 = 42$ sq. units

12. PQ = 5 – 1 = 4, height of $\Delta PQR = (2-m)$ or $(m-2)$.

When the height = $2-m$, $\frac{(2-m)\times 4}{2} = 14$, $2-m = 7$, $\therefore\ m = -5$

When the height = $m-2$, $\frac{(m-2)\times 4}{2} = 14$, $m-2 = 7$, $\therefore\ m = 9$

*Ans.* $m = -5$ *or* $9$.

13. BC forms a vertical line, $\therefore$ $x$-coordinate of D = $x$-coordinate of A = 4.

BC = 2 – (–2) = 4, BC = AD = ($y$-coord. of A) – ($y$-coord. of D),

$\therefore$ $y$-coord. of D = $y$-coord. of A – BC = 3 – 4 = –1.

$\therefore$ Coordinates of D = (4, –1)

14. AB is a horizontal line, $\therefore$ AB = 1 – (–4) = 5.

Let $h$ be height from C to AB, $\therefore$ $\frac{5\times h}{2} = 15$, $h = 6$.

In Quadrant I and Quadrant II, the $y$-coordinate of C = $h + 2 = 6 + 2 = 8$.

In Quadrant III and Quadrant IV, the $y$-coordinate of C = $2 - h = 2 - 6 = -4$.

On the other hand, the $x$-coordinate of C does not affect the area and can be any numbers.

*Ans. In the 1st quadrant, the coordinates of C can be (1, 8).*

*In the 2nd quadrant, the coordinates of C can be (–4, 8).*

*In the 3rd quadrant, the coordinates of C can be (–7, –4).*

*In the 4th quadrant, the coordinates of C can be (6, –4).*

15. (a) Height of ABCD = 2 – (–4) = 6, ∴ BC = $48 \div 6$ = 8 units

(b) $x$-coordinate of D = $x$-coordinate of A = –4,

$y$-coordinate of D = $y$-coordinate of A – BC = 5 – 8 = –3,

∴ Coordinates of D = (–4, –3)

16. (a) QR is vertical and QR = 5 – (–3) = 8,

∴ Area of $\Delta OQR = \frac{8 \times 6}{2}$ = 24 sq. units

(b) Let (0, $y$) be the coordinates of T, then area of OTQR = $\frac{(y+8) \times 6}{2}$ = $3y + 24$,

∴ $3y + 24 = 24(1 + 25\%)$, $3y = 6$, $y = 2$.

*Ans. The coordinates of T are (0, 2).*

17. (a) The base XZ = 17 – (–8) = 25; the height = $k$ – 4;

∴ $\frac{25 \times (k-4)}{2} = 150$, $25(k-4) = 300$, $k - 4 = 12$, ∴ $k = 16$

(b) The base YZ = 15, $\frac{XY \times 15}{2} = 150$, $15XY = 300$, ∴ XY = 20 units

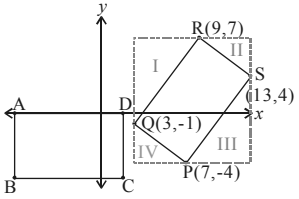
Perimeter of $\Delta XYZ$ = XY + YZ + XZ = 20 + 15 + 25 = 60 units

18. (a) Area of PQRS

= Area of big rectangle – (Area I + Area II + Area III + Area IV)

= $10 \times 11 - (\frac{6 \times 8}{2} + \frac{3 \times 4}{2} + \frac{6 \times 8}{2} + \frac{3 \times 4}{2})$

= 110 – (24 + 6 + 24 + 6) = 50 sq. units




(b) DC = AB = 5, AD = BC = $50 \div 5$ = 10,

∴ Coordinates of D = (–8 + 10, 0) =

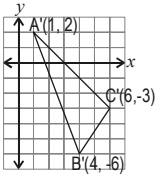
(2, 0), coordinates of B = (–8, –5), coordinates of C = (2, –5).

## Unit 15: Transformation in the coordinate plane

1. (a) (–1, 2) (b) (14, –8) 2. (a) (2, 8) (b) (7, –9)

3. Coordinates of image = (9, 4),

∴ Distance = 9 – (–9) = 18 units

4. 


5. (a) Distance between (7, –1) and the horizontal line = 2 – (–1) = 3 units, ∵ (7, –1) is below the line,

∴ $y$-coordinate of image = –1 + 2(3) = 5,

∴ Coordinates of image = (7, 5)

(b) Distance between the point and the vertical line = 4 – (–1) = 5 units,

∵ (4, 6) is on the right of the line,

∴ $x$-coordinate of image = 4 – 2(5) = –6,

∴ Coordinates of image = (–6, 6)

6. $L_1$ is a horizontal line with $y$-coordinate = –3, $1 - 2[1 - (-3)] = 1 - 2(4) = -7$,

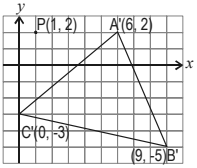
$5 - 2[5 - (-3)] = 5 - 2(8) = -11$, $3 - 2[3 - (-3)] = 3 - 2(6) = -9$.

$\therefore$ After reflection along $L_1$, the images of A, B, C are $(-5, -7)$, $(-1, -11)$ and $(3, -9)$ respectively.

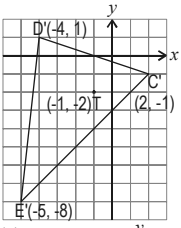
$L_2$ is a vertical line with $x$-coordinate = 4.

$\therefore$ After reflection along $L_2$, coordinates of A' = (13, –7), coordinates of B' = (9, –11), coordinates of C' = (5, –9).

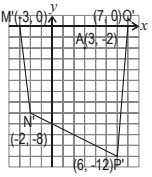
7. P(1, 2), A(1, –3), $1 - 1 = 0$, $2 - (-3) = 5$
$\therefore$ A' = (1+5, 2–0) = (6, 2)
P(1, 2), B(–6, –6), $1 - (-6) = 7$, $2 - (-6) = 8$
$\therefore$ B' = (1+8, 2–7) = (9, –5)
P(1, 2), C(–4, 3), $1 - (-4) = 5$, $3 - 2 = 1$
$\therefore$ C' = (1–1, 2–5) = (0, –3)



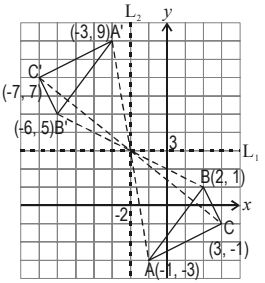

8.



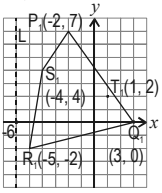

9.



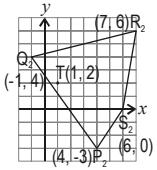

11.



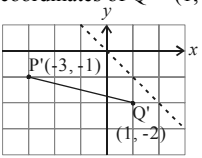

10. (a)



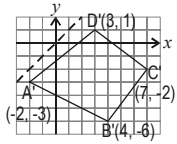

(b)



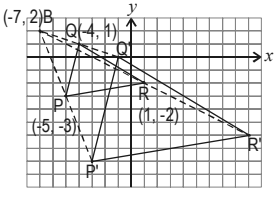

12. Coordinates of P' = (–3, –1), coordinates of Q' = (1, –2)

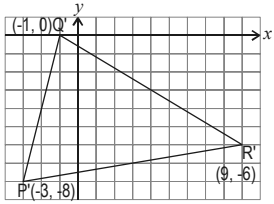



13. Coordinates of A' = (–2, –3), coordinates of B' = (4, –6), coordinates of C' = (7, –2), coordinates of D' = (3, 1)




14. Coordinates of P' = (–3, –8), coordinates of Q' = (–1, 0), coordinates of R' = (9, –6)

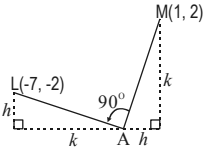



15. (a) Coordinates of $P_1$ = (7, –6), coordinates of $P_2$ = (6, 7), coordinates of $P_3$ = (–7, 6), coordinates of $P_4$ = (–6, –7)

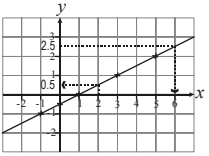
(b) From (a), coordinates of $P_n$ will repeat with a period of 4,
i.e. $P_0 = P_4 = P_8 = \ldots$, $P_1 = P_5 = P_9 = \ldots$, $P_2 = P_6 = P_{10} = \ldots$, $P_3 = P_7 = P_{11} = \ldots$,
$\because$ $101 = 4(25) + 1$, $\therefore$ Coordinates of $P_{101}$ = coordinates of $P_1$ = $(7, -6)$

16. Let $h$ = vertical distance between L and A = horizontal distance between M and A
Let $k$ = horizontal distance between L and A = vertical distance between M and A
$x$-coordinate of A $= -7 + k = 1 - h$, $\therefore$ $k = 8 - h$
$y$-coordinate of A $= -2 - h = 2 - k$, $\therefore$ $k = 4 + h$
$8 - h = 4 + h$ ($\because$ $= k$), $4 = 2h$, $h = 2$, $\therefore$ $k = 8 - 2 = 6$.
$\therefore$ $x$-coordinate of A $= -7 + 6 = -1$,
$y$-coordinate of A $= -2 - 2 = -4$,
*Ans. Coordinates of A are (–1, –4).*

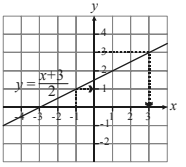



## Unit 16: Linear equations in two unknowns

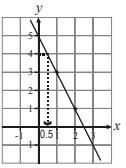
1. (a) –1, –0.5, 0, 1, 2
(b) $m = 0.5$, $n = 6$



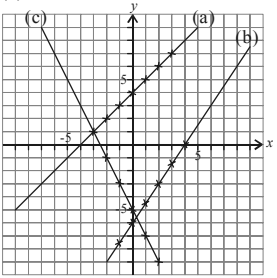

2. (a) 1
(b) 3



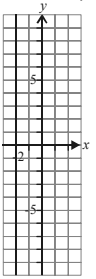

3. (a) 3, 1, –1
(b) $x$-axis: (2.5, 0), $y$-axis: (0, 5)
(c) From the graph, $k = 0.5$
(d) $2(k) + 4 = 5$, $2k = 1$, $\therefore$ $k = 0.5$



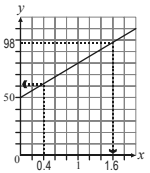

4. (a) 2, 3, 4, 5, 6, 7
(b) –7.5, –6, –4.5, –3, –1.5, 0
(c) 1, –1, –3, –5, –7, –9




6. $x + 2 = 0$, $x = -2$




5. (a) $y = hx + 50$
(b) \$62
(c) 1.6 kg
(d) When x = 1,
y = 80.
$80 = h\,(1) + 50$,
$\therefore$ $h = 30$




7. (a) $2(-7) + k(-6) - 4 = 0$, $-6k = 18$, $\therefore$ $k = -3$

(b) $2h - 3(\frac{5}{2}) - 4 = 0$, $4h - 15 - 8 = 0$, $4h = 23$, $\therefore$ $h = 5\frac{3}{4}$

(c) Let M be $(x, 0)$, $2x - 3(0) - 4 = 0$, $2x = 4$, $\therefore$ $x = 2$.

Let N be $(0, y)$, $2(0) - 3y - 4 = 0$, $-3y = 4$, $\therefore$ $y = -1\frac{1}{3}$.

*Ans. The coodinates of M and N are (2, 0) and (0, $-1\frac{1}{3}$) respectively.*

8. On the $x$-axis, $y = 0$, $\frac{3}{2}x - \frac{1}{3}(0) = \frac{3}{8}$, $\therefore\ x = \frac{1}{4}$. *Ans. The point is* $(\frac{1}{4}, 0)$.

   On the $y$-axis, $x = 0$, $\frac{3}{2}(0) - \frac{1}{3}y = \frac{3}{8}$, $\therefore\ y = -1\frac{1}{8}$. *Ans. The point is* $(0, -1\frac{1}{8})$.

9. (a) $a(2) - 3(4) + 2 = 0$, $2a = 10$, $\therefore\ a = 5$

   (b) When $x = 0$, $5(0) - 3y + 2 = 0$, $y = \frac{2}{3}$

   When $y = 0$, $5x - 3(0) + 2 = 0$, $x = -\frac{2}{5}$

   $\therefore$ The shaded area $= \frac{1}{2} \times \frac{2}{3} \times \frac{2}{5} = \frac{2}{15}$ sq. units

10. (a) $4 = 3(0) + k$, $\therefore\ k = 4$

    (b) $0 = 3x + 4$, $x = -\frac{4}{3}$, $\therefore$ Coordinates of Q $= (-\frac{4}{3}, 0)$

    $\because$ OQ = OR, $\therefore$ Coordinates of R $= (0, \frac{4}{3})$ or $(0, -\frac{4}{3})$

    When R $= (0, \frac{4}{3})$, PR $= 4 - \frac{4}{3} = \frac{8}{3}$,

    $\therefore$ Area of $\triangle$PQR $= \frac{1}{2} \times \frac{8}{3} \times \frac{4}{3} = \frac{16}{9} = 1\frac{7}{9}$ sq.units

    When R $= (0, -\frac{4}{3})$, PR $= 4 + \frac{4}{3} = \frac{16}{3}$,

    $\therefore$ Area of $\triangle$PQR $= \frac{1}{2} \times \frac{16}{3} \times \frac{4}{3} = \frac{32}{9} = 3\frac{5}{9}$ sq. units

11. $3(\frac{1}{2}) + 4(p) - 5 = 0$, $3 + 8p - 10 = 0$, $8p = 7$, $\therefore\ p = \frac{7}{8}$

    $3(\frac{1}{5}) + 4(q) - 5 = 0$, $3 + 20q - 25 = 0$, $20q = 22$, $\therefore\ q = \frac{11}{10}$

    $\therefore\ 2p + q = 2(\frac{7}{8}) + \frac{11}{10} = \frac{35}{20} + \frac{22}{20} = 2\frac{17}{20}$

12. (a) $3(2r) + pr = 0$, $pr = -6r$, $\therefore\ p = -6$ ($\because\ r \neq 0$)

    (b) Put $(-2s, s)$ into $3x - 6y = 0$,
    L.H.S. $= 3(-2s) - 6s = -12s$, R.H.S. $= 0$,
    If $s = 0$, L.H.S. = R.H.S., $\therefore$ D lies on the graph when $s = 0$.
    If $s \neq 0$, L.H.S. $\neq$ R.H.S., $\therefore$ D does not lie on the graph when $s \neq 0$.

13. $\because$ PR // $y$-axis, $\therefore$ Coordinates of R are $(-10, 0)$.

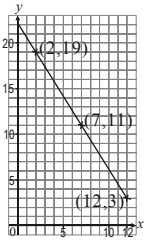
    Let Q be $(0, q)$, $3(q) + 5(0) = 4$, $3q = 4$, $\therefore\ q = \frac{4}{3}$,

    $\therefore$ Area of PROQ $= \frac{(\text{PR} + \text{OQ}) \times \text{OR}}{2} = \frac{1}{2} \times (18 + \frac{4}{3}) \times 10 = 96\frac{2}{3}$ sq. units

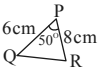
14. (a) $2.4x+1.5y=33.3$, $24x+15y=333$, $\therefore\ 8x+5y=111$

(c) $\because$ $x$ and $y$ must be integers, $\therefore$ From the graph, the possible answers of $(x, y)$ are (2, 19) or (7, 11) or (12, 3).
*Ans.* 2 *$2.4* stamps and 19 *$1.5* stamps; 7 *$2.4* stamps and 11 *$1.5* stamps; or 12 *$2.4* stamps and 3 *$1.5* stamps.

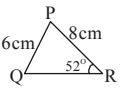
(b)



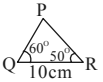

## Unit 17: Congruent triangles (1)

1. (a) $\Delta ABC \cong \Delta PRQ$ (b) $\Delta XYZ \cong \Delta LMN$
2. (a) $a = 55°, b = 11$ (b) $x = 32°, y = 5$
   (c) $x = 90°, y = 20$ (d) $a = 180° - 35° - 28° = 117°, b = 8.5$
3. (a) 14cm (b) 17cm (c) 8cm
4. (a) 53° (b) $\angle X = 180° - 53° - 66° = 61°$ (c) 61°
5. (a) yes, $\Delta ABC \cong \Delta ZXY$ (SAS) (b) yes, $\Delta LMN \cong \Delta ZXY$ (SSS)
   (c) no (d) yes, $\Delta XYZ \cong \Delta LMN$ (AAS)
   (e) yes, $\Delta ABC \cong \Delta EFD$ (RHS) (f) no
6. (a) $\Delta PQR \cong \Delta LMN$ (SAS) (b) $\Delta ABC \cong \Delta YXZ$ (RHS)
   (c) $\Delta PQR \cong \Delta FED$ (AAS) (d) $\Delta MNL \cong \Delta YXZ$ (ASA)
7. (a) $\Delta PQR \cong \Delta PQS$ (ASA) (b) $\Delta ABC \cong \Delta ABD$ (RHS)
   (c) $\angle EGF = 180° - 70° - 60° = 50°$ ; $\angle GHF = 180° - 60° - 50° = 70°$
   $\therefore \Delta EFG \cong \Delta HGF$ (ASA) (d) $\Delta OAB \cong \Delta OXY$ (or: $\Delta OYX$) (SAS)
8. (a) $\Delta ABC \cong \Delta ZYX$ (RHS) (b) $\Delta PQR \cong \Delta XYZ$ (or: $\Delta YXZ$) (SAS)
   (c) $\Delta PQR \cong \Delta ZYX$ (AAS)
9. (a) 

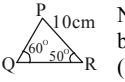
No, the two triangles will be congruent.
($\because$SAS)

(b) 

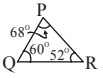
or 

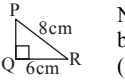

(c) 

No, the two triangles will be congruent.
($\because$ASA)

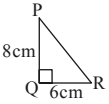
(d) 

No, the two triangles will be congruent.
($\because$AAS)

(e) 


(f) 

No, the two triangles will be congruent.
($\because$RHS)

(g) 

No, the two triangles will be congruent.
($\because$SAS)

10. $\Delta DAB \cong \Delta DEC$ (RHS)
11. QP = QR, PM = RM, QM = QM, $\therefore$ $\Delta PQM \cong \Delta RQM$ (SSS)
12. QP = QR, QM = QM, $\angle QMP = 90° = \angle QMR$, $\therefore \Delta PQM \cong \Delta RQM$ (RHS)
13. AB = CB ($\therefore$AH + HB = CK + KB), BK = BH, $\angle ABK = \angle CBH$,
    $\therefore \Delta AKB \cong \Delta CHB$ (SAS)
14. $\angle CDE = \angle FED$ ($\because \angle CDF + \angle FDE = \angle FEC + \angle CED$),
    $\angle CED = \angle FDE$, DE = ED, $\therefore \Delta CED \cong \Delta FDE$ (ASA)
15. $\angle PNQ = 90° = \angle PMR$, $\angle QPN = \angle RPM$, NP = MP, $\therefore \Delta RMP \cong \Delta QNP$ (ASA)
16. $\angle BAD = 60° = \angle CAE$, BA = CA, AD = AE, $\therefore \Delta BAD \cong \Delta CAE$ (SAS)
17. (a) $\angle CDE + \angle CDF = 90°$, $\angle CDE + \angle ADE = 90°$, $\therefore \angle CDF = \angle ADE$

(b) AD = CD, ∠EAD = 90° = ∠FCD, ∠ADE = ∠CDF [from (a)],
$\therefore \Delta AED \cong \Delta CFD$ (ASA)

## Unit 18: Similar triangles (1)

1. (a) $\angle A = 180^\circ - 80^\circ - 45^\circ = 55^\circ$, $\angle R = 180^\circ - 55^\circ - 45^\circ = 80^\circ$
$\therefore \Delta ABC \sim \Delta QRP$ (A.A.A.)

   (b) No.

   (c) $\frac{10}{15} = \frac{2}{3}$, $\frac{12}{18} = \frac{2}{3}$, $\frac{16}{24} = \frac{2}{3}$, $\therefore \Delta ABC \sim \Delta PRQ$ (3 sides proportional)

   (d) $\frac{25}{10} = \frac{5}{2}$, $\frac{30}{12} = \frac{5}{2}$, $\therefore \Delta ACB \sim \Delta RPQ$ (ratio of 2 sides, inc. ∠ )

   (e) $\frac{4}{3} \neq \frac{5}{8}$ , $\therefore$ No. (f) $\Delta ABC \sim \Delta PRQ$ (A.A.A.)

2. $\angle N = 180^\circ - 75^\circ - 35^\circ = 70^\circ$, $\therefore \angle D = 70^\circ$

3. (a) $40^\circ$ (b) $\angle Q = 180^\circ - 53^\circ - 40^\circ = 87^\circ$, $\therefore \angle ZYX = 87^\circ$

   (c) $\frac{YX}{5} = \frac{6}{4}$, $YX = \frac{6}{4} \times 5 = 7.5$ cm

4. $z = \underline{\underline{86^\circ}}$, $\frac{x}{6} = \frac{10}{4}$, $x = \frac{10}{4} \times 6 = \underline{\underline{15}}$, $\frac{y}{7} = \frac{10}{4}$, $y = \frac{10}{4} \times 7 = \underline{\underline{17.5}}$

5. (a) $\frac{m}{8} = \frac{15}{10}$, $m = \frac{15}{10} \times 8 = \underline{\underline{12}}$ , $\frac{n}{6} = \frac{15}{10}$, $n = \frac{15}{10} \times 6 = \underline{9}$

   (b) $\frac{x}{7} = \frac{7}{3.5}$, $x = \frac{7}{3.5} \times 7 = \underline{\underline{14}}$ , $\frac{9}{y} = \frac{7}{3.5}$, $y = \frac{3.5}{7} \times 9 = \underline{\underline{4.5}}$

   (c) $\frac{x}{6} = \frac{12}{8}$, $x = \frac{12}{8} \times 6 = \underline{9}$ , $\frac{6}{y} = \frac{12}{8}$, $y = \frac{8}{12} \times 6 = \underline{4}$

   (d) $\frac{x}{18} = \frac{6}{9}$, $x = \frac{6}{9} \times 18 = \underline{\underline{12}}$, $\frac{y}{12} = \frac{6}{9}$, $y = \frac{6}{9} \times 12 = \underline{8}$

6. $\frac{XY}{XN} = \frac{XZ}{XM}$, $\therefore \frac{6}{12} = \frac{a}{18}$, $a = \frac{6}{12} \times 18 = 9$

7. (a) $\frac{x}{3} = \frac{6+3}{6}$, $x = \frac{9}{6} \times 3 = 4.5$

   (b) $\frac{y}{y+15} = \frac{8}{20}$, $5y = 2(y+15)$, $3y = 30$, $y = 10$

   (c) $\frac{z}{z+6} = \frac{3}{3+9}$, $4z = z+6$, $3z = 6$, $z = 2$

   (d) $\frac{r}{r+2} = \frac{16}{20}$, $5r = 4(r+2)$, $r = 8$

8. (a) $\frac{x}{5} = \frac{3}{2}$, $x = \frac{3}{2} \times 5 = \underline{\underline{7.5}}$, $\frac{6}{y} = \frac{3}{2}$, $y = \frac{2}{3} \times 6 = \underline{4}$

(b) $\frac{y}{6}=\frac{2}{10}$, $y=\frac{2}{10}\times 6=\underline{\underline{\frac{6}{5}}}$, $\frac{x}{8}=\frac{2}{10}$, $x=\frac{2}{10}\times 8=\underline{\underline{\frac{8}{5}}}$

9. (a) $\Delta AED\sim\Delta ABC$, $\therefore \frac{AD}{AC}=\frac{AE}{AB}$, $\frac{3}{6+a}=\frac{6}{3+15}$, $\frac{3}{6+a}=\frac{1}{3}$, $9=6+a$, $a=3$

(b) $\Delta RNM\sim\Delta RPQ$, $\therefore \frac{RN}{RP}=\frac{NM}{PQ}$, $\frac{r}{50}=\frac{8}{20}$, $r=\frac{8}{20}\times 50=20$

(c) $\Delta ACB\sim\Delta EDB$, $\therefore \frac{AC}{ED}=\frac{AB}{EB}$, $\frac{x}{8}=\frac{18}{12}$, $x=\frac{18}{12}\times 8=12$

(d) $\Delta QPR\sim\Delta QTS$, $\therefore \frac{QR}{QS}=\frac{QP}{QT}$, $\frac{8+u}{12}=\frac{12+2}{8}$, $8+u=\frac{14}{8}\times 12$, $u=21-8=13$

10. (a) $\frac{6}{8}=\frac{c}{c+3}$, $3(c+3)=4c$, $c=9$

(b) $\frac{20}{r+16}=\frac{16}{28}$, $r+16=\frac{7}{4}\times 20$, $r+16=35$, $r=19$

11. Let the other sides be $x$ cm and $y$ cm, $\frac{4}{6}=\frac{x}{9}=\frac{y}{12}$, $x=\frac{4}{6}\times 9=6$, $y=\frac{4}{6}\times 12=8$, $\therefore$The perimeter $=4+6+8=18$ cm.

12. Perimeter of the first triangle $=5+3+4=12$ cm.

$\therefore$The longest side of the second triangle $=5\times\frac{96}{12}=40$ cm.

13. Let the actual length be $x$ m. $\frac{x\text{ m}}{12\text{ cm}}=\frac{6\text{ m}}{5\text{ cm}}$, $x\text{ m}=6\text{ m}\times\frac{12\text{ cm}}{5\text{ cm}}=14.4$ m,

*Ans. The actual length is 14.4 m.*

14. $\frac{y}{24}=\frac{4}{32}$, $y=\frac{4}{32}\times 24=3$

15. $\Delta DEA\sim\Delta CAB$, $\therefore \angle ADE=\angle BCA=45^\circ$, $\angle BAC=\angle AED=75^\circ$

$\angle DAE=180^\circ-\angle ADE-75^\circ=180^\circ-45^\circ-75^\circ=60^\circ$

$\angle DAB=\angle BAC-\angle DAE=75^\circ-60^\circ=\underline{\underline{15^\circ}}$

16. $\Delta PQR\sim\Delta CBA$. $\angle Q=180^\circ-80^\circ-30^\circ=70^\circ$, $\angle A=180^\circ-70^\circ-80^\circ=30^\circ$

$\because \angle P=\angle C$, $\angle Q=\angle B$, $\angle R=\angle A$, $\therefore \Delta PQR\sim\Delta CBA$ (A.A.A.)

$\frac{AC}{RP}=\frac{AB}{RQ}=\frac{CB}{PQ}$, $\frac{z}{10}=\frac{y}{12}=\frac{3}{6}$,

$\therefore \frac{z}{10}=\frac{3}{6}$, $z=\frac{3}{6}\times 10=\underline{\underline{5}}$, $\frac{y}{12}=\frac{3}{6}$, $y=\frac{3}{6}\times 12=\underline{\underline{6}}$

17. $\Delta CAD\sim\Delta EAB$. $\angle ACD=\angle AEB$, $\angle CAD=\angle EAB$,

$\angle CDA=\angle EBA$ ($\because \angle CDA=180^\circ-\angle ACD-\angle CAD$, $\angle EBA=180^\circ-\angle AEB-\angle EAB$.)

$\therefore \Delta CAD\sim\Delta EAB$ (A.A.A.)

$\frac{AD}{AB}=\frac{CA}{EA}$, $\therefore \frac{4+a}{6}=\frac{3+6}{4}$, $4+a=\frac{9}{4}\times 6$, $4+a=13.5$, $a=9.5$

18. $\Delta MNH \sim \Delta NKH$ . In $\Delta MNH$ and $\Delta NKH$ ,

$\frac{MN}{NK}=\frac{40}{50}=\frac{4}{5}$, $\frac{NH}{KH}=\frac{40}{50}=\frac{4}{5}$, $\frac{MH}{NH}=\frac{32}{40}=\frac{4}{5}$

$\therefore \Delta MNH \sim \Delta NKH$ (3 sides proportional)

$r=\angle NKH=\angle MNH=\underline{\underline{48^\circ}}$, $p=\frac{180^\circ-48^\circ}{2}=\underline{\underline{66^\circ}}$, $q=p=\underline{\underline{66^\circ}}$

19. $\Delta EDF \sim \Delta GDH$ . In $\Delta EDF$ and $\Delta GDH$ ,

$\frac{DE}{DG}=\frac{2}{2+4}=\frac{1}{3}$, $\frac{DF}{DH}=\frac{3}{3+6}=\frac{1}{3}$, $\angle EDF=\angle GDH$

$\therefore \Delta EDF \sim \Delta GDH$ (ratio of 2 sides, inc. $\angle$ ), $\frac{x}{10}=\frac{2}{2+4}$, $x=\frac{2}{6}\times 10=3\frac{1}{3}$

20. $\Delta PRQ \sim \Delta QRS$ . In $\Delta PRQ$ and $\Delta QRS$ ,

$\angle QPR=\angle SQR$, $\angle PRQ=\angle QRS$ ,

$\angle PQR=\angle QSR$ ($\because 180^\circ-\angle QPR-\angle PRQ=180^\circ-\angle SQR-\angle QRS$ ),

$\therefore \Delta PRQ \sim \Delta QRS$ (A.A.A.)

$\frac{PQ}{QS}=\frac{QR}{SR}$, $\frac{e}{3}=\frac{6}{4.5}$, $e=\frac{6}{4.5}\times 3=\underline{\underline{4}}$

$\frac{PR}{QR}=\frac{QR}{SR}$, $\frac{c+4.5}{6}=\frac{6}{4.5}$, $c+4.5=\frac{6}{4.5}\times 6$, $c=8-4.5=\underline{\underline{3.5}}$

21. (a) Yes. In $\Delta ECB$ and $\Delta DCA$ , $\angle BEC=\angle ADC$ , $\angle BCE=\angle ACD$ ,

$\angle EBC=\angle DAC$ ($\because 180^\circ-\angle BEC-\angle BCE=180^\circ-\angle ADC-\angle ACD$ ),

$\therefore \Delta ECB \sim \Delta DCA$ (A.A.A.)

(b) $\frac{EC}{DC}=\frac{BC}{AC}$, $\frac{k}{18}=\frac{6+18}{30}$, $k=\frac{24}{30}\times 18=14.4$

22. (a) $\angle BAE=\underline{\underline{90^\circ-a}}$, $\angle BEA=180^\circ-90^\circ-\angle BAE=90^\circ-(90^\circ-a)=\underline{\underline{a}}$,

$\angle CED=180^\circ-90^\circ-\angle BEA=\underline{\underline{90^\circ-a}}$,

$\angle EDC=180^\circ-90^\circ-\angle CED=90^\circ-(90^\circ-a)=\underline{\underline{a}}$,

$\angle ADE=180^\circ-90^\circ-a=\underline{\underline{90^\circ-a}}$ .

(b) Their angles are : $a$, $90^\circ-a$ and $90^\circ$ , $\therefore$ they are similar triangles (A.A.A.)

$\Delta AED \sim \Delta EBA \sim \Delta DCE$ .

(c) $\because \Delta EBA \sim \Delta AED$

$\therefore \frac{EB}{AE}=\frac{BA}{ED}=\frac{EA}{AD}$, $\frac{y}{6}=\frac{x}{8}=\frac{6}{10}$, $y=\frac{6}{10}\times 6=\underline{\underline{3.6}}$, $x=\frac{6}{10}\times 8=\underline{\underline{4.8}}$

$\because \Delta DCE \sim \Delta AED$

$\therefore \frac{DC}{AE}=\frac{CE}{ED}=\frac{DE}{AD}$, $\frac{q}{6}=\frac{p}{8}=\frac{8}{10}$, $q=\frac{8}{10}\times 6=\underline{\underline{4.8}}$, $p=\frac{8}{10}\times 8=\underline{\underline{6.4}}$

*[Remark: Since the opposite sides of a rectangle are equal, you can find p, q without using similar triangles.]*