ANSWERS

Appendices

Unit 1 Approximation

1. (a) 5000 (b) hundred (c) 4839.55 (d) 4840 (e) 1
2.

		2 decimal places	nearest tenth	nearest ten	nearest whole
(a)	130.4449	130.44	130.4	130	130
(b)	16.258	16.26	16.3	20	16
(c)	8.50007	8.50	8.5	10	9
(d)	0.0772	0.08	0.1	0	0

3. (a) 87 (b) 101 (c) 7 (d) 1 (e) 20 (f) 0
4. (a) 4,000,000 (b) 3,960,000
5. (a) 18 cm (b) 210 m (c) 0.20 mm
6. (a) = 0.0345 kg ≈ 0.03 kg (b) = 715.6 g ≈ 720 g (c) = 34.65 cm ≈ 34.7 cm
7.

	39104	208	0.5433	0.00697
The most sig. fig.	3	2	5	6
Number of sig. fig.	5	3	4	3

8. (a) 200 (b)199 (c) 198.5 (d) 198.50
9. (a) 2 sig. fig. (b) 3 sig. fig.
10. (a) 2 (b) 1 (c) 0
11. (a) 2 (b) 1 (c) 2 (d) 0, 1, 2 or 3 (or: can't be determined)
12. (a) 0.081 (b) 180 (c) –26 (d) 400,000
13. (a) 5.70 (b) 0.263 (c) 66.4 (d) 132
14. The cost = 72.8 × 12 × 4 = 3494.4 ≈ \$3490
15. The average weight = $\dfrac{11.074\text{ kg}}{6} = 1.8$ kg (to the nearest 0.1 kg)
16.

	(a)	(b)	(c)	(d)
lower limit	419.5 cm	98.5	256.75 g	6995
upper limit	420.5 cm	99.5	256.85 g	7005

17. (a) The speed = $\dfrac{31.225 + 31.235}{2} = 31.23$ km/h
    (b) 4 sig. fig., or 2 decimal places, or 0.01 km/h
18. The weight lies between 13.95 kg and 14.05 kg.
19. The lower limit = $420000 - \dfrac{100}{2} = \$419,950$, the upper limit = $420000 + \dfrac{100}{2} = \$420,050$
20. The lower limit = $(38.4 - \dfrac{0.1}{2}) \div 58 = 0.6612$ minute,

    the upper limit = $(38.4 + \dfrac{0.1}{2}) \div 58 = 0.6629$ minute
21. 30 m: lower limit = $30 - \dfrac{1}{2} = 29.5$ m, upper limit = $30 + \dfrac{1}{2} = 30.5$ m

    16 m: lower limit = $16 - \dfrac{1}{2} = 15.5$ m, upper limit = $16 + \dfrac{1}{2} = 16.5$ m

    ∴ lower limit of the perimeter = 2 × (29.5+15.5) = 90 m,
    upper limit of the perimeter = 2 × (30.5+16.5) = 94 m
22. 12 cm: lower limit = $12 - \dfrac{0.5}{2} = 11.75$ cm, upper limit = $12 + \dfrac{0.5}{2} = 12.25$ cm

    8 cm: lower limit = $8 - \dfrac{0.5}{2} = 7.75$ cm, upper limit = $8 + \dfrac{0.5}{2} = 8.25$ cm

∴ lower limit of the area = 11.75 × 7.75 = 91.1 cm^2 (3 sig. fig.),
upper limit of the area = 12.25 × 8.25 = 101 cm^2 (3 sig. fig.)

23.

	Max absolute error	Relative error
(a)	0.5 cm^3	$=\frac{0.5}{336}=\frac{1}{672}$
(b)	0.005 kg	$=\frac{0.005}{0.84}=\frac{1}{168}$
(c)	0.05° C	$=\frac{0.05}{19}=\frac{1}{380}$
(d)	\$ 5	$=\frac{5}{780}=\frac{1}{156}$

24.

	Max absolute error	% error (3 sig. fig.)
(a)	$=\frac{5}{2}=2.5$ g	$=\frac{2.5}{165}\times100\%=1.52\%$
(b)	$=\frac{2}{2}=1$ mm	$=\frac{1}{5.8}\times100\%=17.2\%$
(c)	$=\frac{0.01}{2}=0.005$ L	$=\frac{0.005}{22}\times100\%=0.0227\%$
(d)	$=\frac{10}{2}=5$ s	$=\frac{5}{960}\times100\%=0.521\%$

25. (a) The maximum absolute error = $\frac{10}{2}$ = 5 g, relative error = $\frac{5}{540}=\frac{1}{108}$,

% error = $\frac{1}{108}\times 100\% = 0.926\%$ (3 sig. fig.)

(b) The maximum absolute error = $\frac{5}{2}$ = 2.5 cm^2, relative error = $\frac{2.5}{240}=\frac{1}{96}$,

% error = $\frac{1}{96}\times 100\% = 1.04\%$ (3 sig. fig.)

26. The maximum absolute error = $66^\circ\times\frac{1}{12}=5.5^\circ$,

∴ the true value lies between (66 ± 5.5°), that is between 60.5° and 71.5° .

27. (a) The maximum error = 49 × 5% = 2.45 kg
(b) The range of actual weight = (49 ± 2.45) kg, that is between 46.55 kg and 51.45 kg.

28. (a) The maximum absolute error = $\frac{1000}{2}$ = 500,

∴ the % error = $\frac{500}{3000}\times100\%=16.7\%$ (3 sig. fig.)

(b) The maximum absolute error = $\frac{100}{2}$ = 50,

∴ the % error = $\frac{50}{3000}\times100\%=1.67\%$

29. (a) The maximum absolute error = 0.5 m/s,

∴ the % error = $\frac{0.5}{26}\times100\%=1.92\%$ (3 sig. fig.)

(b) The maximum error in 1 s = 0.5 m,
∴ the max error in 20 minutes = 0.5 × 20 × 60 = 600 m

30. The height of the boy = $\frac{154.5+165.5}{2}=165$ cm,

the maximum absolute error = 165.5 – 165 = 0.5 cm,

∴ the % error = $\frac{0.5}{165}\times100\%=0.303\%$ (3 sig. fig.)

31. (a) The maximum % error = $\frac{0.2}{13}\times100\%=1.54\%$ (3 sig. fig.)

(b) The maximum % error = $\frac{15}{185}\times100\%=8.11\%$ (3 sig. fig.)

32. (a) The absolute error = 900 – 879 = 21, $\therefore$ the relative error = $\frac{21}{879}=\frac{7}{293}$,

the % error = $\frac{7}{293}\times 100\% = 2.39\%$ (3 sig. fig.)

(b) The absolute error = 18.237 – 18.2 = 0.037 g,

$\therefore$ the relative error = $\frac{0.037}{18.237}=\frac{37}{18237}$ =0.00203 (3 sig. fig.),

the % error = 0.00203 × 100% = 0.203% (3 sig. fig.)

33. (a) Take the integral part only (or called "the front-end method").
(b) If the number is not an integer, round up to the next integer.
(c) Correct to the nearest \$0.5. (d) Correct to the nearest \$500.
(e) Correct to the nearest 4 cm^3.

34. (a) 1 cm^2 = 100 mm^2, $\therefore$ 817.3 mm^2 = 8.173 $cm^2 \approx$ 8.2 cm^2
(b) 1 m^2 = 10000 cm^2, $\therefore$ 402988 cm^2 = 40.2988 $m^2 \approx$ 40 m^2
(c) 1 km^2 = 1000000 km^2, $\therefore$ 6049095 m^2 = 6.049095 $km^2 \approx$ 6.0 km^2

35. (a)

x	Actual value	Approximate value	Percentage error (3 sig. fig.)
0.4	$\sqrt{0.4+1}=\sqrt{1.4}$	1+0.5(0.4) =1.2	$\frac{1.2-\sqrt{1.4}}{\sqrt{1.4}}\times 100\%$ = 1.42 %
1	$\sqrt{1+1}=\sqrt{2}$	1+0.5(1) =1.5	$\frac{1.5-\sqrt{2}}{\sqrt{2}}\times 100\%$ = 6.07%
4	$\sqrt{4+1}=\sqrt{5}$	1+0.5(4) =3	$\frac{3-\sqrt{5}}{\sqrt{5}}\times 100\%$ = 34.2%

(b) The method is appropriate only when x is a very small number.
Otherwise the error will be too big.

36. (a) It can correct the number of each centre to the nearest hundred before calculating the total. $\therefore$ The total number of students ≈ 100 × 6 = 600.

(b) The actual number = 52 × 6 = 312,

the % error = $\frac{600-312}{312}\times 100\%$ = 92.3 % (3 sig. fig.)

37. Lower limit of the speed = 8.5m/s, upper limit of the speed = 9.5 m/s.
$\therefore$ the least possible distance travelled in 1 day = 8.5 × (24 × 60 × 60)
= 734400 m =734.4 km
the greatest possible distance travelled in 1 day = 9.5 × (24 × 60 × 60)
= 820800 m =820.8 km

38. 8.3 m lies between (8.3 ± 0.05)m; 17.4 m lies between (17.4 ± 0.05)m.
$\therefore$ the shortest length of the remaining rope = 17.35 – 8.35 = 9 m
the longest length of the remaining rope = 17.45 – 8.25 = 9.2 m

39. The area lies between (64 ± 0.5) cm^2; the base lies between (12 ± 0.5) cm.

Area = $\frac{1}{2}$ × base × height, $\therefore$ height = $\frac{2\times \text{area}}{\text{base}}$

$\therefore$ lower limit of the height = $\frac{2\times 63.5}{12.5}$ = 10.16 cm

upper limit of the height = $\frac{2\times 64.5}{11.5}$ = 11.22 cm

40. The shortest time = $\frac{45\times 24}{80+4}$ min + 45 × (9 – 1)s = 18 min 51 s (to the nearest s)

The longest time = $\dfrac{45\times 24}{80-4}$ min + 45 × (9+1)s = 21 min 43 s (to the nearest s)

41. (a) The lower limit = 85 × (1 – 4%) = 81.6 g.
 ∵ 82 g is greater than the lower limit, ∴ it will be accepted.
 (b) The upper limit of the weight a packet of noodles = 85 × (1 + 4%) = 88.4 g.
 ∴ The no. of packets = $\dfrac{2\text{kg}}{88.4\text{g}} = \dfrac{2000}{88.4} \approx 22.6$
 Ans. The maximum no. of packets is 22.

42. The max absolute error = $\dfrac{0.5}{2}$ = 0.25 cm.
 Let the length of the measured object be x cm when the % error is 1%.
 ∴ $\dfrac{0.25}{x}\times 100\% = 1\%$, $x = 0.25 \times 100 = 25$
 If the object is shorter than x, the error will be greater than 1%.
 Ans. He can measure objects with length between 25 cm and 1 m.

43. (a) 3 min 20 s = 3 × 60 + 20 = 200s; the max error = $0.5x$
 ∴ $\dfrac{0.5x}{200}\times 100\% = 1.25\%$, $x = \dfrac{1.25\times 200}{100\times 0.5} = 5$
 (b) The max. error = $\dfrac{x}{2} = \dfrac{5}{2} = 2.5$ s, ∴ the range = 3 min 20 s ± 2.5 s.
 ∴ It lies between 3 min 17.5 s and 3 min 22.5 s.

44. (a) The sum = 41000+42000+39000+41000+4000+40000 = 207,000
 (b) 5 of the numbers are close to 40000, ∴ we can use the clustering method.
 The sum = 40000 × 5 + 4000 = 204000.
 (c) The exact sum = 41208+42371+38894+40759+3977+ 39928 = 207137
 ∴ the % error of the estimated value in (b)
 $= \dfrac{207137-204000}{207137}\times 100\% = \dfrac{3137}{207137}\times 100\% = 1.51\ \%$
 Since the % error is rather small, it is a good estimation.

45. (a) The estimated value = 880 × $\dfrac{9}{72}$ = 880 × $\dfrac{1}{8}$ = 110
 (b) $(9\times 874.9)\div 73$ = 107.8644 (correct to 4 decimal places)
 ∴ the % error = $\dfrac{110-107.8644}{107.8644}\times 100\% = \dfrac{2.1356}{107.8644}\times 100\% = 1.98\%$
 Since the error is rather small, it is a good estimation.

Unit 2 Polynomials

1. (a) 5, 4, 3, 0, –2 (b) 3, 3, 0, –5, 4 (c) 4, 3, –6, 1, 0
2. (a) 5 (b) –1 (c) 0
3. (a) $3x^4 + 3x^2 - x + 5$ (b) $-x^5 + 2x^4 - x^3 + 8x^2 + 1$
4. (a) $8 - 2y + 16y^2 - 9y^3$ (b) $-1 + 4y^2 + 2y^3 - y^4 + y^5$
5. (a) $12x$ (b) $-4a - 2b$ (c) $-m - 3n$ (d) $5x^2 - 4$
6. (a) $-12p + 12$ (b) $-x^2 + x + 11$
 (c) $-4a^2 + 5a - 30$ (d) $13x^3 + 5x^2 - 2x + 14$
7. (a) $-7x^2 + 2x - 3$ (b) $4xy^2 - 4x^2y$ (c) $-x^2 - 10x - 4$
 (d) $3a^2b - ab^2 + 2ab + 4a - 10$

8. (a) $= 4p - 3q - 5p - q = -p - 4q$ (b) $= 18x^2 + 5y^2 - 3x^2 - 2y^2 = 15x^2 + 3y^2$
(c) $= -3k^2 + 2 - 8k + 7 + 8k^2 - 8k = 5k^2 - 16k + 9$
(d) $= 6x - y + 2x - 7y - 2x - 3y = 6x - 11y$
(e) $= x^2 - 8x + 1 - 3x^2 - 5x - 2 + 4x^2 - 3x + 6 = 2x^2 - 16x + 5$
(f) $= -7a^2 - 9ac + 4c^2 - 3a^2 + 8ca + 5c^2 + 5ca - 6a^2 + 11c^2 = -16a^2 + 4ac + 20c^2$
9. $=(-3x^2 + 7x - 4) - (3x^2 - 5x + 11) = -3x^2 + 7x - 4 - 3x^2 + 5x - 11 = -6x^2 + 12x - 15$
10. (a) $1\frac{5}{12}x$ (b) $-\frac{5}{12}a^2$ (c) $-\frac{33}{40}y^3$ (d) $\frac{5}{6}xy^2$
(e) $= -\frac{6}{7}ab + ab = \frac{1}{7}ab$
11. (a) $= -5x^2 + 35x - x + 7 = -5x^2 + 34x + 7$
(b) $= -8a^2 - 12ab - 12ab - 18b^2 = -8a^2 - 24ab - 18b^2$
(c) $= -16y^4 + 2y^3 + 40y^3 - 5y^2 = -16y^4 + 42y^3 - 5y^2$
(d) $= 4x^3 + 12x^2 - 24x - x^2 - 3x + 6 = 4x^3 + 11x^2 - 27x + 6$
(e) $= -10x^4 + 14x^3 - 2x^2 + 15x^2 - 21x + 3 = -10x^4 + 14x^3 + 13x^2 - 21x + 3$
12. (a) $-6y^3 + 2y^2 - 4y$ (b) $15a^4 + 30a^3 - 40a^2$ (c) $2x^2 + 15x + 18$
(d) $4y^2 + y - 3$ (e) $42r^2 - 25rs + 3s^2$ (f) $2x^2 + 5x - 18$
13. (a) $= -3(16x^2 + 38x - 5) = -48x^2 - 114x + 15$
(b) $= 3x^3 + 6x^2 - 2x - 12x^2 - 24x + 8 = 3x^3 - 6x^2 - 26x + 8$
(c) $= 9x^3 + x^2y - 4xy^2 + 18x^2y + 2xy^2 - 8y^3 = 9x^3 + 19x^2y - 2xy^2 - 8y^3$
(d) $= -15x^2 + 30x - 35 - 6x^3 + 12x^2 - 14x = -6x^3 - 3x^2 + 16x - 35$
14. (a) $= 18x - 6 - 8x + 12 = 10x + 6$
(b) $= 32y^2 - 24y - 8y^2 - 28y - 3y^2 + 6y = 21y^2 - 46y$
15. The coefficient of $x^2 = 2\times5+7\times(-4)+(-8)\times(-3) = 6$
16. Total value of the coins = $2(a^2 + 2a - 5) + 5(3a^2 - a + 2) = \$(17a^2 - a)$
17. (a) The perimeter $= 2[(3k + 4) + (15 - 2k)] = 2(k + 19) = (2k + 38)$ cm
(b) The area $= (3k + 4)(15 - 2k) = (-6k^2 + 37k + 60)$ cm^2
18. (a) $= (10x^2 + 19x + 6)(x - 1) = 10x^3 + 19x^2 + 6x - 10x^2 - 19x - 6$
$= 10x^3 + 9x^2 - 13x - 6$
(b) $= (-20y^2 - 17y + 10)(y + 3) = -20y^3 - 17y^2 + 10y - 60y^2 - 51y + 30$
$= -20y^3 - 77y^2 - 41y + 30$
(c) $= 3(-3a^2 + 5ab + 2b^2)(a - 5b) = 3(-3a^3 + 20a^2b - 23ab^2 - 10b^3)$
$= -9a^3 + 60a^2b - 69ab^2 - 30b^3$
19. (a) $9x^2 + 12x + 4$ (b) $4y^2 - 20y + 25$
(c) $16a^2 - 24ab + 9b^2$ (d) $9x^2 + 42x + 49$
20. $= (2x - 1)(2x - 1)(2x - 1) = (2x - 1)(4x^2 - 4x + 1) = 8x^3 - 12x^2 + 6x - 1$
21. (a) $= 16x^2 - 4x(8 - 10x) = 16x^2 - 32x + 40x^2 = 56x^2 - 32x$
(b) $= (3y^2 - 6y + 15) - (3y^2 - 17y - 6) = 11y + 21$
(c) $= (2x^2 - 2x - 24) - (3x^2 - 13x - 10) = -x^2 + 11x - 14$
(d) $= 8k^2 + 3k - (2k - 1)(9k - 7) = 8k^2 + 3k - (18k^2 - 23k + 7) = -10k^2 + 26k - 7$
(e) $= (3a + 2)(a - 1) - 3(10 - 2a)(1 + a) = 3a^2 - a - 2 - 3(10 + 8a - 2a^2)$
$= 9a^2 - 25a - 32$
22. Coefficient of $y^2 = (13)(4) + (-5)(-6) + (4)(3) - [(1)(15) + (9)(-8)] = 94 - (-57) = 151$
23. Area of the trapezium $= \frac{1}{2}[(2a+b)+(4b-3a)](5b-a) = \frac{1}{2}(5b-a)(5b-a)$
$= \frac{1}{2}(25b^2 - 10ab + a^2)$ sq. units
24. (a) Width $= \frac{1}{2}(24x-8) - 5x = 12x - 4 - 5x = (7x - 4)$ cm
(b) Decrease in area $= 5x(7x - 4) - (5x - x)(7x - 4 - 1)$

$= 35x^2 - 20x - 4x(7x - 5) = 7x^2$ cm^2

25. (a) Length $= (20 - x) - 2x = 20 - 3x$, Width $= (5x + 2) - 2x = 3x + 2$, Height $= x$,
$\therefore$ Dimensions of the box are $(20 - 3x)$ cm $\times$ $(3x + 2)$ cm $\times$ x cm
(b) Capacity $= (20 - 3x)(3x + 2)x = (40 + 54x - 9x^2)x = (40x + 54x^2 - 9x^3)$ cm^3

26. (a) $(x + y)(x^2 - xy + y^2) = x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3$
(b) From (a), when $x = a$, $y = 1$, $(a^3 + 1) \div (a + 1) = a^2 - a(1) + 1^2 = a^2 - a + 1$
(c) From (b), when $a = 200$, $800001 \div 201 = 200^2 - 200 + 1 = 39801$

Unit 3 Simple factorization and grouping terms

1. (a) $= 6(3y^2 + 1)$ (b) $= 7x(x - 7)$
(c) $= 3x(3y - 4t + 5ty)$ (d) $= 4x(4x^2 - 2x + 1)$
(e) $= 12bc(3a^2 + 2ac + 5)$ (f) $= 15r^3s^2(-3r - 4s + 5s^2)$

2. (a) $= (x + y)(1 - k)$ (b) $= 10(a^2 + b^2)(3p + 5q)$
(c) $= (m + n)^3[8(m + n) - 1] = (m + n)^3(8m + 8n - 1)$
(d) $= (a + b)(c + d)(1 + c + d)$

3. (a) $= (y^2 - y) + (xy - x) = y(y - 1) + x(y - 1) = (y - 1)(x + y)$
(b) $= (na + ny) + (a + y) = n(a + y) + (a + y) = (a + y)(n + 1)$
(c) $= (3k^3 + 21k^2) - (k + 7) = 3k^2(k + 7) - (k + 7) = (k + 7)(3k^2 - 1)$
(d) $= (mn - 2nr) - (my - 2ry) = n(m - 2r) - y(m - 2r) = (m - 2r)(n - y)$
(e) $= (2ab^2c + 6a^2b) - (5bc^2 + 15ac) = 2ab(bc + 3a) - 5c(bc + 3a)$
$= (bc + 3a)(2ab - 5c)$
(f) $= (e + f)^2 - (e + f) = (e + f)(e + f - 1)$

4. (a) $= r(x - y) + s(x - y) = (x - y)(r + s)$
(b) $= 8x^2(4a - 3b) - 4x(4a - 3b) = 4x(4a - 3b)(2x - 1)$

5. (a) $= (8ay + 4a^2) + (6xy + 3ax) = 4a(2y + a) + 3x(2y + a) = (2y + a)(4a + 3x)$
(b) $= (rs - 3r) - (5s - 15) = r(s - 3) - 5(s - 3) = (s - 3)(r - 5)$
(c) $= (p^2 - p) - (pq - q) = p(p - 1) - q(p - 1) = (p - 1)(p - q)$
(d) $= (c + b) - (ac^2 + abc) = (c + b) - ac(c + b) = (c + b)(1 - ac)$
(e) $= (pq + r) + (r^2pq + rp^2q^2) = (pq + r) + rpq(r + pq) = (r + pq)(1 + rpq)$
(f) $= (xz - 2x^2z) - (3yz^2 - 6xyz^2) = xz(1 - 2x) - 3yz^2(1 - 2x)$
$= z(1 - 2x)(x - 3yz)$

6. (a) $= (6x^2 + 9) + (2ax^3 + 3ax) + (2x^2y + 3y)$
$= 3(2x^2 + 3) + ax(2x^2 + 3) + y(2x^2 + 3) = (2x^2 + 3)(3 + ax + y)$
(b) $= (ah^2 + bhn - hp) - (ahn + bn^2 - np) = h(ah + bn - p) - n(ah + bn - p)$
$= (ah + bn - p)(h - n)$
(c) $= (x^3 - x^2y + x^2) - (3x - 3y + 3) = x^2(x - y + 1) - 3(x - y + 1)$
$= (x - y + 1)(x^2 - 3)$
(d) $= (pt^2 - p^2t + 3rp^2t) + (6rt - 6rp + 18r^2p) = pt(t - p + 3rp) + 6r(t - p + 3rp)$
$= (t - p + 3rp)(pt + 6r)$

7. (a) $-x + 1 = 1 - x$; $\therefore$ $x - 1$ is different.
(b) $-a + b = -(a - b)$; $\therefore$ $a + b$ is different.
(c) $(x - y)(a - b) = [-(y - x)][-(b - a)] = (y - x)(b - a)$; $\therefore$ $(x + y)(a + b)$ is different.
(d) $(m - n)^2 = [-(n - m)]^2 = (n - m)^2$; $\therefore$ $-(n - m)^2$ is different.

(e) $(x-y)^3 = [-(y-x)]^3 = (-1)^3(y-x)^3 = -(y-x)^3$; $\therefore$ $(y-x)^3$ is different.

8. (a) $= 2(a+b)(p-q) - 4(p-q)(x+y) = 2(p-q)(a+b-2x-2y)$
(b) $=18(p-q)^3 - 12(p-q)^2 = 6(p-q)^2(3p-3q-2)$ (c) $= (y+3)^2(5y+1)$
(d) $= (y-x)^2(m-n)^3 + (y-x)^3(m-n)^4 = (y-x)^2(m-n)^3\,[1 + (y-x)(m-n)]$

9. (a) $= (8p+8q)+(p+q)^2 = 8(p+q)+(p+q)^2 = (p+q)(8+p+q)$
(b) Cannot be factorized. (c) Cannot be factorized. (d) Cannot be factorized.
(e) $= 4rs^2 - 4t^2 + st - 16rst = (4rs^2 - 16rst) + (st - 4t^2)$
$= 4rs(s-4t) + t(s-4t) = (s-4t)(4rs+t)$
(f) Cannot be factorized.
(g) $= (3n+4mn) - (6p+8mp) + (18m+24m^2)$
$= n(3+4m) - 2p(3+4m) + 6m(3+4m) = (3+4m)(n-2p+6m)$

10. $= 4[-(a+b)]^2 - (a+b) = 4(a+b)^2 - (a+b) = (a+b)[4(a+b)-1] = (a+b)(4a+4b-1)$

11. $= (x-y)^3 + 9(x-y)^2 - (x-y) = (x-y)[(x-y)^2 + 9(x-y) - 1]$
$= (x-y)(x^2 - 2xy + y^2 + 9x - 9y - 1)$

12. $= \ell(s^2t + \ell s^2t + s + \ell s - st^2 - t) = \ell[(s^2t+s) + (\ell s^2t + \ell s) - (t + st^2)]$
$= \ell[s(st+1) + \ell s(st+1) - t(st+1)] = \ell(st+1)(s+\ell s - t)$

13. $= (1+x^2+x^4+x^6) + x(1+x^2+x^4+x^6) = (1+x^2+x^4+x^6)(1+x)$
$= [(1+x^2) + x^4(1+x^2)](1+x) = (1+x^4)(1+x^2)(1+x)$
Ans. Its 3 factors are $(1+x)$, $(1+x^2)$, *and* $(1+x^4)$.

14. $= 1 + x + 2x + 2x^2 - x^2 - x^3 = (1+x) + 2x(1+x) - x^2(1+x) = (1+x)(1+2x-x^2)$

15. The area of David's rectangle $= 15(2y^3 + y^2 + 2y + 1) = 15[\,y^2(2y+1) + (2y+1)\,]$
$= 15(2y+1)(y^2+1)$ cm^2
The width of Donna's rectangle $= (30y + 15) = 15(2y+1)$ cm
$\because$ the two rectangles have the same area,
$\therefore$ the length of Donna's rectangle $= \dfrac{15(2y+1)(y^2+1)}{15(2y+1)} = (y^2+1)$ cm

16. (a) When $n = 1$, $S = 1^2 + 1 + 41 = 43$; When $n = 2$, $S = 2^2 + 2 + 41 = 47$;
When $n = 3$, $S = 3^2 + 3 + 41 = 53$; When $n = 4$, $S = 4^2 + 4 + 41 = 61$;
When $n = 5$, $S = 5^2 + 5 + 41 = 71$
(b) $S = n^2 + n + 41 = n(n+1) + 41$
When $n = 41$, $S = 41(42)+41 = 41(42+1)$ which is a composite number.
When $n = 40$, $S = 40(41)+41 = 41(40+1)$ which is also a composite number.
Ans. The two values of n are 40 and 41.

Unit 4 Algebraic identities & factorization

1. $x^2 + 6x + p \equiv x^2 + 2qx + q^2$, $\therefore$ $2q = 6$ and $p = q^2$, $q = 3$ and $p = 3^2 = 9$
2. $x^2 + 4x + 4 - x^2 + x + 2 \equiv Ax + (B - A)$, $5x + 6 \equiv Ax + (B - A)$,
$\therefore$ A = 5 and B – A = 6; $\therefore$ B – 5 = 6, B = 11. *Ans. A = 5 and B = 11.*
3. $x^2 - 6x + 9 + Ax - 3A \equiv x^2 + B$, $x^2 + (A-6)x + (9 - 3A) \equiv x^2 + B$, $\therefore$ A – 6 = 0
and 9 – 3A = B. A = 6 and 9 – 3(6) = B, B = –9. *Ans. A = 6 and B = –9.*
4. $3Ax^2 + (3B + A)x + B \equiv 6x^2 + Cx - 2$, $\therefore$ 3A = 6, 3B + A = C and B = –2,
A = 2 and 3(–2) + 2 = C, C = –4. *Ans. A = 2, B = –2 and C = –4.*
5. L.H.S.$= [(x+3y) + (3y-x)][(x+3y) - (3y-x)] = (6y)(2x) = 12xy$ = R.H.S.

$\therefore$ It is an identity.

6. (a) L.H.S. $= 3x + 18 + 2x - 16 = 5x + 2$, R.H.S.$= 7x + 14 - 5x - 9 = 2x + 5 \neq$ L.H.S.
$\therefore$ It is not an identity.
(b) L.H.S. $= x^2 - 3xy$, R.H.S.$= x^2 - 3xy - 4y^2 + 4y^2 = x^2 - 3xy =$ L.H.S.
$\therefore$ It is an identity.
(c) L.H.S.$= 8x - 3y - 2y - 3z - 3x + 2z = 5x - 5y - z$,
R.H.S.$= 5x - 5y - 5z + 4z = 5x - 5y - z =$ L.H.S., $\therefore$ It is an identity.
(d) L.H.S. $= y^2 + 6y + 9 - 10 = y^2 + 6y - 1$, R.H.S. $= y^2 - 1 \neq$ L.H.S.
$\therefore$ It is not an identity.
(e) R.H.S. $= 2x^2 - (2x^2 - 3x - 9) = 3x + 9 = 3(x + 3) =$ L.H.S., $\therefore$ It is an identity.

7. L.H.S. $= 2[36x^2 + 12x + 1 - (36x^2 + 9x - 10)] = 2(3x + 11) = 6x + 22$,
R.H.S. $= 10x - 18 + 55x + 40 - 60x + x = 6x + 22 =$ L.H.S.
$\therefore$ It is an identity.

8. (a) $= 25x^2 + 40xy + 16y^2$ (b) $= 49m^2 - 4n^2$ (c) $= 9x^2 - 48x + 64$
(d) $= 1 - 16x^2$ (e) $= 36a^2b^2 - 25$
(f) $= (4t - 9)^2 = 16t^2 - 72t + 81$ (g) $= 16p^4 - 56p^2 + 49$

9. (a) $= (10 + 2x)^2 = 100 + 40x + 4x^2$ (b) $= 4x^2 - 2(2x)(\frac{1}{x}) + \frac{1}{x^2} = 4x^2 - 4 + \frac{1}{x^2}$
(c) $= 2(36x^2 + 60xy + 25y^2) = 72x^2 + 120xy + 50y^2$
(d) $= (\frac{2}{m})^2 + 2(\frac{2}{m})(\frac{3}{n}) + (\frac{3}{n})^2 = \frac{4}{m^2} + \frac{12}{mn} + \frac{9}{n^2}$
(e) $= (7a + \frac{3}{a})(7a - \frac{3}{a}) = 49a^2 - \frac{9}{a^2}$
(f) $= 3(16 - y^4) = 48 - 3y^4$ (g) $= (-6x)^2 - (13y)^2 = 36x^2 - 169y^2$

10. (a) $= (3 - 5y)^2$ (b) $= (2n - 7)^2$ (c) $= (x + 8)^2$
(d) $= (3y + 1)(3y - 1)$ (e) $= (5m + 2n)^2$ (f) $= (7xy + 2)^2$
(g) $= (9a - 10b)(9a + 10b)$ (h) $= 121b^2 - 16a^2 = (4a + 11b)(11b - 4a)$

11. (a) $= 3(x^2 - 25) = 3(x - 5)(x + 5)$ (b) $= 5(y^2 - 6y + 9) = 5(y - 3)^2$
(c) $= (ab + 6pq)^2$ (d) $= 1 - (4y^2)^2 = (1+4y^2)(1-4y^2) = (1+4y^2)(1+2y)(1-2y)$
(e) $= 3(x^4 - 9y^4) = 3(x^2 - 3y^2)(x^2 + 3y^2)$ (f) $= 9(16 - 9k^2) = 9(4 - 3k)(4 + 3k)$
(g) $= 4(4m^2 - 9) = 4(2m - 3)(2m + 3)$
(h) $= [3y - (5y + x)][3y + (5y + x)] = -(2y + x)(8y + x)$
(i) $= p(p^2 - 10pq + 25q^2) = p(p - 5q)^2$
(j) $= (x^4 - 1)(x^4 + 1) = (x^2 - 1)(x^2 + 1)(x^4 + 1) = (x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$
(k) $= [(x - 2y) + 7]^2 = (x - 2y + 7)^2$
(l) $= [2(a - b)]^2 - [5(a + b)]^2 = [(2a - 2b) - (5a + 5b)][(2a - 2b) + (5a + 5b)]$
$= -(3a + 7b)(7a + 3b)$

12. (a) not possible $(not: 4x^2 + 4x + 1)$ (b) not possible $(not: y^2 + 14y + 49)$
(c) $(1 - 3a)^2$ (d) not possible $(not: n^2 - 12n + 36)$
(e) not possible $(not: x^2 - 9y^2)$ (f) $= 5(4k^2 - 1) = 5(2k + 1)(2k - 1)$
(g) not possible $(not: 16y^2 + 8y + 1)$ (h) $= m^2 + 6m + 9 = (m + 3)^2$
(i) $= (\frac{a}{3})^2 - 2(\frac{a}{3})(\frac{b}{2}) + (\frac{b}{2})^2 = (\frac{a}{3} - \frac{b}{2})^2$ (j) $= (x)^2 + 2(x)(\frac{1}{x}) + (\frac{1}{x})^2 = (x + \frac{1}{x})^2$

13. (a) $= [(a + b) + c]^2 = (a + b)^2 + 2c(a + b) + c^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
(b) $= [(x + 3) - 4y][(x + 3) + 4y] = (x + 3)^2 - (4y)^2 = x^2 + 6x + 9 - 16y^2$
(c) $= [(2x - y) + 6]^2 = (2x - y)^2 + 12(2x - y) + 36$
$= 4x^2 + y^2 - 4xy + 24x - 12y + 36$

(d) $= [p+(3q-r)][p-(3q-r)] = p^2-(3q-r)^2$

$= p^2-(9q^2-6qr+r^2) = p^2-9q^2+6qr-r^2$

14. $= (\frac{m-1}{m}+\frac{m+1}{m})(\frac{m-1}{m}-\frac{m+1}{m}) = (\frac{2m}{m})(\frac{-2}{m}) = \frac{-4}{m}$

15. $= \sqrt{(10001+9999)(10001-9999)} = \sqrt{20000\times 2} = \sqrt{40000} = \sqrt{(200)^2} = 200$

16. $= (x^2+2x+1)+(y+xy) = (x+1)^2+y(x+1) = (x+1)(x+y+1)$

17. $= (m^2+2mn+n^2)+(m^2n+mn^2) = (m+n)^2+mn(m+n) = (m+n)(m+n+mn)$

18. $= y^2-4+\frac{4}{y^2} = y^2-2(y)(\frac{2}{y})+(\frac{2}{y})^2 = (y-\frac{2}{y})^2$

19. $= mn+4-2m-2n = (mn-2m)-(2n-4) \quad = m(n-2)-2(n-2) \quad = (n-2)(m-2)$

20. $= (s^2-4rs+4r^2)-\frac{s^2}{4} = (s-2r)^2-(\frac{s}{2})^2$

$= [(s-2r)+\frac{s}{2})][(s-2r)-\frac{s}{2}] = (\frac{3s}{2}-2r)(\frac{s}{2}-2r)$

21. $= y(x^2-y^2-2x+1) = y[(x^2-2x+1)-y^2] = y[(x-1)^2-y^2] = y(x-1+y)(x-1-y)$

22. $= (2x+1+x^2)(2x+1-x^2) = (x+1)^2(2x+1-x^2)$

23. $= (x^2+9+6x)(x^2+9-6x) = (x+3)^2(x-3)^2$

24. $= x^2-y^2-6x+6y = (x+y)(x-y)-6(x-y) = (x-y)(x+y-6)$

25. $= x^2+y^2+2xy+2xy-1-x^2y^2 = (x^2+2xy+y^2)-(1-2xy+x^2y^2) = (x+y)^2-(1-xy)^2$

$= [(x+y)+(1-xy)][(x+y)-(1-xy)] = (x+y+1-xy)(x+y-1+xy)$

26. $(a+\frac{1}{a})^2 = 2^2, a^2+2a(\frac{1}{a})+\frac{1}{a^2} = 4, \quad a^2+2+\frac{1}{a^2} = 4, a^2+\frac{1}{a^2} = 2$

27. $= h^2-k^2-h+k = (h^2-k^2)-(h-k) = (h-k)(h+k)-(h-k) = (h-k)(h+k-1)$

28. $= [(a^2+6a)+9]^2 = [(a+3)^2]^2 = (a+3)^4$

29. (a) $(p^2+8q^2)^2$ (b) $= (p^4+16p^2q^2+64q^4)-16p^2q^2$

$= (p^2+8q^2)^2-(4pq)^2 = (p^2+8q^2-4pq)(p^2+8q^2+4pq)$

30. (a) $(a^2+b^2)^2$ (b) $= (a^4+2a^2b^2+b^4)-a^2b^2 = (a^2+b^2)^2-(ab)^2$

$= (a^2+b^2+ab)(a^2+b^2-ab)$

31. (a) $= \frac{1}{3}(x^2+6x+9) = \frac{1}{3}(x+3)^2$

(b) $= \frac{1}{3}(p+q)^2+2p+2q+3 = \frac{1}{3}(p+q)^2+2(p+q)+3$

Put $p+q=x$ in part (a), the result = $\frac{1}{3}[(p+q)+3]^2 = \frac{1}{3}(p+q+3)^2$

32. (a) $= [(\frac{3}{x}+\frac{x}{3})+(\frac{3}{x}-\frac{x}{3})][(\frac{3}{x}+\frac{x}{3})-(\frac{3}{x}-\frac{x}{3})] \quad = (\frac{6}{x})(\frac{2x}{3}) = 4$

(b) From (a), $(\frac{3}{x}+\frac{x}{3})^2-(\frac{3}{x}-\frac{x}{3})^2 = 4$,

$\therefore \ (\frac{3}{x}+\frac{x}{3})^2-(\sqrt{5})^2 = 4, (\frac{3}{x}+\frac{x}{3})^2 = 9, \quad \frac{3}{x}+\frac{x}{3} = 3 \text{ or } -3$

33. (a) R.H.S. $= (x+y)(x^2-xy+y^2) = x^3-x^2y+xy^2+x^2y-xy^2+y^3 = x^3+y^3$ =L.H.S.

$\therefore$ It is an identity.

(b) R.H.S. $= (x-y)(x^2+xy+y^2) = x^3+x^2y+xy^2-x^2y-xy^2-y^3 = x^3-y^3$ =L.H.S.

$\therefore$ It is an identity.

(c) $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x^3 + y^3)(x - y)(x^2 + xy + y^2)$ [from (b)]
$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$ [from (a)]
Ans. Its factors are $(x + y)$, $(x - y)$, $(x^2 - xy + y^2)$ *and* $(x^2 + xy + y^2)$.

34. Dividend $= 16x^2 + kx + 4$, divisor $= 2x + 5$, quotient $= mx + n$, remainder $= -1$.
$\because$ Dividend $\equiv$ divisor $\times$ quotient $+$ remainder
$\therefore\ 16x^2 + kx + 4 \equiv (2x + 5)(mx + n) - 1$, $16x^2 + kx + 4 \equiv 2mx^2 + 2nx + 5mx + 5n - 1$
Comparing the coefficients, we get: $16 = 2m$, $4 = 5n - 1$ and $k = 2n + 5m$,
$\therefore\ m = \frac{16}{2} = 8$, $n = \frac{4+1}{5} = 1$, $k = 2(1) + 5(8) = 42$. *Ans.* $m = 8, n = 1, k = 42$.

35. (a) $x^2 - 33 = 16y^2$, $x^2 - 16y^2 = 33$, $(x + 4y)(x - 4y) = 33$;
but $1 + 4y = x$, $1 = x - 4y$; $\therefore$ $(x + 4y)(1) = 33$, $x + 4y = 33$
(b) $x = 33 - 4y$ ….. (i), and $x = 1 + 4y$ ….. (ii). From (i) and (ii), $1 + 4y = 33 - 4y$,
$8y = 32$, $y = 4$; $\therefore$ $x = 4(4) + 1 = 17$ *Ans.* $x = 17, y = 4$.

36. $x^2 - 2x - y^2 + 4y - 3 = (x^2 - 2x + 1) - (y^2 - 4y + 4) = (x - 1)^2 - (y - 2)^2$
$= [(x - 1) + (y - 2)]\,[(x - 1) - (y - 2)] = (x + y - 3)(x - y + 1)$

37. $= x^4 + 18^2 - 100x^2 = [x^4 - 2(18)x^2 + 18^2] + 2(18)x^2 - 100x^2$
$= (x^2 - 18)^2 - 64x^2 = (x^2 - 18)^2 - (8x)^2 = (x^2 - 18 + 8x)(x^2 - 18 - 8x)$

Unit 5 Algebraic fractions (1)

1. (a) $\frac{3}{5y^4}$ (b) $\frac{3x^4}{2}$ (c) $\frac{3b^2}{7}$ (d) $\frac{3(x+2y)}{x-2y}$
(e) $\frac{5x^2(z-4)}{z}$ (f) $\frac{4}{3(2a+b)}$

2. (a) $= \frac{9x^2(6x-1)}{9x} = x(6x - 1)$ (b) $= \frac{2(3m-n)}{10x(3m-n)} = \frac{1}{5x}$
(c) $= \frac{a-b}{(a-b)(a+b)} = \frac{1}{a+b}$
(d) $= \frac{4(x^2-2x+1)}{x(x-a)-(x-a)} = \frac{4(x-1)^2}{(x-a)(x-1)} = \frac{4(x-1)}{x-a}$
(e) $= \frac{3x(x^2-a)+(x^2-a)}{7x(3x+1)} = \frac{(3x+1)(x^2-a)}{7x(3x+1)} = \frac{x^2-a}{7x}$

3. (a) $= \frac{2(3-2a)}{-3(3-2a)} = -\frac{2}{3}$ (b) $= \frac{8(1-k)}{-k^2(1-k)} = -\frac{8}{k^2}$
(c) $= \frac{-y(y^2+5)}{y^2+5} = -y$ (d) $= \frac{9(x-y)^2}{12(x-y)} = \frac{3}{4}(x-y)$
(e) $= \frac{-3x^2(5y-2x)}{7(5y-2x)} = -\frac{3x^2}{7}$

4. (a) $\frac{5}{6}$ (b) $= \frac{12m}{7} \times \frac{21}{4m} = 9$ (c) $= \frac{x^4}{3(x-2)} \times \frac{3}{x^3} = \frac{x}{x-2}$
(d) $= 9ab^3 \times \frac{6}{a^2b^2} = \frac{54b}{a}$ (e) $= \frac{1}{5}ab^2 \times \frac{b^2}{5a} \times a^2 = \frac{a^2b^4}{25}$

(f) $=n^2x^3\times\frac{3}{2nx^2}\times\frac{3}{2n}=\frac{9x}{4}$

(g) $=\frac{3(b+2)}{6(b-2)}\times\frac{4(b-2)}{5(2+b)}=\frac{2}{5}$

(h) $=\frac{m(1-a)}{b(x-1)}\div\frac{c(1-a)}{n(1-x)}=\frac{m(1-a)}{b(x-1)}\times\frac{-n(x-1)}{c(1-a)}=-\frac{mn}{bc}$

5. (a) $=\frac{4(b+1)}{a-3}\times\frac{18(a-3)}{4(a-1)}\times\frac{1}{6(b+1)}=\frac{3}{a-1}$

(b) $=\frac{(x-y)(x+y)}{x^2+y^2}\times\frac{1}{(x+y)^2}\times[(x^2)^2-(y^2)^2]$

$=\frac{(x-y)}{(x+y)(x^2+y^2)}\times(x^2+y^2)(x^2-y^2)=\frac{(x-y)(x+y)(x-y)}{x+y}=(x-y)^2$

(c) $=\frac{b(a+b)}{a(2a-b)}\times\frac{5(2a-b)}{b^2(3a+1)}\times\frac{1}{a+b}=\frac{5}{ab(3a+1)}$

(d) $=\frac{x(y+1)-(y+1)}{x^2(x-1)-(x-1)}\times\frac{-12(x-1)}{8(y+1)}=\frac{(x-1)(y+1)}{(x^2-1)(x-1)}\times\frac{-3(x-1)}{2(y+1)}=\frac{-3(x-1)}{2(x-1)(x+1)}=\frac{-3}{2(x+1)}$

6. (a) $=\frac{10y-9y}{12}=\frac{y}{12}$

(b) $=\frac{12r-(2r+3)}{9}=\frac{10r-3}{9}$

(c) $=\frac{5(a-3)+2(a+2)}{30}=\frac{5a-15+2a+4}{30}=\frac{7a-11}{30}$

(d) $=\frac{y-7+5}{5}=\frac{y-2}{5}$

(e) $=\frac{4x-(x+2)}{4}=\frac{4x-x-2}{4}=\frac{3x-2}{4}$

7. (a) $\frac{1+5x}{xy}$

(b) $=\frac{3-(4+b)}{3a}=\frac{3-4-b}{3a}=-\frac{b+1}{3a}$

(c) $=\frac{3(2x+a)-2(a-3x)-(x+a)}{6x}=\frac{6x+3a-2a+6x-x-a}{6x}=\frac{11x}{6x}=\frac{11}{6}$

(d) $\frac{15m-2}{5m}$

(e) $=\frac{8x-(1-6x)}{4x}=\frac{8x-1+6x}{4x}=\frac{14x-1}{4x}$

8. (a) $\frac{12-y}{3(y+5)}$

(b) $=\frac{14-12}{21(x-y)}=\frac{2}{21(x-y)}$

(c) $=\frac{b}{a-b}+\frac{a}{a-b}=\frac{b+a}{a-b}$

(d) $=\frac{8-x}{6-x}-\frac{2}{6-x}=\frac{8-x-2}{6-x}=\frac{6-x}{6-x}=1$

(e) $=\frac{10y+9y-6y}{12(a-b)}=\frac{13y}{12(a-b)}$

9. (a) $=\frac{x(x-1)-2x^2}{x-1}=\frac{x^2-x-2x^2}{x-1}=\frac{x(1+x)}{1-x}$

(b) $=\frac{m+n+m-n}{m+n}=\frac{2m}{m+n}$

(c) $=\frac{3y(x+y)}{3xy}-\frac{x(x-y)}{3xy}=\frac{3xy+3y^2-x^2+xy}{3xy}=\frac{4xy+3y^2-x^2}{3xy}$

(d) $=\frac{2(k+2)-3(k-3)}{(k-3)(k+2)}=\frac{2k+4-3k+9}{(k-3)(k+2)}=\frac{13-k}{(k-3)(k+2)}$

(e) $=\frac{y(2y-x)-x(2x-y)}{(2x-y)(2y-x)}=\frac{2y^2-xy-2x^2+xy}{(2x-y)(2y-x)}=\frac{2(y+x)(y-x)}{(2x-y)(2y-x)}$

10. (a) $=\frac{x-4+3x}{x(x-1)(x-4)}=\frac{4(x-1)}{x(x-1)(x-4)}=\frac{4}{x(x-4)}$

(b) $=\frac{y-2(y+3)}{(y+3)^2}=\frac{y-2y-6}{(y+3)^2}=-\frac{y+6}{(y+3)^2}$

(c) $=\dfrac{4(1-x)-2(x-2)}{(3x-4)(x-2)(1-x)}=\dfrac{4-4x-2x+4}{(3x-4)(x-2)(1-x)}=\dfrac{8-6x}{(3x-4)(x-2)(1-x)}$

$=\dfrac{-2(3x-4)}{(3x-4)(x-2)(1-x)}=\dfrac{2}{(x-2)(x-1)}$

(d) $=\dfrac{5(a+2)-4(a+1)}{(a+1)(a+6)(a+2)}=\dfrac{5a+10-4a-4}{(a+1)(a+6)(a+2)}$

$=\dfrac{a+6}{(a+1)(a+6)(a+2)}=\dfrac{1}{(a+1)(a+2)}$

(e) $=\dfrac{2(1+k)+2(1-k)-2}{(1+k)(1-k)}=\dfrac{2+2k+2-2k-2}{(1+k)(1-k)}=\dfrac{2}{(1+k)(1-k)}$

(f) $=\dfrac{4(x-2)+3(x+2)-7x}{(x+2)(x-2)}=\dfrac{4x-8+3x+6-7x}{(x+2)(x-2)}=\dfrac{-2}{(x+2)(x-2)}$

11. (a) $=\dfrac{5}{2(a-4b)}+\dfrac{a-b}{b(a-4b)}=\dfrac{5b+2(a-b)}{2b(a-4b)}=\dfrac{5b+2a-2b}{2b(a-4b)}=\dfrac{3b+2a}{2b(a-4b)}$

(b) $=\dfrac{2}{(x+3)(x-4)}-\dfrac{1-x}{(4-x)(4+x)}=\dfrac{2(x+4)+(1-x)(x+3)}{(x+3)(x-4)(x+4)}$

$=\dfrac{2x+8+3-2x-x^2}{(x+3)(x-4)(x+4)}=\dfrac{11-x^2}{(x+3)(x-4)(x+4)}$

(c) $=\dfrac{2}{x(x+2)}+\dfrac{4}{(x+2)(x+6)}=\dfrac{2(x+6)+4x}{x(x+2)(x+6)}=\dfrac{2x+12+4x}{x(x+2)(x+6)}$

$=\dfrac{6(x+2)}{x(x+2)(x+6)}=\dfrac{6}{x(x+6)}$

(d) $=\dfrac{a+b}{b(b-a)}-\dfrac{a+b}{a(b-a)}=\dfrac{a+b}{b-a}(\dfrac{1}{b}-\dfrac{1}{a})=\dfrac{a+b}{b-a}(\dfrac{a-b}{ab})=-\dfrac{a+b}{ab}$

12. $=\dfrac{x^2(x-y)+y^2(x-y)}{(x^2)^2-(y^2)^2}=\dfrac{(x^2+y^2)(x-y)}{(x^2+y^2)(x^2-y^2)}=\dfrac{x-y}{(x+y)(x-y)}=\dfrac{1}{x+y}$

13. $=\dfrac{2a-b}{(3a+b)(x-y)}+\dfrac{3-x}{4x(x-y)}=\dfrac{4x(2a-b)}{4x(3a+b)(x-y)}+\dfrac{(3a+b)(3-x)}{4x(x-y)(3a+b)}$

$=\dfrac{8ax-4bx+9a-3ax+3b-bx}{4x(3a+b)(x-y)}=\dfrac{5ax-5bx+9a+3b}{4x(3a+b)(x-y)}$

14. $=\dfrac{m-3n}{(3m-5n)^2}-\dfrac{m+n}{(3m-5n)(4+n)}=\dfrac{(4+n)(m-3n)}{(4+n)(3m-5n)^2}-\dfrac{(3m-5n)(m+n)}{(3m-5n)^2(4+n)}$

$=\dfrac{4m-12n+mn-3n^2-(3m^2-2mn-5n^2)}{(4+n)(3m-5n)^2}=\dfrac{4m-12n+3mn+2n^2-3m^2}{(4+n)(3m-5n)^2}$

15. $8ab^2+12+8b+16ab+4b^2+24a=4(2ab^2+3+2b+4ab+b^2+6a)$

$=4[(3+6a)+(2b+4ab)+(b^2+2ab^2)]=4(1+2a)(3+2b+b^2)$

$\therefore$ The given expression $=\dfrac{5(3+2b+b^2)}{4(1+2a)(3+2b+b^2)}-\dfrac{7a-4}{3(2a-1)(2a+1)}$

$=\dfrac{5}{4(1+2a)}-\dfrac{7a-4}{3(2a-1)(2a+1)}=\dfrac{5\cdot 3(2a-1)-4(7a-4)}{12(1+2a)(2a-1)}$

$=\dfrac{30a-15-28a+16}{12(1+2a)(2a-1)}=\dfrac{2a+1}{12(2a+1)(2a-1)}=\dfrac{1}{12(2a-1)}$

16. $= (\frac{p^2-q^2}{pq}) \div (\frac{q-p}{pq}) = \frac{(p+q)(p-q)}{pq} \times \frac{pq}{q-p} = -(p+q)$

17. $= \frac{2ab+a^2+b^2}{ab} \div \frac{a^2-b^2}{ab} = \frac{(a+b)^2}{ab} \cdot \frac{ab}{(a+b)(a-b)} = \frac{a+b}{a-b}$

18. $= 1 \div \left(1+\frac{(2y)\cdot y}{(\frac{1}{y}-y)\cdot y}\right) = 1 \div \left(1+\frac{2y^2}{1-y^2}\right) = 1 \div \left(\frac{1-y^2+2y^2}{1-y^2}\right) = 1 \div \frac{1+y^2}{1-y^2} = \frac{1-y^2}{1+y^2}$

19. (a) The other adjacent side = area × 2 ÷ side = 2 $(4x^2 - 20x + 25) \div (2x-5)$
$= 2(2x-5)^2 \div (2x-5) = 2(2x-5)$ cm

(b) $2(2x-5) > (2x-5)$, $\therefore$ $2(2x-5) = 18$, $2x-5 = 9$, $\therefore$ $x = 7$
$\therefore$ Its area $= 4(7)^2 - 20(7) + 25 = 81$ cm^2

Unit 6 Use of formulae

1. (a) $A = lw$
(b) $A = 6x^2$
(c) $T = \frac{D}{S}$
(d) $C = \frac{a}{12} \times m + b \times n$, $\therefore$ $C = \frac{am}{12} + bn$

2. (a) $a = \frac{180(12-2)}{12} = \frac{1800}{12} = 150$
(b) $99 = \frac{1}{2}(11)(4+b)$, $18 = 4+b$, $\therefore$ $b = 14$
(c) $D = (-6)^2 - 4(3)(-2) = 36 + 24 = 60$
(d) $\frac{1}{6} = \frac{1}{4} - \frac{1}{v}$, $\frac{1}{v} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$, $\therefore$ $v = 12$
(e) $7600 = 4000(1+0.15t)$, $1.9 = 1+0.15t$, $0.15t = 0.9$, $\therefore$ $t = 6$

3. (a) $s = (10)(3) + \frac{1}{2}(8)(3)^2 = 30 + 36 = 66$
(b) $84 = (6)(4) + \frac{1}{2}a(4)^2$, $60 = 8a$, $\therefore$ $a = 7.5$

4. $2s = 5 + 12 + 13 = 30$, $s = 15$, $\therefore$ $A = \sqrt{15(15-5)(15-12)(15-13)} = \sqrt{900} = 30\,\text{cm}^2$

5. (a) $p - b = 2ax$, $\therefore$ $x = \frac{p-b}{2a}$
(b) $ax + bx = c$, $x(a+b) = c$, $\therefore$ $x = \frac{c}{a+b}$
(c) $\frac{x}{a} = c - b$, $\therefore$ $x = a(c-b)$
(d) $m - x = n^2$, $\therefore$ $x = m - n^2$
(e) $2rx = q - k$, $\therefore x = \frac{q-k}{2r}$
(f) $2x = 5b - e$, $\therefore$ $x = \frac{5b-e}{2}$
(g) $8m = h - 4x$, $4x = h - 8m$, $\therefore$ $x = \frac{h-8m}{4}$
(h) $3y + 3 = x + u$, $\therefore$ $x = 3y + 3 - u$
(i) $\frac{f}{x} = g - 1$, $\therefore$ $x = \frac{f}{g-1}$
(j) $ax + b = ct$, $ax = ct - b$, $x = \frac{ct-b}{a}$

6. (a) $m + n = x - ax$, $m + n = x(1-a)$, $\therefore$ $x = \frac{m+n}{1-a}$
(b) $ax + 2a = bx - 3b$, $2a + 3b = bx - ax$, $2a + 3b = x(b-a)$, $\therefore$ $x = \frac{2a+3b}{b-a}$
(c) $nx + mx = mns$, $x(n+m) = mns$, $\therefore$ $x = \frac{mns}{n+m}$

(d) $xy+a=b$, $xy=b-a$, $\therefore\ x=\dfrac{b-a}{y}$

(e) $sh+sx=kr-rx$, $sx+rx=kr-sh$, $x(s+r)=kr-sh$, $\therefore\ x=\dfrac{kr-sh}{s+r}$

(f) $x^2-cx=x^2-ax+bx-ab$, $ab=cx-ax+bx$, $ab=x(c-a+b)$,

$\therefore\ x=\dfrac{ab}{c-a+b}$

(g) $mqx-3pq=4pnx$, $mqx-4pnx=3pq$, $x(mq-4pn)=3pq$,

$\therefore\ x=\dfrac{3pq}{mq-4pn}$

(h) $\dfrac{6nx+3m}{2k}=-x$, $6nx+3m=-2kx$, $6nx+2kx=-3m$, $x(6n+2k)=-3m$,

$\therefore\ x=\dfrac{-3m}{6n+2k}$

7. (a) $\dfrac{s}{180}=n-2$, $\therefore\ n=\dfrac{s}{180}+2$

(b) $2A=bh$, $\therefore\ b=\dfrac{2A}{h}$

(c) $\dfrac{A}{\pi}=r^2$, $r=\sqrt{\dfrac{A}{\pi}}$

(d) $y-c=mx$, $\therefore m=\dfrac{y-c}{x}$

(e) $A-2\pi r^2=2\pi rh$, $\therefore\ h=\dfrac{A-2\pi r^2}{2\pi r}$

(f) $3v=\pi r^2h$, $\therefore\ h=\dfrac{3v}{\pi r^2}$

(g) $x-a=(n-1)d$, $\dfrac{x-a}{d}=n-1$, $\therefore\ n=\dfrac{x-a}{d}+1$

8. (a) $b^2=c^2-a^2$, $\therefore\ b=\sqrt{c^2-a^2}$

(b) $\dfrac{2T}{n}=a+b$, $\therefore\ a=\dfrac{2T}{n}-b$

(c) $s-ut=\dfrac{1}{2}at^2$, $\therefore\ a=\dfrac{2(s-ut)}{t^2}$

(d) $\dfrac{v+u}{uv}=\dfrac{1}{f}$, $\therefore f=\dfrac{uv}{u+v}$

(e) $\dfrac{1}{u}-\dfrac{1}{f}=\dfrac{1}{v}$, $\dfrac{f-u}{uf}=\dfrac{1}{v}$, $\therefore v=\dfrac{uf}{f-u}$

(f) $bx+ay=ab$, $ay=ab-bx$, $ay=b(a-x)$, $\therefore\ b=\dfrac{ay}{a-x}$

(g) $s(R-1)=a(R^n-1)$, $\therefore\ a=\dfrac{s(R-1)}{R^n-1}$

(h) $aH=bH-bh$, $bh=bH-aH$, $bh=H(b-a)$, $\therefore\ H=\dfrac{bh}{b-a}$

(i) $kx-ky=mn+p-y$, $y-ky=mn+p-kx$, $y(1-k)=mn+p-kx$,

$\therefore\ y=\dfrac{mn+p-kx}{1-k}$

(j) $40as-40bs=abc$, $40as=abc+40bs$, $40as=b(ac+40s)$,

$\therefore\ b=\dfrac{40as}{ac+40s}$

(k) $2s=2an+n(n-1)d$, $2s-2an=n(n-1)d$, $\therefore\ d=\dfrac{2s-2an}{n(n-1)}$

9. (a) $\dfrac{T}{2\pi}=\sqrt{\dfrac{L}{g}}$, $(\dfrac{T}{2\pi})^2=\dfrac{L}{g}$, $\therefore\ L=\dfrac{gT^2}{4\pi^2}$

(b) $2Lf = \sqrt{\frac{T}{m}}, (2Lf)^2 = \frac{T}{m}, \quad \therefore \ m = \frac{T}{4L^2 f^2}$

(c) $x - xt^2 = 1 + t^2, \ x - 1 = t^2 + xt^2, \ x - 1 = t^2(1 + x), \ t^2 = \frac{x-1}{x+1}, \quad \therefore \ t = \sqrt{\frac{x-1}{x+1}}$

(d) $\frac{c}{2} = \sqrt{a+b}, \ (\frac{c}{2})^2 = a + b, \quad \therefore \ b = \frac{c^2}{4} - a$

(e) $c^2 = a^2 + b^2 - 2abk, \ 2abk = a^2 + b^2 - c^2, \quad \therefore \ k = \frac{a^2 + b^2 - c^2}{2ab}$

(f) $x - p = \sqrt{q^2 - r^2}, \ (x-p)^2 = q^2 - r^2, \ r^2 = q^2 - (x-p)^2,$

$\therefore \ r = \sqrt{q^2 - (x-p)^2}$

10. (a) $C = \frac{5}{9}(F - 32), \frac{9C}{5} = F - 32, \quad \therefore \ F = \frac{9C}{5} + 32$

(b) Water boils at 100°C. When $C = 100$, $F = \frac{9(100)}{5} + 32 = 212$.

Ans. Water boils at 100°C and 212°F.

(c) Water freezes at 0°C. When $C = 0$, $F = \frac{9(0)}{5} + 32 = 32$.

Ans. Water freezes at 0°C and 32°F.

(d) When $F = 99.5$, $C = \frac{5}{9}(99.5 - 32) = 37.5$.

Ans. The normal body temperature is 37.5°C.

(e) When $C = 40.5$, $F = \frac{9(40.5)}{5} + 32 = 104.9$.

Ans. His body temperature is 104.9°F.

11. When $n = 1$, $S = 9$. When $n = 2$, $S = 13 = 9 + 4(1)$.
When $n = 3$, $S = 17 = 9 + 4(2)$. $\therefore \ S = 9 + 4(n-1) = 4n + 5$

12. When $w = 0$, $l = 15$. When $w = 10$, $l = 18 = 15 + \frac{3}{10}(10)$.

When $w = 20$, $l = 21 = 15 + \frac{3}{10}(20)$. When $w = 30$, $l = 24 = 15 + \frac{3}{10}(30)$.

$\therefore \ l = 15 + \frac{3}{10}w$

13. (a) $B = \frac{A+C}{2}, \quad \therefore$ C = 2B – A.

(b) All the values of Z are even numbers. $X^2 + Y^2 = \frac{Z}{2},$

$\therefore \ X^2 = \frac{Z}{2} - Y^2, \quad X = \sqrt{\frac{Z}{2} - Y^2}$

14. (a) $\because$ $(n-1)$ cards should be sent by each of the n students. $\therefore \ T = n(n-1)$.

(b) When $n = 36$, $T = 36(36-1) = 1260$. *Ans. The number of cards sent is 1260.*

15. (a) $k = an + b$ (b) $k = an + b, \ k - b = an, \quad \therefore \ n = \frac{k-b}{a}$

(c) $k = c(n-1) + d, \ k = c(\frac{k-b}{a} - 1) + d, \ k = \frac{kc - bc}{a} - c + d,$

$ak = kc - bc - ac + ad, \ ak - kc = ad - bc - ac,$

$k(a-c) = ad - bc - ac$, $\quad \therefore \; k = \dfrac{ad - bc - ac}{a-c}$

16. $x = \dfrac{t+1}{2}$, $2x = t+1$, $t = 2x-1$; $y = \sqrt{t-1}$, $y = \sqrt{2x-1-1}$, $\quad \therefore \; y = \sqrt{2x-2}$

17. When $V = 1000$, $1000 = \dfrac{4}{3}\pi r^3$, $750 = \pi r^3$, $r^3 = \dfrac{750}{\pi}$, $r = \sqrt[3]{\dfrac{750}{\pi}}$,

    $\therefore \; A = 4\pi(\sqrt[3]{\dfrac{750}{\pi}})^2 = 483.60$ *Ans.* *The total surface area is 483.60 cm².*

18. (a) $1+2+3+.......+1000 = \dfrac{1000(1000+1)}{2} = 500500$

    (b) $2+4+6+......+1000 = 2(1+2+3+......+500) = 2\times\dfrac{500(500+1)}{2} = 250500$

    (c) $1+3+5+......+999 = (1+2+3+......+1000) - (2+4+6+......+1000)$
    $= 500500 - 250500 = 250000$

    (d) $500+501+502+......+1000 = (1+2+3+......+1000) - (1+2+3+......+499)$
    $= 500500 - \dfrac{499(499+1)}{2} = 500500 - 124750 = 375750$

    (e) Sum $= 3+6+9+......+999 = 3(1+2+3+......+333) = 3\times\dfrac{333(333+1)}{2} = 166833$

19. (a) When $t = 0$, $s = -5(0-3)^2 + 80 = -45 + 80 = 35$
    Ans. The height of the building is 35 m.

    (b) $\because \; (t-3)^2 \geq 0$, $\quad \therefore \; -5(t-3)^2 \leq 0$, $-5(t-3)^2 + 80 \leq 80$, $s \leq 80$
    Ans. The maximum height the stone can reach is 80 m.

    (c) The height is maximum when $(t-3)^2 = 0$, i.e. $t = 3$
    Ans. The stone takes 3 s to reach the maximum height.

20. (a) $\dfrac{1}{a}$ of the hall will be painted. (b) $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{x}$

    (c) When $a = 24$, $x = 8$, $\dfrac{1}{24} + \dfrac{1}{b} = \dfrac{1}{8}$, $\dfrac{1}{b} = \dfrac{1}{12}$, $b = 12$,
    Ans. It takes 12 hours for Simon to complete the work alone.

21. (a) By Formula A, when $P = 81$, $N = 10\sqrt{81} = 10(9) = 90$
    By Formula B, when $P = 81$, $N = 0.85(81) + 15 = 83.85$

    (b) By Formula A, when $N = 50$, $50 = 10\sqrt{P}$, $5 = \sqrt{P}$, $P = 5^2 = 25$,
    By Formula B, when $N = 50$, $50 = 0.85P + 15$, $35 = 0.85P$, $P = 41.2$,
    Ans. Jane's original score is 25, and Susan's original score is 41.2.

    (c) From the result of (b), original scores of 25 and 41.2 will become 50 using Formula A and B respectively. Therefore, more students will pass the examination if Formula A is adopted.

22. (a) $\dfrac{24}{24-h} = \dfrac{8}{x}$, $3x = 24 - h$, $\quad \therefore \; h = 24 - 3x$

    (b) $V = \dfrac{(x+8)h}{2} \times 18 = 9(x+8)(24-3x) = 9(192 - 3x^2)$, $\quad \therefore \; V = 1728 - 27x^2$

    (c) When $x = 6$, $V = 1728 - 27(6)^2 = 1728 - 972 = 756$.
    Ans. The volume of water is 756 cm³.

(d) Let H cm be the height of the container, $\frac{2}{3}H = 24 - 3(4) = 12$, $H = 18$.

When the container is full of water, $h = 18$ and $x = a$,

$\therefore$ $18 = 24 - 3a$, $3a = 6$, $a = 2$, $\therefore$ The upper base is 2 cm.

When $x = 2$, $V = 1728 - 27(2)^2 = 1728 - 108 = 1620$

Ans. *The capacity of the container is 1620 cm³.*

23. (a) Height of the larger pyramid = $(h + y)$, base area = b^2

Height of the smaller pyramid = h, base area = a^2

$\therefore$ volume of the frustum $= \frac{1}{3}b^2(h+y) - \frac{1}{3}a^2h = \frac{1}{3}(b^2h - a^2h + b^2y)$

(b) $\frac{a}{b} = \frac{h}{h+y}$, $\therefore$ $bh = ah + ay$, $bh - ah = ay$, $\therefore$ $h = \frac{ay}{b-a}$

(c) Combining (a) and (b), volume of the frustum $= \frac{1}{3}[(b^2 - a^2)h + b^2y]$

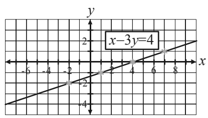
$= \frac{1}{3}[(b+a)(b-a)\frac{ay}{b-a} + b^2y] = \frac{1}{3}[(b+a)ay + b^2y] = \frac{1}{3}(aby + a^2y + b^2y)$

$= \frac{1}{3}y(a^2 + ab + b^2)$

Unit 7 Simultaneous linear equations

1

x	–2	1	4	7
y	–2	–1	0	1



2. (a) When $x = 5$, $y = -7$, $3(5) + k(-7) - 1 = 0$, $14 = 7k$,

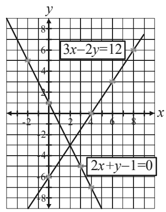
$\therefore$ $k = 2$

(b) When $x = h$, $y = \frac{5}{2}$, $3(h) + 2(\frac{5}{2}) - 1 = 0$, $3h + 4 = 0$, $\therefore$ $h = -\frac{4}{3}$

3 (a), (b)

(a)	x	–2	0	3	4
	y	5	1	–5	–7
(b)	x	0	4	6	8
	y	–6	0	3	6

(c) From the graph, $x = 2$, $y = -3$



4. (a) $\begin{cases} y = 8x - 19 \text{......(1)} \\ y = 6x - 15 \text{......(2)} \end{cases}$ Put (1) into (2), $8x - 19 = 6x - 15$, $x = 2$......(3)

Put (3) into (1), $y = 8 \times 2 - 19 = -3$. *Ans.* $x = 2, y = -3.$

(b) $\begin{cases} x = 2y + 13 \text{......(1)} \\ 3x + y = 4 \text{......(2)} \end{cases}$ Put (1) into (2), $3(2y + 13) + y = 4$, $6y + 39 + y = 4$,

$y = -5$......(3). Put (3) into (1), $x = 2(-5) + 13 = 3$. *Ans.* $x = 3, y = -5.$

(c) $\begin{cases} 3y - 7x = 1 \text{......(1)} \\ y + 2x = 9 \text{.......(2)} \end{cases}$ From (1), $3y = 1 + 7x$, $y = \frac{1+7x}{3}$......(3)

Put (3) into (2), $\frac{1+7x}{3} + 2x = 9$, $1 + 7x + 6x = 27$, $x = 2$......(4)

Put (4) into (3), $y=\frac{1+7\times2}{3}=\frac{15}{3}=5$. *Ans.* $x = 2, y = 5$.

(d) $\begin{cases}2y=5x+5......(1)\\2y=6x+8......(2)\end{cases}$. Put (1) into (2), $5x + 5 = 6x + 8$, $x = -3......(3)$

Put (3) into (1), $2y=5(-3)+5$, $y=\frac{-10}{2}=-5$. *Ans.* $x = -3, y = -5$.

(e) $\begin{cases}8x+8=3y......(1)\\2y=11-6x......(2)\end{cases}$. From (1), $y=\frac{8x+8}{3}......(3)$. Put (3) into (2),

$2\left(\frac{8x+8}{3}\right)=11-6x$, $16x + 16 = 33 - 18x$, $34x = 17$, $x=\frac{1}{2}......(4)$

Put (4) into (3), $y=\frac{8\times\frac{1}{2}+8}{3}=\frac{4+8}{3}=\frac{12}{3}=4$ *Ans.* $x=\frac{1}{2}, y = 4$.

(f) $\begin{cases}5x+3y=7......(1)\\5y-3x=6......(2)\end{cases}$ From (1), $y=\frac{7-5x}{3}......(3)$. Put (3) into (2),

$5\left(\frac{7-5x}{3}\right)-3x=6$, $35 - 25x - 9x = 18$, $-34x = -17$, $x=\frac{1}{2}......(4)$

Put (4) into (3), $y=\frac{1}{3}\left(7-5\times\frac{1}{2}\right)=\frac{1}{3}\times\frac{9}{2}=\frac{3}{2}$ *Ans.* $x=\frac{1}{2}$, $y=\frac{3}{2}$.

5. (a) $\begin{cases}x+y=11......(1)\\x-y=7......(2)\end{cases}$.
(1) + (2), $2x = 18$, $x = 9......(3)$
Put (3) into (1), $9 + y = 11$, $y = 2$
Ans. $x = 9, y = 2$.

(b) $\begin{cases}x+2y=3......(1)\\x-y=9......(2)\end{cases}$
(2) × 2, $2x - 2y = 18......(3)$
(1) + (3), $3x = 21$, $x = 7......(4)$
Put (4) into (2), $7 - y = 9$, $7 - 9 = y$, $y = -2$. *Ans.* $x = 7, y = -2$

(c) $\begin{cases}4x-y=18......(1)\\x+2y=-9......(2)\end{cases}$
(1) × 2, $8x - 2y = 36......(3)$
(2) + (3), $9x = 27$, $x = 3......(4)$
Put (4) into (2), $3 + 2y = -9$, $2y = -12$, $y = -6$. Ans. $x = 3$, $y = -6$

(d) $\begin{cases}5x+3y=2......(1)\\3x+9y=-6......(2)\end{cases}$
(1) × 3, $15x + 9y = 6......(3)$
(3) − (2), $12x = 12$, $x = 1......(4)$
Put (4) into (1), $5 + 3y = 2$, $3y = -3$, $y = -1$. *Ans.* $x = 1, y = -1$

(e) $\begin{cases}3y+2x=2......(1)\\2y-3x=-16......(2)\end{cases}$
(1) × 3, $9y + 6x = 6......(3)$
(2) × 2, $4y - 6x = -32......(4)$
(3) + (4), $13y = -26, y = -2......(5)$
Put (5) into (1), $-6 + 2x = 2$, $2x = 8$, $x = 4$. *Ans.* $x = 4, y = -2$

(f) $\begin{cases}5x+5y=1......(1)\\3x-2y=5......(2)\end{cases}$
(1) × 2, $10x + 10y = 2......(3)$
(2) × 5, $15x - 10y = 25......(4)$
(3) + (4), $25x = 27, x = \frac{27}{25}......(5)$

Put (5) into (1), $\frac{5\times27}{25}+5y=1$, $5y = 1 - \frac{27}{5}$, $5y=\frac{-22}{5}$, $y=\frac{-22}{25}$

Ans. $x = \frac{27}{25}$, $y=\frac{-22}{25}$

6. (a) $3x + y - 120 = 3x - 2y$, $y - 120 = -2y$, $3y = 120$, $y = 40$, $3x - 2y = 10 + 2x$,
$3x - 2\times40 = 10 + 2x$, $x = 10 + 80$, $x = 90$. *Ans.* *(90, 40)*

(b) $9x + 3y + 5 = x - 2y + 7$, $8x + 5y = 2......(1)$

$x - 2y + 7 = 7x - 3y - 4$, $6x - y = 11$......(2)

(2) × 5, $30x - 5y = 55$......(3). (1) + (3), $38x = 57$, $x = \frac{3}{2}$ ...... (4)

Put (4) into (2), $9 - y = 11$, $y = -2$. *Ans.* $(\frac{3}{2}, -2)$

(c) $\frac{5x}{3} - y = 6$......(1), $\frac{x}{3} + y = 6$......(2). (1) + (2), $\frac{5x}{3} + \frac{x}{3} = 12$, $5x + x = 36$,

$6x = 36$, $x = 6$......(3). Put (3) into (2), $\frac{6}{3} + y = 6$, $2 + y = 6$, $y = 4$.

Ans. *(6, 4)*

(d) $\frac{6x-3y}{4} = 6$ ......(1), $\frac{12x+5y}{30} = 6$ ......(2).

From (1), $6x - 3y = 24$, $12x - 6y = 48$ ......(3)

From (2), $12x + 5y = 180$ ......(4)

(4) – (3), ∴ $5y + 6y = 180 - 48$, $11y = 132$, $y = 12$ ......(5)

Put (5) into (4), $12x + 5(12) = 180$, $12x = 120$, $x = 10$. *Ans.* *(10, 12)*

7. (a) $\begin{cases} 9x - 5y - 11 = 0 \text{......(1)} \\ 3x - 5y + 13 = 0 \text{......(2)} \end{cases}$ (1) – (2), $6x - 24 = 0$, $6x = 24$, $x = 4$......(3)

Put (3) into (1), $36 - 5y - 11 = 0$, $-5y = -25$, $y = 5$. *Ans. (4, 5)*

(b) $\begin{cases} 0.35x - 0.15y = 1 \text{......(1)} \\ 0.2x + 0.1y = 1.5 \text{.....(2)} \end{cases}$ (1) × 2, $0.7x - 0.3y = 2$......(3)
(2) × 3, $0.6x + 0.3y = 4.5$.....(4)
(3) + (4), $1.3x = 6.5$, $x = 5$......(6)

Put (6) into (2), $1 + 0.1y = 1.5$, $0.1y = 0.5$, $y = 5$. *Ans.* *(5, 5)*

(c) $\begin{cases} 10x + y = 3(x+y),\ 10x + y = 3x + 3y,\ 7x - 2y = 0 \text{ ......(1)} \\ 9(x-y) = x - y - 32,\ 9x - 9y = x - y - 32,\ x - y = -4 \text{ .....(2)} \end{cases}$

From (2), $x = y - 4$......(3). Put (3) into (1), $7(y - 4) - 2y = 0$, $5y = 28$,

$y = \frac{28}{5}$ ......(4). Put (4) into (3), $x = \frac{28}{5} - 4 = \frac{28-20}{5} = \frac{8}{5}$. *Ans.* $(\frac{8}{5}, \frac{28}{5})$

(d) $\begin{cases} 3(x-y) = -(7+2y), \\ 3(2x-y) = 2 + y, \end{cases}$ $3x - 3y = -7 - 2y$, $3x - y = -7$,
$y = 3x + 7$......(1)
$6x - 3y = 2 + y$, $6x - 4y = 2$......(2)

Put (1) into (2), $6x - 4(3x + 7) = 2$, $6x - 12x - 28 = 2$, $-6x = 30$,
$x = -5$......(3). Put (3) into (1), $y = 3(-5) + 7 = -8$. *Ans.* *(–5, –8)*

(e) $\begin{cases} 3x + 2y = 4(4+x) + 1, \\ 5x + 3y = 7(4+x) - 2, \end{cases}$ $3x + 2y = 16 + 4x + 1$, $x = 2y - 17$......(1)
$5x + 3y = 28 + 7x - 2$, $2x - 3y = -26$......(2)

Put (1) into (2), $2(2y - 17) - 3y = -26$, $4y - 34 - 3y = -26$, $y = 8$......(3)
Put (3) into (1), $x = 2(8) - 17 = -1$. *Ans.* *(–1, 8)*

(f) $\begin{cases} -2(2+4y) = 5x - 5y, \\ 3(1-x) + 2(y+1) = y - x, \end{cases}$ $-4 - 8y = 5x - 5y$, $5x + 3y = -4$......(1)
$3 - 3x + 2y + 2 = y - x$, $y = 2x - 5$.....(2)

Put (2) into (1), $5x + 3(2x - 5) = -4$, $5x + 6x - 15 = -4$, $11x = 11$
$x = 1$......(3). Put (3) into (2), $y = 2(1) - 5 = -3$. *Ans.* *(1, –3)*

(g) $\begin{cases} 4(x-y) + 2(x+y) = -(19+y), \\ 5(x+y) - (2x+y) = -5, \end{cases}$ $6x - 2y = -19 - y$, $6x + 19 = y$.....(1)
$5x + 5y - 2x - y = -5$,
$3x + 4y = -5$......(2)

Put (1) into (2), $3x + 4(6x + 19) = -5$, $3x + 24x + 76 = -5$, $27x = -81$,
$x = -3$.....(3). Put (3) into (1), $y = 6(-3) + 19 = -1$. *Ans.* *(–3, –1)*

8. (a) $\begin{cases} x = 2y + 8 \text{......(1)} \\ \frac{x}{3} + 4 = y \text{......(2)} \end{cases}$ Put (2) into (1), $x = 2(\frac{x}{3} + 4) + 8 = \frac{2x}{3} + 16$,

$3x = 2x + 48$, $x = 48$......(3)

Put (3) into (1), $48 = 2y + 8$, $2y = 40$, $y = 20$. *Ans.* *(48, 20)*

(b) $\begin{cases} y = \frac{3}{2}x - 4 \text{......(1)} \\ x = -\frac{2}{3}y \text{......(2)} \end{cases}$ Put (1) into (2), $x = -\frac{2}{3}(\frac{3}{2}x - 4) = -x + \frac{8}{3}$,

$2x = \frac{8}{3}$, $x = \frac{4}{3}$......(3)

Put (3) into (2), $-\frac{4}{3} = -\frac{2}{3}y$, $y = \frac{4}{3} \times (-\frac{3}{2}) = -2$. *Ans.* $(\frac{4}{3}, -2)$

(c) $\begin{cases} \frac{2y + x}{3} = 3 \text{......(1)} \\ y - \frac{2}{3}x = 1 \text{......(2)} \end{cases}$ From (1), $2y + x = 9$......(3)
From (2), $3y - 2x = 3$......(4)

(3)×2 + (4), $2(2y + x) + (3y - 2x) = 9 \times 2 + 3$,

$7y = 21, y = 3$, $\therefore$ $2(3) + x = 9$, $x = 3$. *Ans. (3, 3)*

(d) $\begin{cases} \frac{x}{5} - \frac{3y}{2} = 3 \text{......(1)} \\ \frac{3x}{8} - \frac{5y}{2} = 5 \text{......(2)} \end{cases}$ (1) × 10, $2x - 15y = 30$......(3)
(2) × 8, $3x - 20y = 40$......(4)

(3)×3 – (4)×2, $3(2x - 15y) - 2(3x - 20y) = 90 - 80$, $-45y + 40y = 10$,
$-5y = 10$, $y = -2$, $\therefore$ $2x - 15(-2) = 30$, $2x = 0$, $x = 0$. *Ans.* *(0, –2)*

(e) $\begin{cases} \frac{5x}{3} - y = -17 \text{......(1)} \\ \frac{x}{3} - \frac{y}{2} = -4 \text{......(2)} \end{cases}$ (1) × 3, $5x - 3y = -51$......(3)
(2) × 6, $2x - 3y = -24$......(4)
(3) – (4), $3x = -27$, $x = -9$......(5)

Put (5) into (3), $5(-9) - 3y = -51$, $3y = 6$, $y = 2$ *Ans.* *(–9, 2)*

(f) $\begin{cases} \frac{x}{3} - y = -5 \text{......(1)} \\ \frac{3y}{2} + \frac{2x}{3} = \frac{26}{3} \text{......(2)} \end{cases}$ From (1), $y = \frac{x}{3} + 5$......(3)

Put (3) into (2), $\frac{3}{2}(\frac{x}{3} + 5) + \frac{2x}{3} = \frac{26}{3}$,

$\frac{x}{2} + \frac{2x}{3} = \frac{26}{3} - \frac{15}{2}$, $\frac{7x}{6} = \frac{52 - 45}{6}$, $\frac{7x}{6} = \frac{7}{6}$, $x = 1$, $\therefore$ $y = \frac{1}{3} + 5 = \frac{16}{3}$

Ans. $(1, \frac{16}{3})$

9. (a) Let $m = \frac{1}{x}$, $n = \frac{1}{y}$

$\therefore \begin{cases} 2m + n = 2 \text{......(1)} \\ 3m - 2n = 10 \text{......(2)} \end{cases}$

(1)×2 + (2), $\therefore$ $2(2m + n) + (3m - 2n) = 4 + 10$,
$7m = 14$, $m = 2$ .....(3)
Put (3) into (1), $n = 2 - 4 = -2$ ......(4)
$x = \frac{1}{m} = \frac{1}{2}$, $y = \frac{1}{n} = -\frac{1}{2}$. *Ans.* $(\frac{1}{2}, -\frac{1}{2})$

(b) $\begin{cases} \frac{2}{x} - \frac{1}{y} = \frac{3}{2} \text{......(1)} \\ \frac{1}{x} + \frac{3}{y} = 2 \text{......(2)} \end{cases}$ (1) × 3, $\frac{6}{x} - \frac{3}{y} = \frac{9}{2}$......(3)

(2) + (3), $\frac{6}{x} + \frac{1}{x} = \frac{9}{2} + 2$, $\frac{7}{x} = \frac{13}{2}$, $x = \frac{14}{13}$......(4)

Put (4) into (2), $\frac{1}{\frac{14}{13}} + \frac{3}{y} = 2$, $\frac{13}{14} + \frac{3}{y} = 2$, $\frac{3}{y} = 2 - \frac{13}{14}$,

$\frac{3}{y}=\frac{15}{14}$, $y=\frac{14}{5}$. *Ans.* $(\frac{14}{13}, \frac{14}{5})$

10. Let the bigger number be y and the smaller number be x.
∴ $x + y = 91$ and $y - x = 17$. Solving the equations, we get $x = 37$ and $y = 54$.
Ans. The numbers are 37 and 54.

11. Let the bigger number be y and the smaller number be x.
∴ $\frac{1}{4}(x+y)=16$ and $2(y-x)=36$.
Solving the equations, we get $x = 23$ and $y = 41$.
Ans. The numbers are 23 and 41.

12. Let the bigger number be y and the smaller number be x. Solving the equations, we get $x = 14$ and $y = 56$.
∴ $\begin{cases} y-x=\frac{3}{5}(x+y),\ 8x-2y=0,\ 4x-y=0 \\ y=3x+14 \end{cases}$
Ans. The numbers are 14 and 56.

13. Let the numerator be y and the denominator be x. Solving the equations, we get $x = 12$ and $y = 7$.
∴ $\begin{cases} \frac{y+9}{2x}=\frac{2}{3},\ 3y+27=4x \\ \frac{y-1}{x}=\frac{1}{2},\ 2y-2=x \end{cases}$
Ans. The fraction is $\frac{7}{12}$.

14. Let the prices of a table and a chair be x and y respectively.
∴ $3x + 4y = 2800$ and $4x + 9y = 4650$.
Solving the equations, we get $x = 600$ and $y = 250$.
Ans. The prices of a table and a chair are \$600 and \$250 respectively .

15. $2m - 3n + 10 = 4m - 5n + 6 = m + n - 9$.
Solving the equations, we get $m = 9$ and $n = 7$.
∴ The perimeter = 3(9 + 7 – 9) = 3(7) = 21 units.

16. (a) Opposite sides of a rectangle are equal, Solving the equations, we get $x = 5$, $y = 3$.
$\begin{cases} x+3=y+5,\ x-y=2 \\ x-y+4=3x-4y+3,\ 2x-3y=1 \end{cases}$
(b) AB = 5 + 3 = 8, BC = 5 – 3 + 4 = 6, ∴ perimeter = 2 (8 + 6) = 28 units.
(c) Area = 8 × 6 = 48 square units.

17. Let the numbers of students and pencils be x and y respectively.
∴ $y = 6x + 5$ and $y = 7x - 8$. Solving the equations, we get $x = 13$ and $y = 83$.
Ans. The number of students and pencils are 13 and 83 respectively .

18. Let the numbers of \$2 coins and \$5 coins be x and y respectively.
∴ $x + y = 30$ and $2x + 5y = 90$. Solving the equations, we get $x = 20$ and $y = 10$.
Ans. The numbers of \$2 and \$5 coins are 20 and 10 respectively.

19. Let the tens' digit and the units' digit be x and y respectively.
The original no. = $10x + y$,
the reversed no. = $10y + x$
∴ $\begin{cases} 10x+y=8(x+y),\ 2x-7y=0 \\ 10y+x=\frac{1}{3}(10x+y)+3,\ 7x-29y+9=0 \end{cases}$
Solving the equations, we get $x = 7$ and $y = 2$. *Ans. The number is 72.*

20. $x^2 + Ax - A \equiv x^2+3x-Bx-3B-11 \equiv x^2+x(3-B)-(3B+11)$
Comparing the coefficients, we get: $A = 3 - B$, and $A = 3B + 11$.
Solving the equations, we get $A = 5$, $B = -2$.

21. Let the present ages of David and Bobby be x and y respectively. Solving the equations, we get $x = 50$ and $y = 37$.
∴ $\begin{cases} x=y+13 \\ y-11=\frac{2}{3}(x-11),\ 2x-3y=-11 \end{cases}$
Ans. The present ages of David and Bobby are 50 and 37 respectively .

22. Let the present ages of A and B be x and y respectively. $\therefore \begin{cases} \frac{x}{y}=\frac{5}{3}, & 3x=5y \\ \frac{x+9}{y+9}=\frac{4}{3}, & 3x-4y=9 \end{cases}$
Solving the equations, we get $x = 15$ and $y = 9$.
Ans. The present ages of A and B are 15 and 9 respectively.

23. Let the speeds of the cars be x km/h and y km/h. $\therefore \begin{cases} y=x+12 \\ 0.5(x+y)=79, & x+y=158 \end{cases}$
Solving the equations, we get $x = 73$ and $y = 85$.
Ans. The speeds of the cars are 73 km/h and 85 km/h.

24. Let the speeds of the cars be x km/h and y km/h. $\therefore \begin{cases} 4(x+y)=280, & x+y=70 \\ 14(y-x)=280, & y-x=20 \end{cases}$
Solving the equations, we get $x = 25$ and $y = 45$.
Ans. The speeds of two cars are 25 km/h and 45 km/h.

25. Let the speeds of the boat and the water current be x km/h and y km/h respectively. Solving the equations, we get $x = 15$ and $y = 3$. $\therefore \begin{cases} 6(x+y)=108, & x+y=18 \\ 9(x-y)=108, & x-y=12 \end{cases}$
Ans. The speeds of the boat and the current are 15 km/h and 3km/h respectively.

26. Let x km/h and y km/h be the speeds of the ship in still water and the current of water respectively. $22.5(x-y)=360$, $x-y=16$........(1)
$20(x+y)=360$, $x+y=18$.........(2)
Solving (1) and (2), we get $x = 17$ and $y = 1$.
Ans. The speed of the ship against the current = 17 – 1 = 16 km/h.

27. $\frac{4x+3y}{10}-\frac{2x-y}{5}=\frac{x-y}{2}$, $4x+3y-4x+2y=5x-5y$, $x-2y=0$....(1)
$8y-\frac{5x-2}{3}=2x+y$, $24y-5x+2=6x+3y$, $11x-21y=2$.....(2)
Solving (1) and (2), we have $x = 4$ and $y = 2$.

28. Let $x=\frac{1}{u}$ and $y=\frac{1}{v}$. $\therefore$ $12x-5y+\frac{1}{10}=0$, and $2x+6y-\frac{34}{10}=0$
Solving the equations, we get $x=\frac{1}{5}$ and $y=\frac{1}{2}$. *Ans.* $u=\frac{1}{x}=5$, $v=\frac{1}{y}=2$.

29. $\begin{cases} \frac{6}{x}-\frac{10x}{y}=-5\text{.........(1)}, & (1)\times 2,\ \frac{12}{x}-\frac{20x}{y}=-10\text{..........(3)} \\ \frac{9}{x}-\frac{4x}{y}=9\text{..........(2)}, & (2)\times 5,\ \frac{45}{x}-\frac{20x}{y}=45\text{.........(4)} \end{cases}$
(4) – (3), $\therefore$ $\frac{45}{x}-\frac{12}{x}=45+10, \frac{33}{x}=55, x=\frac{33}{55}=\frac{3}{5}$.........(5)
Put (5) into (1), $6(\frac{5}{3})-\frac{1}{y}(10)(\frac{3}{5})=-5, 10-\frac{6}{y}=-5, 15=\frac{6}{y}, y=\frac{6}{15}=\frac{2}{5}$
[Or: Let $\frac{1}{x}=a, \frac{x}{y}=b$, then solve for a and b first.] *Ans.* $(\frac{3}{5},\frac{2}{5})$

30. Let the equations be (1) and (2) respectively. From (1), $x=-2y$......(3)
Sub. (3) into (2), $\therefore$ $-2y=\frac{1}{1-\frac{1}{1-(\frac{-2y}{y})}}=\frac{1}{1-\frac{1}{3}}=\frac{1}{\frac{2}{3}}=\frac{3}{2}$,
$\therefore$ $y=-\frac{3}{4}$, $\therefore$ $x=-2(-\frac{3}{4})=\frac{3}{2}$. *Ans.* $(\frac{3}{2},-\frac{3}{4})$

31. (a) $x-(7+3x)+1=0$, $-2x=6$, $\therefore$ $x=-3$, $\therefore$ $y=7+3(-3)=-2$

(b) Let $y = \frac{1}{p+q}, x = \frac{1}{p}$. From (a), $-3 = \frac{1}{p}$, $\therefore\ p = -\frac{1}{3}$

$-2 = \frac{1}{p+q}$, $-2p - 2q = 1$, $-2(-\frac{1}{3}) - 2q = 1$, $-2q = \frac{1}{3}$, $\therefore\ q = -\frac{1}{6}$

32. (a) $A(2) + B(1) = 18, 2A + B = 18$..........(1)

$A(-6) + B(15) = 18$, $-2A + 5B = 6$..........(2)

Solving (1) and (2), we have $A = 7$ and $B = 4$.

(b) $7(5) + 4(k) = 18$, $4k = -17$, $\therefore\ k = -\frac{17}{4}$

33. (a) Two straight lines will never intersect when they are parallel to each other.

(b) $6x - 3y + 2 = 0, 12x - 6y + 4 = 0$..........(1)

$4x + ry + 4 = 0, 12x + 3ry + 12 = 0$..........(2)

They are parallel when $3r = -6$, $\therefore\ r = -2$

34. (a) $5x + y - 4 = 0$...........(1); $2x + y - 10 = 0$..........(2)

Solving (1) and (2), we have $x = -2$ and $y = 14$.

Ans. The coordinates of P are (–2 , 14).

(b) $a(-2) + b(14) + 16 = 0$, $a - 7b = 8$......(1)

$7a(-2) - 2b(14) - 14 = 0, a + 2b = -1$......(2)

Solving (1) and (2), we get $a = 1$ and $b = -1$.

35. $a + b = 2m$..........(1); $a - b = 2n$..........(2)

(1) + (2), $2a = 2m + 2n$, $a = m + n$………..(3)

Put (3) into (1), $(m + n) + b = 2m$, $b = m - n$, $\therefore\ \frac{a}{b} = \frac{m+n}{m-n}$

36. (a) Let the equations be (1), (2), (3) and (4) respectively.

(1) + (2) + (3), $\therefore\ (x_1 + x_2) + (x_2 + x_3) + (x_3 + x_1) = 10 + (-3) + 9$,

$2(x_1 + x_2 + x_3) = 16$, $x_1 + x_2 + x_3 = 8$.....(5). Put (5) into (4), $y = 8$.

(b) Put (2) into (5), $\therefore\ x_1 + (-3) = 8$, $x_1 = 11$.

Put (3) into (5), $\therefore\ x_2 + 9 = 8$, $x_2 = -1$.

Put (1) into (5), $10 + x_3 = 8$, $x_3 = -2$

Ans. $x_1 = 11$, $x_2 = -1$, $x_3 = -2$.

37. (a) $16500 = a + k(1000)$, $a + 1000k = 16500$.........(1)

$34500 = a + k(5000)$, $a + 5000k = 34500$..........(2)

Solving (1) and (2), we get $a = 12000$ and $k = 4.5$

(b) When $n = 40000$, $C = 12000 + 4.5(40000) = 192000$

Ans. The average printing cost of 1 copy $= 192000 \div 40000 = \$4.8$.

38. Let $x and $y be the amounts that A and B originally had.

$2(x - 240) = y + 240, 2x - y = 720$..........(1)

$x + 1010 = 3(y - 1010), x - 3y = -4040$........(2)

Solving (1) and (2), we get $x = 1240$ and $y = 1760$.

Ans. The amount they have altogether = 1240 + 1760 = $3000

39. Let x and y be the numbers of coins in P and Q respectively.

$2(x - 22) = y + 22, 2x - y = 66$..........(1)

$2(x - 22) + 46 = 5y + 2(22)$, $2x - 5y = 42$..........(2)

Solving (1) and (2), we get $x = 36$ and $y = 6$.

Ans. There are 36 coins in Box P and 6 coins in Box Q originally.

40. Let x and y be the present ages of Adam and his sister respectively.

$x - 4 = 4(y - 4), x - 4y = -12$..........(1)

$x+8=2(y+8)$, $x-2y=8$..........(2)

Solving (1) and (2), we get $x = 28$ and $y = 10$.

Ans. The sum of their present ages = 28 + 10 = 38.

41. Let x and y be the present ages of the elder daughter and the woman respectively,

$\therefore$ the present age of the younger daughter is $(x - 4)$.

$x+(x-4)+y=50$, $2x+y=54$..........(1)

$y-x=63-y$, $x-2y=-63$..........(2)

Solving (1) and (2), we get $x = 9$ and $y = 36$; $\therefore$ $x-4 = 9 - 4 = 5$.

Ans. The woman is 36 years old and the daughters are 9 and 5 years old.

42. Let x m/s and y m/s be A's speed and B's speed respectively,

$\therefore$ the speed of C is $(y + 1.5)$ m/s. $x\times12\times60=y\times10\times60$, $6x-5y=0$..........(1)

$x\times18\times60=(y+1.5)\times(18-6)\times60$, $3x-2y=3$..........(2)

Solving (1) and (2), we get $x = 5$ and $y = 6$; $\therefore$ $y+1.5 = 6 + 1.5 = 7.5$.

Ans. The speed of A, B and C are 5 m/s, 6 m/s and 7.5 m/s respectively.

43. The speeds of the boat against the current and along the current are $(x - y)$ km/h and $(x + y)$ km/h respectively.

$\therefore$ $\dfrac{315}{x-y}-\dfrac{315}{x+y}=2$ .....(1), and $\dfrac{28}{x-y}=\dfrac{36}{x+y}$ .....(2)

Let $u=\dfrac{1}{x-y}$ and $v=\dfrac{1}{x+y}$.

From (1), $315u-315v=2$..........(3), from (2) $7u-9v=0$.........(4)

Solving (3) and (4), we get $u=\dfrac{1}{35}$ and $v=\dfrac{1}{45}$.

$\therefore$ $\dfrac{1}{x-y}=\dfrac{1}{35}$, $x-y=35$.......(5), and $\dfrac{1}{x+y}=\dfrac{1}{45}$, $x+y=45$.......(6)

Solving (5) and (6), we get $x = 40$ and $y = 5$.

44. Let x h and y h be the time taken by A and B respectively.

$\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{3}$..........(1), $\dfrac{1}{x}\times4+(\dfrac{1}{x}+\dfrac{1}{y})\times2=1$.........(2)

Put (1) into (2), $\dfrac{4}{x}+\dfrac{2}{3}=1$, $\dfrac{1}{x}=\dfrac{1}{12}$, $\therefore$ $x=12$. $\dfrac{1}{12}+\dfrac{1}{y}=\dfrac{1}{3}$; $\dfrac{1}{y}=\dfrac{1}{4}$, $\therefore$ $y=4$

Ans. A and B take 12 h and 4 h respectively to fill up the pool alone.

45. Let x days and y days be the time taken by A and B respectively.

$x=2y$.........(1); $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{14}$.........(2)

Put (1) into (2), $\therefore$ $\dfrac{1}{2y}+\dfrac{1}{y}=\dfrac{1}{14}$, $\dfrac{3}{2y}=\dfrac{1}{14}$, $y=\dfrac{3}{2}\times14=21$; $\therefore$ $x = 2(21) = 42$

Ans. A and B take 42 days and 21 days respectively.

46. Let x and y be the original numbers of workers and days required respectively,

$(x+8)(y-10)=xy$, $-10x+8y=80$..........(1)

$(x-8)(y+20)=xy$, $20x-8y=160$.........(2)

Solving (1) and (2), we get $x = 24$ and $y = 40$.

Ans. The numbers of workers and days required are 24 and 40 respectively.

47. (a) Let \$$x$, \$$y$ and \$$z$ be the costs of an apple, an orange and a pear respectively.

$x+3y+2z=22.6$..........(1); $5x+8y+3z=62.6$........(2)

$(1)\times5-(2)$, $7y+7z=50.4$, $y+z=7.2$.........(3)

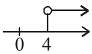
Ans. The total cost of 1 orange and 1 pear is \$7.2.

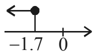
(b) $5y = 4z, 5y - 4z = 0$..........(4). Solving, (3) and (4), we have y = 3.2, z = 4.
From (1), $x + 3(3.2) + 2(4) = 22.6, \quad x = 5$

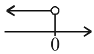
Ans. The costs of an apple, an orange and a pear are \$5, \$3.2 and \$4 respectively.

48. Let x and y be the numbers of males and females in project A respectively,
$\therefore$ in project B, the no. of males = 181 – x, and the no. of females = 154 – y.
$x = 2y + 3, \quad x - 2y = 3$.....(1); $\quad 3(181 - x) = 154 - y, \quad 3x - y = 389$......(2)
Solving (1) and (2), we get x = 155 and y = 76.
Ans. There are 155 males and 76 females in project A, while there are 26 males and 78 females in project B.

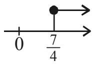
Unit 8 Inequalities

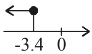
1. (a) 

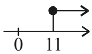
(b) 

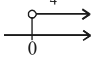
(c) 

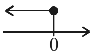
2. (a) $x > -2$ (b) $x \le 1.5$ (c) $x \ge 0$ (d) $x < -3\frac{1}{7}$

3. (a) $x \ge \frac{7}{4}$ 

(b) $x \le -3.4$ 

(c) $x \ge 11$ 

(d) $x > 0$ 

(e) $x \le 0$ 

4. (a) $x \le 13$ (b) $y \le 7$ (c) $5x + 3 > 13$
(d) $\frac{y}{2} - 2 \le 7$ (e) $\frac{x+10}{4} \ge 8$ (f) $\frac{2}{5}m - 1 \le 11$

5. (a) $-9x < 9, \; x > -1$ (b) $4x \ge -14, \quad \therefore \; x \ge -\frac{7}{2}$
(c) $-8x \le 0, \quad \therefore \; x \ge 0$ (d) $7 - x < 7, \; -x < 0, \quad \therefore \; x > 0$
(e) $6x - 9 \le 4x, \quad 2x \le 9, \quad \therefore \; x \le \frac{9}{2}$
(f) $-20x + 5 > 2x - 6, \quad -22x > -11, \quad \therefore \; x < \frac{1}{2}$
(g) $12x + 8 \ge 3x + 14 - 7x, \quad 16x \ge 6, \quad \therefore \; x \ge \frac{3}{8}$
(h) $5 - 4x < 8 - 6x + 3, \; 2x < 6, \quad \therefore \; x < 3$
(i) $2(5x - 7x - 14) > 12 - 3x, \; 2(-2x - 14) > 12 - 3x,$
$-4x - 28 > 12 - 3x, \; -x > 40, \quad \therefore \; x < -40$

6. (a) $3x + 4 < 54 - 12x, \; 15x < 50, \quad \therefore \; x < \frac{10}{3}$
(b) $3(3x + 4) \ge 5(2 - x), \quad 9x + 12 \ge 10 - 5x, \quad 14x \ge -2, \quad \therefore \; x \ge -\frac{1}{7}$
(c) $(5x - 1) - 9 \le 0, \quad 5x \le 10, \quad \therefore \; x \le 2$
(d) $(8x + 12) - 5 > 50x, \; -42x > -7, \quad \therefore \; x < \frac{1}{6}$
(e) $4(2x + 11) + 3(6 - x) \ge 12, \quad 8x + 44 + 18 - 3x \ge 12, \quad 5x \ge -50, \quad \therefore \; x \ge -10$
(f) $-3(x - 5) + 24 < 2(2x - 3)$, $-3x + 15 + 24 < 4x - 6$, $-7x < -45$, $\quad \therefore \; x > \frac{45}{7}$
(g) $x + 6 - 2(x - 3) \le 20, \quad x + 6 - 2x + 6 \le 20, \quad -x \le 8, \quad \therefore \; x \ge -8$

(h) $6(1+2x)-2(4x+7) < 3(9-x)$, $6+12x-8x-14 < 27-3x$,
$4x-8 < 27-3x$, $7x < 35$, $\therefore$ $x < 5$

7. $2(15+x) \le 18$, $15+x \le 9$, $x \le -6$. *Ans. The greatest value of x is –6.*

8. $\frac{y}{3}+13 \le y$, $y+39 \le 3y$, $-2y \le -39$, $y \ge 19.5$. *Ans. The least value of y is 20.*

9. Let x be the smallest integer. $x+(x+1)+(x+2) < 15$, $3x < 12, x < 4$.
Ans. The maximum value of the smallest number is 3.

10. Let x be the larger odd number. $x+(x-2) > 28$, $2x > 30$, $x > 15$
Ans. The least value of the larger odd number is 17.

11. Let h cm be David's height. $h+(h-14) \ge 280$, $2h \ge 294$, $\therefore h \ge 147$.
Ans. The height of David is at least 147 cm.

12. Let x be the number of hotdogs. $16x+8.4\times 5 \le 150$, $16x \le 108, x \le 6.75$.
Ans. She can buy 6 hotdogs at most.

13. Let x be the number of \$2 coins. $2x+0.5(x-8) < 56$, $2x+0.5x-4 < 56$,
$2.5x < 60, x < 24$. *Ans. The maximum number of \$2 coins is 23.*

14. $2(y+15) > 3y$, $2y+30 > 3y$, $30 > y$, $y < 30$. Besides, y must be a positive number.
Ans. y must be a positive number smaller than 30.

15. Let x be the number of incorrect answers, $3(20-x)-2x > 50$,
$60-3x-2x > 50$, $-5x > -10$, $x < 2$.
Ans. The maximum number of incorrect answers is 1.

16. $2(9+a) \ge 40$, $9+a \ge 20$, $a \ge 11$
Minimum area = Minimum width $\times 9$ = 11×9 = 99 cm^2

17. Let x be the smaller number. $x > \frac{x+4}{2}$, $2x > x+4$, $x > 4$; and x must be a multiple of 4.
Ans. The least value of the smaller number is 8.

18. Let x be the largest number. $x+(x-3)+(x-6) \le 30$, $3x \le 39$, $x \le 13$; and x must be a multiple of 3. *Ans. The greatest value of the largest number is 12.*

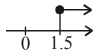
19. (a) Let x be the smaller number. $x+(x+7) < 19$, $2x < 12$, $\therefore$ $x < 6$.
Ans. The smaller number is smaller than 6.
(b) $\because$ $0 < x < 6$, $\therefore$ $7 < x+7 < 13$, and x is an integer.
Ans. The possible values of the larger number are 8, 9, 10, 11 and 12.

20. $a < -3$, $\therefore$ $a+b < b-3$……(i); $b < 15$, $b-3 < 12$………(ii);
From (i) and (ii), $a+b < b-3 < 12$, $\therefore$ $a+b < 12$

21. (a) Let $a = \frac{1}{2}$, $a^2 = (\frac{1}{2})^2 = \frac{1}{4}$, $a^2 < a$. $\therefore$ The statement is not correct.
(b) If $a = -5$, $b = -3$, $a < b$, but $a^2 = (-5)^2 = 25$, $b^2 = (-3)^2 = 9$, $a^2 < b^2$
$\therefore$ The statement is not correct.
(c) If $c = 2$, $d = -2$, $c < d$, but $\frac{1}{c} = \frac{1}{2}$, $\frac{1}{d} = -\frac{1}{2}$, $\frac{1}{c} > \frac{1}{d}$
$\therefore$ The statement is not correct.
(d) If $a = -4$, $b = 3$, $c = -6$, $d = -2$, $a < b$, and $c < d$,
but ac = $(-4)(-6) = 24$, $bd = 3(-2) = -6$, $a \times c > b \times d$.
$\therefore$ The statement is not correct.

22. (a) $6x+9-1-5x > x-5$, $x+8 > x-5$, $8 > -5$,
Ans. x can be any real numbers.
(b) $-\frac{3-4x}{2} > 2x-1$, $-3+4x > 4x-2, -3 > -2$ *Ans. There is no solution.*

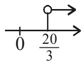
23. (a) $0.81 - 3.24x + 6.97x \le 1.05(3x + 1.6x - 0.8)$,



$0.81 + 3.73x \le 1.05(4.6x - 0.8)$

$0.81 + 3.73x \le 4.83x - 0.84$, $-1.1x \le -1.65$, $x \ge \frac{1.65}{1.1}$, $\therefore$ $x \ge 1.5$

(b) $60 \times \frac{1}{3}[-\frac{9}{4} + \frac{8x}{5} + \frac{1}{4}(9x - 1)] > 60 \times (\frac{6x-1}{5} + \frac{6-x}{6} + \frac{1}{30})$



$20(-\frac{9}{4} + \frac{8x}{5} + \frac{9x}{4} - \frac{1}{4}) > 12(6x - 1) + 10(6 - x) + 2$

$-45 + 32x + 45x - 5 > 72x - 12 + 60 - 10x + 2$

$77x - 50 > 62x + 50$, $15x > 100$, $\therefore$ $x > \frac{20}{3}$

24. (a) Not true. The product of two negative numbers must be positive.
(b) True. $\because$ $a < b$, $\therefore$ $a + 5 < b + 5$, but $b + 5 < b + 6$, $\therefore$ $a + 5 < b + 6$.
(c) True. $kx + h^2 < hx + k^2$, $h^2 - k^2 < hx - kx$, $(h + k)(h - k) < (h - k)x$,
$\because h - k > 0$, $\therefore h + k < x$, $x > h + k$

25. (a) Not true. For example, when $p = -1$, $q = -2$, $(-1) + (-2) = -3 < 0$.
(b) True. $\because$ $q < p$, $\therefore$ $q - p < p - p$, i.e. $q - p < 0$.
(c) True. $\because$ $8 > p > q > -6$, $\therefore$ $8 > p$ and $q > -6$, i.e. $0 > (p - 8)$ and $(q + 6) > 0$. Since $(p - 8)$ is negative and $(q + 6)$ is positive,
$\therefore$ their product must be negative.

26. In a triangle, the sum of the lengths of any two sides must be greater than that of the third side.
$\therefore$ $x < 6 + 3$, $x < 9$ …..(i); $6 < x + 3$, $x > 3$ …..(ii);
$3 < 6 + x$, $x > -3$ …..(iii); $\therefore$ x must be integers from 3 to 9.
Ans. The possible values of x are 4, 5, 6, 7 and 8.

27. $6x < -y$, $\frac{6x}{y} > -1$ ($\because$ $y < 0$), $\therefore$ $\frac{x}{y} > -\frac{1}{6}$

28. $k > 5$, i.e. $5 - k < 0$. $5y + k - ky \le 8 - 3k$, $(5 - k)y \le 8 - 4k$,
$\therefore$ $y \ge \frac{8-4k}{5-k}$ ($\because$ $5 - k < 0$)

29. (a) $m = 1 - n$, and $m > -8$, $\therefore$ $1 - n > -8$, $-n > -9$, $n < 9$
(b) $n = 2m + 4$, $\frac{n-4}{2} = m$, $\therefore$ $\frac{n-4}{2} > -8$, $n - 4 > -16$, $n > -12$
(c) $2n = 7 - 3m$, $m = \frac{7-2n}{3}$,
$\therefore$ $\frac{7-2n}{3} > -8$, $7 - 2n > -24$, $-2n > -31$, $n < \frac{31}{2}$

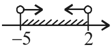
30. (a) $24 + y = 3x$, $x = \frac{24+y}{3}$, $\because$ $x > 0$, $\therefore$ $\frac{24+y}{3} > 0$, $24 + y > 0$, $y > -24$
(b) $y = 3x - 24$, $\because$ $y \le -15$, $\therefore$ $3x - 24 \le -15$, $3x \le 9$, $x \le 3$
(c) $\because$ $x > 7$, $\therefore$ $\frac{24+y}{3} > 7$, $24 + y > 21$, $y > -3$, $\therefore$ y can be 0.

31. $\frac{1}{x} > \frac{1}{y}$, $\frac{1}{x} - \frac{1}{y} > 0$, $\therefore$ $\frac{y-x}{xy} > 0$. But $x > y$, $0 > y - x$, i.e. $(y - x)$ is a negative number. Since $\frac{y-x}{xy}$ is positive but $(y - x)$ is negative, $\therefore$ xy must be negative, that is, x and y must be of opposite signs. But $x > y$, $\therefore$ $x > 0$ and $y < 0$.

32. (a) Greatest value = 3(6) + 2(–2) = 18 – 4 = 14
(b) The smallest value of $x^2 = 0^2 = 0$, the smallest value of $y^2 = (-2)^2 = 4$,
$\therefore$ the smallest value $x^2 + y^2 = 0 + 4 = 4$.
The greatest value of $x^2 = 6^2 = 36$, the greatest value of $y^2 = (-12)^2 = 144$,
$\therefore$ the greatest value $x^2 + y^2 = 36 + 144 = 180$. $\therefore$ $4 \le x^2 + y^2 \le 180$
(c) The smallest value of $y - x = y_{smallest} - x_{biggest} = -12 - 6 = -18$.
The greatest value of $y - x = y_{biggest} - x_{smallest} = -2 - (-3) = 1$.
$\therefore$ $-18 \le y - x \le 1$
(d) Least value = $\dfrac{6}{-2} = -3$, greatest value = $\dfrac{-3}{-2} = \dfrac{3}{2}$, $\therefore$ $-3 \le \dfrac{x}{y} \le \dfrac{3}{2}$

33. (a) $12 - 3(2x + 1) \ge 4(6x - 4)$, $12 - 6x - 3 \ge 24x - 16$, $-30x \ge -25$, $\therefore$ $x \le \dfrac{5}{6}$
(b) Let $x = \dfrac{6y+1}{3}$. The inequality becomes: $1 - \dfrac{1}{4}(2x+1) \ge \dfrac{1}{3}(6x-4)$
From (a), $x \le \dfrac{5}{6}$, $\therefore$ $\dfrac{6y+1}{3} \le \dfrac{5}{6}$, $12y + 2 \le 5$, $y \le \dfrac{1}{4}$

34. (a) $4x + 1 > -19$, $4x > -20$, $\therefore$ $x > -5$
(b) $4 - x < 18 - 8x$, $7x < 14$, $\therefore$ $x < 2$
(c) $\because$ $k > -5$ and $k < 2$, $\therefore$ k are numbers from –5 to 2.



35. (a) Let n be the number of sides. $3n \le 45$, $n \le 15$;
but the smallest number of sides is 3, $\therefore$ $n \ge 3$.
Each interior angle $= \dfrac{(n-2) \times 180°}{n} = \dfrac{180°n - 360°}{n} = 180° - \dfrac{360°}{n}$,
it is greatest when n = 15, and smallest when n = 3.
$\therefore$ The greatest angle $= 180° - \dfrac{360°}{15} = 156°$,
and the smallest angle $= 180° - \dfrac{360°}{3} = 60°$.
(b) Least possible size of an interior angle = 60° (equilateral triangle);
Let x be the size of an exterior angle, $180° - 172° \le x \le 180° - 60°$,
$\therefore$ $8° \le x \le 120°$

36. (a) Let x be the number of copies. For Shop A, $1500 + 1.2x \le 2000$,
$1.2x \le 500$, $x \le 416\dfrac{2}{3}$, $\therefore$ its maximum number of copies is 416.
For Shop B, $900 + 1.5x \le 2000$, $1.5x \le 1100$, $x \le 733\dfrac{1}{3}$,
$\therefore$ its maximum number of copies is 733.
Ans. Print Shop B should be chosen.
(b) $1500 + 1.2x < 900 + 1.5x$, $-0.3x < -600$, $\therefore$ $x > 2000$
Ans. It is cheaper to choose A when the number of copies is more than 2000.

37. (a) Let x be the no. of \$2 coins. $\therefore$ no. of \$5 coins = $\dfrac{140-2x}{5} = 28 - \dfrac{2}{5}x$.
$\because$ the no. of coins must be an integer, $\therefore$ $\dfrac{2}{5}x$ must be an integer,
$\therefore$ x must be a multiple of 5, i.e. the no. of \$2 coins must be a multiple of 5.
(b) $\dfrac{140-2x}{5} - x > 3$, $140 - 2x - 5x \ge 15$, $-7x \ge -125$, $x \le 17.9$;

$\because$ x must be a multiple of 5, $\therefore$ $x = 15$.

Ans. *The maximum number of \$2 coins is 15.*

Unit 9 Angles & parallel

1. (a) $2x + 30^o = 4x - 50^o$ (vert. opp.$\angle$s), $80^o = 2x$, $\therefore$ $x = 40^o$

   (b) $(x+10^o) + 90^o + (x-10^o) = 180^o$ (adj.$\angle$s on st. line), $2x = 90^o$, $\therefore$ $x = 45^o$

   (c) $2x + 3x + x + 90^o = 360^o$ ($\angle$s at a pt.), $6x = 270^o$, $\therefore$ $x = 45^o$

   (d) $\angle$ AOC $= 3x - 13^o$ (vert. opp.$\angle$s), $\angle$ FOB $= 2x + 10^o$ (vert. opp.$\angle$s),

   $\therefore$ $(3x - 13^o) + 98^o + (2x + 10^o) = 180^o$ (adj.$\angle$s on st. line), $5x = 85^o$, $x = 17^o$

2. Let $\theta = \angle$ DBE. $\angle$ DBA $= 133^o - \theta$, and $\angle$ EBC $= 101^o - \theta$,

   $\therefore$ $(133^o - \theta) + \theta + (101^o - \theta) = 180^o$ (adj.$\angle$ s on st. line), $\theta = 54^o$, $\therefore$ $\angle$DBE $= 54^o$

3. $\angle$QSP $= 180^o - 64^o - 47^o = 69^o$ ($\angle$sum of Δ)

   $\angle$QSR $= 180^o - 24^o - 21^o = 135^o$ ($\angle$sum of Δ)

   $\therefore$ $69° + 135° + a° = 360°$ ($\angle$s at a pt.), $\therefore$ $a^o = 156^o$, $a = 156$

4. (a) $m = 32^o$ (alt.$\angle$s, AB//DE). $\angle$DCE $= 180^o - 70^o - m$ ($\angle$sum of Δ),

   $\therefore$ $\angle$DCE $= 180^o - 70^o - 32^o = 78^o$. $n = \angle$ DCE $= 78^o$ (vert. opp.$\angle$s)

   (b) $x^o = 73^o$ (alt.$\angle$s, SR//PQ), $\therefore$ $x = 73$

   $\angle$SQR $= 180^o - 58^o - x^o$ ($\angle$sum of Δ), $\therefore$ $\angle$SQR $= 122^o - 73^o = 49^o$

   $y^o = \angle$ SQR $= 49^o$ (alt.$\angle$s, PS//QR), $\therefore$ $y = 49$

   (c) $\angle$BDF = $\angle$ABD (alt.$\angle$s, DF//AB), $\angle$BDE + $m = 18^o + \angle$ CBD;

   but $\angle$BDE = $\angle$CBD (alt.$\angle$s, DE//CB), $\therefore$ $m = 18^o$

   (d) $y = 55^o$ (corr.$\angle$s, AB//EC). $x + 75^o + y = 180^o$ (adj.$\angle$s on st. line),

   $\therefore x = 180^o - 75^o - 55^o = 50^o$

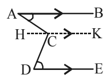
   (e) $\angle$QSR $+ 145^o = 180^o$ (int.$\angle$s, PQ//RS), $\therefore$ $\angle$QSR $= 180^o - 145^o = 35^o$

   $y + \angle$ QSR $+ 90^o = 360^o$ ($\angle$s at a pt.), $\therefore$ $y = 360^o - 90^o - 35^o = 235^o$

   (f) $\angle$ACD $+ 116^o = 180^o$ (int.$\angle$s, AB//CD), $\therefore$ $\angle$ACD $= 64^o$

   $(a + \angle$ACD$) + 47^o = 180^o$ (int.$\angle$s, EC//BD), $\therefore$ $a + 64^o = 133^o$, $a = 69^o$

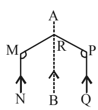
5. (a) Draw HK through C, such that HK//AB//DE.

   $\angle$ACH $= 56^o$ (alt.$\angle$s, AB//HK); $\angle$HCD $= 68^o$ (alt.$\angle$s, HK//DE)

   $x = \angle$ ACH $+ \angle$ HCD $= 56^o + 68^o = 124^o$

   (b) Draw AB through R, such that AB//MN//PQ.

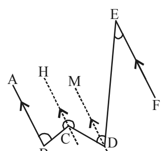
   

   $\angle$ARM $= 120^o$ (alt.$\angle$s, AB//MN),

   $\therefore$ $\angle$ARP $= 228^o - 120^o = 108^o$

   $y = \angle$ ARP $= 108^o$ (alt.$\angle$s, AB//PQ)

   (c) Draw HC//MD//AB//EF. $\angle$MDE $= 32^o$ (alt.$\angle$s, MD//EF),

   $\therefore$ $\angle$ MDC $= 68^o - 32^o = 36^o$.

   $\angle$HCD $+ \angle$MDC $= 180^o$ (int.$\angle$s, HC//MN),

   $\therefore$ $\angle$HCD $= 180^o - 36^o = 144^o$

   $\angle$HCB $+ 82^o = 180^o$ (int.$\angle$s, HC//AD), $\therefore$ $\angle$HCB $= 98^o$

   $m = \angle$ HCB $+ \angle$ HCD $= 98^o + 144^o = 242^o$

6. $a = 37^o$ (alt.$\angle$s, // lines). $d = a = 37^o$ (alt.$\angle$s, // lines). $b = 25^o$ (alt.$\angle$s, // lines)

   $(25^o + d) + c = 180^o$ (int.$\angle$s, // lines), $\therefore$ $25^o + 37^o + c = 180^o$, $c = 118^o$

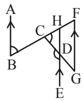
$e+(25^o+d)=180^o$ (int.$\angle$s, // lines), $\therefore\ e+25^o+37^o=180^o$, $e=118^o$

7. $a=69^o$ (corr.$\angle$s, AB//DF), $\angle$BEG $=180^o-69^o-45^o=66^o$ (adj.$\angle$s on st. line)
$\therefore\ b=180^o-66^o=114^o$ (int.$\angle$s, BC//EG)

8. $\angle$CBE $=38^o$ (alt.$\angle$s, AC//DE). $\angle$CED $=74^o$ (corr.$\angle$s, DE//FG)
$\therefore\ \angle$CEB $+38^o=74^o$, $\angle$CEB $=36^o$. *Ans. $\angle$CBE is bigger.*

9. $x^o+85^o=180^o$ (int.$\angle$s, AB//CD), $\therefore\ x=180-85=95$
$x^o+(x-30)^o=y^o$ (vert. opp.$\angle$s), $\therefore\ y=95+95-30=160$

10. Produce ED to H. $\angle$DHC $=59^o$ (alt.$\angle$s, AB//HE);
$\angle$CDH $=180^o-137^o=43^o$ (adj.$\angle$s on st. line)
$q=180^o-\angle$ DHC $-\angle$ CDH ($\angle$sum of Δ),
$\therefore\ q=180^o-59^o-43^o=78^o$



11. $\angle$TSQ $+99^o=112^o$ (alt.$\angle$s, TU//RS), $\therefore\ \angle$TSQ $=13^o$
$x^o+\angle$ TSQ $=101^o$ (alt.$\angle$s, PQ//SV), $\therefore\ x=101-13=88$

12. (a) $\angle$ABE $=180^o-83^o=97^o$ (adj.$\angle$s on st. line)
$\because$ ABE $\neq$ DEH, $\therefore$ AC is not parallel to DF (corr.$\angle$s not equal)
(b) $\angle$DBE $=180^o-110^o-23^o=47^o$ (adj.$\angle$s on st. line)
$\because\ \angle$DBE $=47^o=\angle$ BEC, $\therefore$ DB//EC (alt.$\angle$s equal)

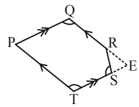
13. (a) $\angle$RTS $=44^o$ (vert. opp.$\angle$s), $\therefore\ 44^o+62^o+x=180^o$ ($\angle$sum of Δ), $x=74^o$
(b) $\because\ \angle$RST $=74^o=\angle$ TPQ, $\therefore$ PQ//RS (alt.$\angle$s equal)

14. (a) $(2y-11)^o+(3y+26)^o=180^o$ (adj.$\angle$s on st. line), $\therefore\ 5y=165$, $y=33$
(b) $\angle$DRS $=(3\times33+26)^o=125^o$, $\angle$BQR $=(158-33)^o=125^o$,
$\therefore$ AB//CD (corr.$\angle$s equal)

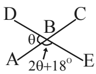
15. If the corresponding angles are equal, then AB//CD, $\therefore\ \angle$DCE $=3x+19^o$
$\angle$CDE $=68^o$ (alt.$\angle$s, BC//DE), $\angle$CDE $+\angle$DCE $+(2x-27^o)=180^o$ ($\angle$sum of Δ),
$\therefore\ 68^o+(3x+19^o)+(2x-27^o)=180^o$, $5x=120^o$, $x=24^o$

16. $\angle$DAB $=47^o$ (alt.$\angle$s, AB//EF). $p+55^o+\angle$ DAB $=180^o$ ($\angle$sum of Δ)
$\therefore\ p+55^o+47^o=180^o$, $p=78^o$. $s=p=78^o$ (corr.$\angle$s, CD//AB)
$q+s+34^o=180^o$ ($\angle$sum of Δ), $\therefore\ q+78^o+34^o=180^o$, $q=68^o$
$r+q=180^o$ (int.$\angle$s, CD//EF), $\therefore r+68^o=180^o$, $r=112^o$

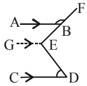
17. Produce QR and TS to meet at E.
$x+\angle$ E $=180^o$ (int.$\angle$s, PQ//TE), $y+\angle$ E $=180^o$ (int.$\angle$s, PT//QE)
$\therefore\ x=y$, $x:y=1:1$



18. (a) Let DBA $=\theta$.
(b) $\theta+(2\theta+18^o)=180^o$ (adj.$\angle$s on st. line), $3\theta=162^o$, $\theta=54^o$
$\angle$DBC $=2\theta+18^o$ (vert. opp.$\angle$s), $\therefore\ \angle$DBC $=2\times54^o+18^o=126^o$



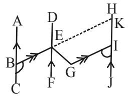
19. (a) Draw GE//CD. $\angle$DEG $=180^o-58^o=122^o$ (int.$\angle$s, GE//CD)
$\angle$BEG $=360^o-\angle$ DEG $-98°$ ($\angle$s at a pt.),
$\therefore\ \angle$BEG $=360^o-122^o-98^o=140^o$
$\because\ \angle$BEG $=140^o=\angle$FBA, $\therefore$ AB//GE (corr.$\angle$s equal);
but GE//CD (construction), $\therefore$ AB//CD



(b) $\angle$CED $=360^o-232^o-64^o=64^o$ ($\angle$s at a pt.)
$\angle$CDE $=180^o-73^o-\angle$CED $=180^o-73^o-64^o=43^o$ ($\angle$sum of Δ)

$\therefore$ $\angle CDB = 43° + 56° = 99°$. $\angle EBD = 180^o - 64^o - 56^o = 60^o$ ($\angle$sum of Δ)
$\therefore$ $\angle ABD = 60° + 27° = 87°$. $\angle ABD + \angle CDB = 87° + 99° \neq 180°$
$\because$ The interior angles are not supplementary, $\therefore$ AB is not parallel to CD.

20. Produce BE to the point K on HJ. $\angle BKI = 180^o - x$ (int.$\angle$s, AC//HJ),
but $\angle BKI = y$ (corr.$\angle$s, BK//GI), $\therefore$ $y = 180^o - x$



21. (a) $\angle PAB = b$ (alt.$\angle$s, PQ//BC); $\angle QAC = c$ (alt.$\angle$s, PQ//BC)
$\angle PAB + a + \angle QAC = 180^o$ (adj.$\angle$s on st. line), $\therefore$ $b + a + c = 180^o$,
i.e. the angle sum of a triangle is 180^o.
(b) $\angle DCA = a$ (alt.$\angle$s, AB//DC). $\angle DCB + b = 180^o$ (int.$\angle$s, AB//DC)
$\therefore$ $(\angle DCA + c) + b = 180^o$, $a + c + b = 180^o$, i.e. the angle sum of a triangle is 180^o.

22. $\angle BAC + \angle ACD = (p + 180^o - r) + q = (p + q) - r + 180^o = r - r + 180^o = 180^o$
$\therefore$ AB//CD (int.$\angle$s supp.)

23. (a) $(x+10)^o = 2(x-10)^o$ (alt.$\angle$s, AB//CD), $\therefore$ $x + 10 = 2x - 20$, $x = 30$
(b) $\angle DCF = 360^o - 2(x-10)^o - 255^o$ ($\angle$s at a pt.)
$\therefore$ $\angle DCF = 360^o - 2(30-10)^o - 255^o = 65^o$
$\angle CFE = (2x+5)^o = (2 \times 30 + 5)^o = 65^o$
$\because$ $\angle CFE = 65^o = \angle DCF$, $\therefore$ CD//EF (alt.$\angle$s equal); but CD//AB, $\therefore$ AB//EF

24. $b = 2a + 24^o$ (corr.$\angle$s, AB//CD) ------ (i)
$b + (2b + 4a - 2^o) = 180^o$ (int.$\angle$s, DF//CE), $3b = 182^o - 4a$ ------ (ii)
Put (i) into (ii), $\therefore$ $3(2a + 24^o) = 182^o - 4a$, $6a + 72^o = 182^o - 4a$, $10a = 110^o$,
$\therefore$ $a = 11^o$, $b = 2(11^o) + 24^o = 46^o$

25. $x + y - 7^o + 135^o = 180^o$ ($\angle$sum of Δ), $\therefore$ $x + y = 52^o$ ------ (i)
$\angle DFE = 360^o - 135^o - 118^o = 107^o$ ($\angle$s at a pt.)
$\therefore$ $107^o + y + 2x - 1^o = 180^o$ ($\angle$sum of Δ), $2x + y = 74^o$ ------ (ii)
(ii) – (i), $\therefore$ $2x - x = 74^o - 52^o$, $x = 22^o$, $\therefore$ $y = 52^o - 22^o = 30^o$

26. $\angle EPN = 180^o - 137^o = 43^o$ (adj.$\angle$s on st. line)
$m = \angle EPN = 43^o$ (corr.$\angle$s, AB//EF). $n + 115^o = 180^o$ (int.$\angle$s, BC//DE), $\therefore$ $n = 65^o$

27. $\angle QPT = \frac{x}{2} + 60^o$ (corr.$\angle$s, PQ//WS). $\angle QPT + \angle PQS = 180^o$ (int.$\angle$s, PT//QS)
$\therefore$ $(\frac{x}{2} + 60^o) + (84^o + x) = 180^o$, $\frac{3}{2}x = 36^o$, $\therefore$ $x = 24^o$
$\angle TYV = x = 24^o$ (corr.$\angle$s, PT//QS). $\angle TWU = \angle TYV = 24°$ (corr.$\angle$s, WU//YV)
$\therefore$ $15° + 24° + \frac{24}{2} + 60° + y = 180°$ (adj.$\angle$s on st. line), $y = 69°$

28. (a) (i) $\angle BQR = 35^o$ (angle of reflection). $\angle BRQ = 180^o - 108^o - 35^o$ ($\angle$sum of Δ)
$\therefore \angle BRQ = 37^o$, $\therefore$ $\angle CRS = 37^o$ (angle of reflection)
(ii) $\angle PQR = 180^o - 35^o - 35^o$ (adj.$\angle$s on st. line), $\therefore$ $\angle PQR = 110^o$
$\angle QRS = 180^o - \angle PQR$ (int.$\angle$s, QP//RS), $\therefore$ $\angle QRS = 180^o - 110^o = 70^o$
$\angle QRB + 70^o +$ CRS $= 180^o$ (adj.$\angle$s on st. line),
but $\angle CRS = \angle QRB$ ($\angle$of reflection), $\therefore$ $2\angle QRB + 70^o = 180^o$,
$\angle QRB = 55^o$ $\angle BQR = 35^o$ ($\angle$of reflection),
$\therefore$ $x = 180^o - 35^o - 55^o$ ($\angle$sum of Δ), $x = 90°$
(b) $\angle BQR = a$ and $\angle BRQ = b$

$\therefore\ x+a+b=180^o$ ($\angle$sum ofΔ), $90^o+a+b=180^o$, $a+b=90^o$

$\angle$PQR $=180^o-2a$ and $\angle$QRS $=180^o-2b$ (adj.$\angle$s on st. line),

$\angle$PQR + $\angle$QRS $=(180^o-2a)+(180^o-2b)=360^o-2(a+b)=360^o-2(90^o)=180^o$,

$\therefore$ QP//RS (int.$\angle$s supp.)

Unit 10 Angles and polygon

1. (a) ΔGFE, ΔGCE, ΔDCE, ΔDAE (b) ΔBCF, ΔDCE
   (c) No. (d) Yes, it is an ext.$\angle$ofΔDCG andΔDCE. (e) No.
2. (a) $y=180^o-135^o=45^o$ (adj.$\angle$s on st. line)
   $x+y=128^o$ (ext.$\angle$ofΔ), $\therefore\ x=128^o-45^o=83^o$
   (b) $\angle C=120^o-35^o=85^o$ (ext.$\angle$ofΔ). $x=\angle C=85^o$ (alt.$\angle$s, AB//CD)
   (c) $\angle$STR $=180^o-40^o-80^o=60^o$ ($\angle$sum ofΔ)
   $y=\angle$STR $=60^o$ (vert. opp.$\angle$s).
   $x+y=110^o$ (ext.$\angle$ofΔ), $\therefore\ x=110^o-60^o=50^o$
   (d) $\angle$CEF $=180^o-108^o=72^o$ (adj.$\angle$s on st. line),
   $\therefore$ $\angle$BCA $=72^o+35^o=107^o$ (ext.$\angle$ofΔ),
   $\therefore\ \theta=54^o+107^o=161^o$ (ext.$\angle$ofΔ)
   (e) $\angle$BAD $=y$ (base$\angle$s, isos.Δ), $\angle$BDC $=32^o$ (base$\angle$s, isos.Δ)
   $\angle$BAD $+y=\angle$BDC (ext. $\angle$ofΔ), $\therefore\ y+y=32^o$, $y=16^o$
   (f) reflex$\angle$EFG $=360^o-\theta$ ($\angle$s at a pt.)
   $28^o+56^o+46^o+(360^o-\theta)=360^o$ ($\angle$sum of quad.), $\therefore\ \theta=130^o$
3. (a) The angle sum $=180^o(4-2)=360^o$ (b) The angle sum $=180^o(5-2)=540^o$
   (c) The angle sum $=180^o(6-2)=720^o$ (d) The angle sum $=180^o(8-2)=1080^o$
   (e) The angle sum $=180^o(15-2)=2340^o$ (f) The angle sum $=180^o(18-2)=2880^o$
4. (a) $x+2x+118^o+(x+23^o)+151^o=180^o\times3$ ($\angle$sum of polygon), $4x=248^o$, $x=62^o$
   (b) $y+(y-65^o)+119^o+108^o+90^o=180^o\times3$ ($\angle$sum of polygon), $2y=288^o$, $y=144^o$
   (c) $x+50^o+(180^o-88^o)+100^o=360^o$ (sum of ext. $\angle$s), $x=118^o$
   (d) $43^o+a+130^o+39^o+(360^o-172^o)=180^o\times3$ ($\angle$sum of polygon), $a=140^o$
5. (a) Let the no. of sides be n. $180(n-2)=2520$, $n=\dfrac{2520}{180}+2=16$
   (b) The no. of side $=\dfrac{3060}{180}+2=19$ (c) The no. of side $=\dfrac{4860}{180}+2=29$
6. Let x be the no. of sides. $180(x-2)=16\times90$, $x-2=8$, $x=10$
   Ans. *The number of sides is 10.*
7. Let n be the no. of sides. $180(n-2)=2\times180(14-2)$, $n-2=24$, $n=26$
   Ans. *The number of sides is 26.*
8. (a) A polygon is a closed area made up of straight lines. A regular polygon is a polygon with all sides equal and all angles equal.
   (b) Equilateral triangle, square, regular octagon.
9. (a) Each exterior angle $=180^o-140^o=40^o$, $\therefore$ no. of sides $=\dfrac{360}{40}=9$
   (b) No. of sides $=\dfrac{360}{180-156}=\dfrac{360}{24}=15$

(c) No. of sides $=\dfrac{360}{180-172}=\dfrac{360}{8}=45$

10. $37+88+96+101+144+x+(\frac{2}{3}x-11)=180\times(7-2)$, $\frac{5}{3}x=445$, $x=267$

11. Let the size of the remaining angle be *x*. $\therefore$ Each of the equal angles $=x+42^o$.
$\therefore$ $5(x+42^o)+x=180^o(6-2)$, $6x=510^o$, $x=85^o$. *Ans. The remaining angle is* 85^o.

12. The sum of interior angles $=360^o$,
$\therefore$ the largest angle $=360^o\times\dfrac{6}{2+3+5+6}=360^o\times\dfrac{6}{16}=135^o$

13. (a) 360^o
(b) 360^o

14. (a) no. of sides $=\dfrac{360}{45}=8$ (b) no. of sides $=\dfrac{360}{30}=12$
(c) no. of sides $=\dfrac{360}{15}=24$

15. Let the no. of sides be *n*. $180(n-2)=360$, $(n-2)=2$, $n=4$. *Ans. It has 4 sides.*

16. Let the no. of sides be *n*. $180(n-2)=4\times360$, $(n-2)=8$, $n=10$
$\therefore$ The size of each interior angle $=\dfrac{4\times360^o}{10}=144^o$

17. Let each exterior angle be *x*. $\therefore$ Each interior angle $=6x+12^o$
$\therefore$ $x+(6x+12^o)=180^o$, $7x=168^o$, $x=24^o$. $\therefore$ The no. of sides $=\dfrac{360}{24}=15$

18. (a) In Δ ACI, $\angle$ JIE = $b+e$ (ext. $\angle$ of Δ). In Δ BDJ, $\angle$ IJE = $a+d$ (ext. $\angle$ of Δ).
In Δ JIE, $\angle$ JIE + $\angle$ IJE + $c=180^o$ ($\angle$ sum of Δ), $\therefore$ $b+e+a+d+c=180^o$
(b) In Δ ACE, $a+c+e=180^o$ ($\angle$ sum of Δ). In Δ BDF, $b+d+f=180^o$ ($\angle$ sum of Δ).
$\therefore$ $a+b+c+d+e+f=180^o+180^o=360^o$
(c) $\angle$ AIG = $e+f$ (ext. $\angle$ of Δ); $\angle$ EGH = $a+b$ (ext. $\angle$ of Δ);
$\angle$ CHI = $d+c$ (ext. $\angle$ of Δ).
$\angle$ AIG + $\angle$ EGH + $\angle$ CHI $=360^o$ (sum of ext. $\angle$s), $\therefore$ a + b + c + d + e + f = 360°
(d) $a+b+c+d+e+f+g+h+i=180^o(10-2)=1440^o$ ($\angle$ sum of polygon)
(e) The sum of interior angles of the hexagon $=180^o(6-2)=720^o$
$a+b+c+d+e+f=360^o\times6-720^o=1440^o$

19. The smallest exterior angle $=360^o\times\dfrac{1}{1+2+3+4+5}=360^o\times\dfrac{1}{15}=24^o$
$\therefore$ The largest interior angle $=180^o-24^o=156^o$

20. $\angle$ AFE = $a+b$ (ext. $\angle$ of Δ), $\angle$ AFE + $c=d$ (ext. $\angle$ of Δ)
$\therefore$ $(a+b)+c=d$, $c=d-a-b$

21.

	no. of sides	Each interior angle
original	n	$\dfrac{180^o(n-2)}{n}$
new	$2n$	$\dfrac{180^o(2n-2)}{2n}=\dfrac{180^o(n-1)}{n}$

$\dfrac{180^o(n-1)}{n}:\dfrac{180^o(n-2)}{n}=4:3$, $\therefore$ $\dfrac{n-1}{n-2}=\dfrac{4}{3}$, $3n-3=4n-8$, $n=5$,
$\therefore$ $2n=10$. *Ans. The no. of sides of the new polygon is 10.*

22. Each interior angle of a regular hexagon $= \dfrac{180^o \times (6-2)}{6} = 120^o$

    Each interior angle of a regular octagon $= \dfrac{180^o (8-2)}{8} = 135^o$

    Angles at a point $= 360^o$; to tessellate, the interior angle must be a factor of 360°.
    $\because$ 120 is a factor of 360, but 135 is not a factor of 360,
    $\therefore$ regular hexagons can tessellate, but not regular octagons.
23. $b = 60^o$ (alt. $\angle$ s, // lines), $2a + 60^o = 2b$ (ext. $\angle$ of Δ),
    $\therefore\ 2a + 60^o = 60^o \times 2$, $a = 30^o$
24. Let $a = \angle$ QPS $= \angle$ SPR, $b = \angle$ QRS $= \angle$ SRP
    $\therefore\ 2a + 54^o + 2b = 180^o$ ($\angle$ sum of Δ), $a + b = 63^o$
    $\theta + a + b = 180^o$ ($\angle$ sum of Δ), $\therefore\ \theta + 63^o = 180^o$, $\theta = 117^o$
25. Let $x = \angle$ ABE $= \angle$ EBC, $y = \angle$ ACE $= \angle$ ECD
    $\angle$ACD $= 80^o + 2x$ (ext. $\angle$ of Δ), $\therefore\ 2y = 80^o + 2x$, $y = 40^o + x$, $y - x = 40^o$
    EBC + $\angle$ BEC $= \angle$ ECD (ext. $\angle$ of Δ), $\therefore x + \angle$ BEC $= y$, $\angle$ BEC $= y - x = 40^o$
26. $(x - 5^o) + 2(x + 10^o) + x + (x + 30^o) + [180^o - (x + 25^o)] = 360^o$ (sum of ext.$\angle$s)
    $\therefore\ 5x - x + 200^o = 360^o$, $4x = 160^o$, $x = 40^o$
27. $\angle C = y$ (corr. $\angle$ s, DE//AC), $\therefore\ x + y + 90^o = 180^o$ ($\angle$ sum of Δ),
    $x + y = 90^o$ …. (i). $x + 2y = 125^o$ (ext. $\angle$ of Δ) …. (ii)
    (ii) – (i), $\therefore\ 2y - y = 125^o - 90^o$, $y = 35^o$; $x = 90^o - 35^o = 55^o$
28. $2x + y + 100^o + x + y = 180^o \times 3$ ($\angle$ sum of polygon), $\therefore\ 3x + 2y = 440^o$ ……. (i)
    $60^o + 100^o + x + y = 360^o$ ($\angle$ sum of quad.), $x + y = 200^o$ ……. (ii)
    (i) – (ii)$\times 2$, $\therefore\ 3x + 2y - 2(x + y) = 440^o - 2 \times 200^o$, $x = 40^o$
    $y = 200^o - x = 200^o - 40^o = 160^o$
29. $\angle$FAE $= \angle$ AFE $= 60^o$ (equil. Δ). $\angle$BAE $= \dfrac{180^o \times (5-2)}{5} = 108^o$ ($\angle$ of regular polygon)

    $\therefore\ \angle$BAF $= 108^o - 60^o = 48^o$. $\because$ AB = AE, AE = AF, $\therefore$ AB = AF

    In Δ ABF, $\angle$ AFB $= \dfrac{180^o - 48^o}{2} = 66^o$ ($\angle$ sum of isos. Δ)

    Similarly, $\angle$ EFD $= 66^o$, $\angle$ AFB $+ 60^o + \angle$ EFD $+ x = 360^o$ ($\angle$ s at a pt.)
    $\therefore\ x = 360^o - 60^o - 66^o \times 2 = 168^o$
30. Let $\angle$ A $= \theta$. $\angle$ C $= \angle$ A $= \theta$ (base $\angle$ s, isos. Δ), $\angle$ B $= \angle$ A $= \theta$ (base $\angle$ s, isos. Δ)
    $\angle$BFC $= \angle$A $+ \angle$B $= \theta + \theta = 2\,\theta$ (ext.$\angle$ ofΔ).
    $\because$ CE = CF , $\therefore\ \angle$CEF $= \angle$BFC = $2\,\theta$ (base$\angle$s, isos.Δ).
    $\angle$C +$\angle$CEF+$\angle$BFC = 180° ($\angle$ sum of Δ), $\angle\,\theta + 2\,\theta + 2\,\theta$ = 180°, $\theta = 36^o$, $\therefore\ \angle$ A $= 36^o$
31. (a) $\angle$ EAB $= \angle$ ABC $= \dfrac{180^o \times (5-2)}{5} = 108^o$ ($\angle$ of regular polygon)

    AB = BC, $\therefore\ \angle$BCA $= \dfrac{180^o - 108^o}{2} = 36^o$ ($\angle$ sum of isos. Δ)

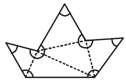
    Similarly, $\angle$ ABE $= 36^o$, $\therefore\ \angle$FBC $= 108^o - 36^o = 72^o$
    $\angle$EFC $= \angle$ FBC $+ \angle$ BCA (ext. $\angle$ of Δ), $\therefore\ \angle$EFC $= 72^o + 36^o = 108^o$

    (b) $\angle$ AED $= 108^o$ (regular pentagon)
    $\angle$AEB $= \angle$ ABE $= 36^o$ (base $\angle$ s, isos. Δ), $\therefore\ \angle$ FED $= 108^o - 36^o = 72^o$

$\angle EFC + \angle FED = 108^o + 72^o = 180^o$, $\therefore$ ED//AC (int.$\angle$s supp.)

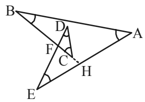
32. $\angle QPT + \angle PTS + (180°-49°) + 103° + (180° - \theta) = 180° \times (5-2)$ ($\angle$ sum of polygon),
but $\angle QPT + \angle PTS = 180^o$, (int.$\angle$s, QP//ST),
$\therefore$ $180° + 131° + 103° + 180° - \theta = 540°$, $\theta = 54°$

33. (a) A concave polygon has at least one interior angle greater than 180^o (reflex angle). Convex polygons have all interior angles smaller than 180^o.
(b) Yes. Any n-sided concave polygons can be divided into $(n-2)$ triangles. For example, the given 7-sided concave polygon can be divided into 5 triangles.



(c) An exterior$\angle$ = 180°– its adjacent interior$\angle$,
$\therefore$ the reflex $\angle$of a concave polygon has no adjacent exterior $\angle$,
$\therefore$ we can't find the sum of exterior angles of a concave polygon.

34. Let the equal angles be x. Produce BC to H.



$\angle BHE = \angle B + \angle A = x + x = 2x$ (ext.$\angle$ ofΔ)
$\angle EFC = \angle D + \angle DCF = x + x = 2x$ (ext. $\angle$ ofΔ)
$\angle E + \angle BHE + \angle EFC = 180^o$ ($\angle$ sum ofΔ),
$\therefore$ $x + 2x + 2x = 180^o$, $5x = 180^o$, $x = 36^o$
Ans. The size of each of these equal angles is 36^o

35. In ABND $= a + b + d + \angle MND = 360^o$ ($\angle$ sum of quad.)
In CMN $= c + \angle CMN + \angle MNC = 180^o$ ($\angle$ sum of Δ)
In EFGM $= e + f + g + \angle LMN = 360^o$ ($\angle$ sum of quad.)
$\therefore$ $a + b + c + d + e + f + g + \angle MND + \angle MNC + \angle CMN + \angle LMN$
$= 360^o + 180^o + 360^o = 900^o$,
but $\angle MND + \angle MNC = 180^o$ and $\angle CMN + \angle LMN = 180^o$ (adj. $\angle$s on st. line),
$\therefore$ $a + b + c + d + e + f + g + 180^o + 180^o = 900^o$
sum of the marked angles $= 900^o - 180^o - 180^o = 540^o$

36. In ΔACS, $a + \frac{a}{2} + b = 180^o$ ($\angle$ sum ofΔ), $b = 180^o - \frac{3}{2}a$ ……. (i)
In ΔBER, $\angle SRD = \frac{b}{3} + (a + 4^o)$ (ext.$\angle$ ofΔ)
In ΔSRD, $b = \angle SRD + 3a - 14^o$ (ext.$\angle$ ofΔ)
$\therefore$ $b = [\frac{b}{3} + (a + 4^o)] + 3a - 14^o$, $\frac{2}{3}b = 4a - 10^o$ ……. (ii)
Put (i) into (ii), $\frac{2}{3}(180^o - \frac{3}{2}a) = 4a - 10^o$, $120^o - a = 4a - 10^o$, $5a = 130^o$,
$a = 26^o$, $\therefore$ $b = 180^o - \frac{3}{2}(26^o) = 141^o$

37. In ABCDEJ, $a + b + c + d + e +$ reflex$\angle$ EJA $= 180^o \times 4$ ($\angle$ sum of polygon)
In FGHIJ, $f + g + h + i + \angle FJI = 180^o \times 3$ ($\angle$ sum of polygon)
$\therefore$ $a + b + c + d + e + f + g + h + i +$ reflex$\angle$ EJA $+ \angle$ FJI
$= 180^o \times 4 + 180^o \times 3 = 180^o \times 7$
But $\angle$FJI $=\angle$EJA (vert. opp.$\angle$s),
$\therefore$ reflex$\angle$EJA $\angle$FJI = reflex$\angle$EJA +$\angle$EJA = 360° ($\angle$s at a pt.)
$\therefore$ Sum of the marked angles $= 180^o \times 7 - 360^o = 900^o$

Unit 11 Congruent triangles (2)

1. $\Delta ACB \cong \Delta PQR$
2. (a) $\angle P = \angle Y = 90°$ (corr. $\angle$s, $\cong\Delta$s); $y = XY = RP = 20$ cm (corr. sides, $\cong\Delta$s)
   (b) $x = \angle R = \angle C = 38°$ (corr. $\angle$s, $\cong\Delta$s)
   y cm = AB = PQ = 6 cm (corr. sides, $\cong\Delta$s), $\therefore$ $y = 6$
   (c) $z = NM = QO$ (corr. sides, $\cong\Delta$s), QO = QP = 5 cm (corr. sides, $\cong\Delta$s),
   $\therefore$ $z = 5$cm. $\angle QOP = 53°$ (base $\angle$s, isos. Δ);
   $\angle Q = 180° - 53° \times 2 = 74°$ ($\angle$ sum of isos. Δ)
   $x = \angle NMO = \angle QOP = 53°$ (corr. $\angle$s, $\cong\Delta$s); $y = \angle Q = 74°$ (corr. $\angle$s, $\cong\Delta$s)
   (d) AB = CD (corr. sides, $\cong\Delta$s), $\therefore$ $2a - 1 = \frac{3}{2}a + 4$, $\frac{1}{2}a = 5$, $a = 10$
   BC = DA (corr. sides, $\cong\Delta$s), $\therefore$ $4b + 1 = 15$, $4b = 14$, $b = 3.5$
   (e) BE = BD (corr. sides, $\cong\Delta$s), $\therefore$ $x = 12$
   AB = CB (corr. sides, $\cong\Delta$s), $\therefore$ $y + 12 = 9 + x = 9 + 12 = 21$, $y = 21 - 12 = 9$
3. (a) LM = BA = 9 cm (corr. sides, $\cong\Delta$s) (b) CA = NM = 13 cm (corr. sides, $\cong\Delta$s)
   (c) NL = CB = 18 cm (corr. sides, $\cong\Delta$s)
4. (a) $\angle ZXY = \angle PRQ = 49°$ (corr. $\angle$s, $\cong\Delta$s)
   (b) $\angle YZX = 180° - \angle ZXY - \angle XYZ = 180° - 49° - 74° = 57°$ ($\angle$sum of Δ)
   (c) $\angle RPQ = \angle YZX = 57°$ (corr. $\angle$s, $\cong\Delta$s)
5. (a) $\Delta ABC \cong \Delta ZXY$ (S.A.S.) (b) $\Delta MLN \cong \Delta XZY$ (S.S.S.)
6. (a) No. (not the included $\angle$) (b) No. (not corr. sides)
   (c) $\Delta BCA \cong \Delta DFE$ (R.H.S.) (d) $\Delta CAB \cong \Delta RPQ$ (A.S.A.)
   (e) No. (not corr. sides) (f) $\Delta PQR \cong \Delta DFE$ (A.A.S.)
7. (a) $\Delta BAC \cong \Delta PQR$ (S.A.S.) (b) $\Delta RPQ \cong \Delta XZY$ (A.A.S.)
   (c) $\Delta BCA \cong \Delta YXZ$ (R.H.S.)
8. (a) ΔSPQ. $\angle RPQ = \angle SPQ$ and $\angle PQR = \angle PQS$ (given), PQ = PQ (common),
   $\therefore$ $\Delta RPQ \cong \Delta SPQ$ (A.S.A.)
   (b) PS = PR (corr. sides, $\cong\Delta$s) (c) $\angle QRP = \angle QSP$ (corr. $\angle$s, $\cong\Delta$s)
9. (a) ΔBOA (or: ΔAOB). OB = OX and OA = OY (radii),
   $\angle BOA = \angle XOY$ (given), $\therefore$ $\Delta BOA \cong \Delta XOY$ (S.A.S.)
   (b) BA = XY (corr. sides, $\cong\Delta$s)
10. (a) ΔQNP. In ΔRMP and ΔQNP, PM = PN (given), PR = 2PN = PQ (given),
    $\angle RPM = \angle QPN$ (common), $\therefore$ $\Delta RMP \cong \Delta QNP$ (S.A.S.)
    (b) $\angle PMR = \angle QNP$ (corr. $\angle$s, $\cong\Delta$s)
11. In ΔABC and ΔADC, AB = AD and BC = DC (given), AC = AC (common),
    $\therefore$ $\Delta ABC \cong \Delta ADC$ (S.S.S.), $\therefore$ $\angle B = \angle D$ (corr. $\angle$s, $\cong\Delta$s)
12. (a) ΔDEC. In ΔDAB and ΔDEC, AB = EC and AD = ED (given),
    $\angle BAD = \angle CED = 90°$ (given). $\therefore$ $\Delta DAB \cong \Delta DEC$ (R.H.S.)
    (b) $\angle ECD = \angle ABD$ (corr. $\angle$s, $\cong\Delta$s); $\angle ABD = 180° - 90° - 68° = 22°$ ($\angle$sum of Δ),
    $\therefore$ $\angle ECD = 22°$
13. Join QS. In ΔQPS and ΔSRQ, PS = RQ and PQ = RS (given),
    QS = SQ (common). $\therefore$ $\Delta QPS \cong \Delta SRQ$ (S.S.S.), $\therefore$ $x = y$ (corr. $\angle$s, $\cong\Delta$s)
14. $\angle EGF = 180° - 73° - 59° = 48°$. In ΔFGE and ΔGFH, FG = GF (common),
    $\angle EFG = \angle HGF = 59°$ (given), $\angle EGF = \angle HFG = 48°$ (proved),
    $\therefore$ $\Delta FGE \cong \Delta GFH$ (A.S.A.), $\therefore$ EF = HG (corr. sidess, $\cong\Delta$s)
15. (a) In ΔADB and ΔBCA, AB = BA (common), AD = BC and BD = AC (given),

$\therefore$ $\Delta ADB \cong \Delta BCA$ (S.S.S.)

(b) $\angle DAB = \angle CBA$ (corr. $\angle$s, $\cong\Delta$s) (c) $\angle BAC = \angle ABD$ (corr. $\angle$s, $\cong\Delta$s)

$\therefore$ $\angle DAB - \angle BAC = \angle CBA - \angle ABD$, i.e. $\angle DAE = \angle CBE$

16. $\angle CED + \angle FEC = \angle FDE + \angle CDF$ (given), i.e. $\angle FED = \angle CDE$
In ΔEDC and ΔDEF, $\angle CED = \angle FDE$ (given), $\angle FED = \angle CDE$ (proved),
ED = DE (common), $\therefore$ $\Delta EDC \cong \Delta DEF$ (A.S.A.), $\therefore$ EC = DF (corr. sides, $\cong\Delta$s)

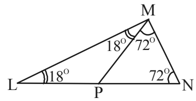
17. In ΔPQN and ΔPRM, PN = PM (given), $\angle PNQ = \angle PMR = 90°$ (given),
$\angle QPN = \angle RPM$ (common), $\therefore$ $\Delta PQN \cong \Delta PRM$ (A.S.A.),
$\therefore$ $\angle PQN = \angle PRM$ (corr. $\angle$s, $\cong\Delta$s), i.e. $\angle MQH = \angle NRH$

18. In ΔPQM and ΔRQM, QP = QR and PM = RM (given), QM = QM (common),
$\therefore$ $\Delta PQM \cong \Delta RQM$ (S.S.S.), $\therefore$ $\angle QMP = \angle QMR$ (corr. $\angle$s, $\cong\Delta$s);
$\angle QMP + \angle QMR = 180°$ (adj. $\angle$s on st. line),
$\therefore$ $\angle QMP = \angle QMR = \dfrac{180°}{2} = 90°$, i.e. $QM \perp PR$.

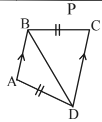
19. RS = PS = 16 cm (corr. sides, $\cong\Delta$s)
$QS = \sqrt{QR^2 + RS^2} = \sqrt{12^2 + 16^2} = \sqrt{400} = 20$ cm (Pythagoras' Theorem)

20. In ΔABC, $AC^2 + BC^2 = 24^2 + 10^2 = 676$, $AB^2 = 26^2 = 676$
$\because$ $AC^2 + BC^2 = AB^2$, $\therefore$ $\angle B = 90°$ (Converse of Pyth. Thm.)
$\angle B = \angle Q = 90°$ (proved), AB = PQ and BC = QR (given), $\therefore$ $\Delta ABC \cong \Delta PQR$ (S.A.S.)

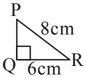
21. No, because the two triangles do not satisfy any of the conditions for the test of congruent triangles. For example, ΔMPL and ΔMPN below match the given information, but they are not congruent.



22. (a) $\angle ABD = \angle CDB$ (alt. $\angle$s, AB // DC)
(b) AD = BC (given), BD = BD (common),
but we don't know whether their included angles are equal,
$\therefore$ the two triangles may not be congruent.
(c)

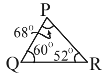


23. (a)

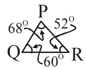


No, $\because$ they must be congruent Δs (RHS)

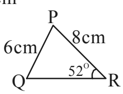
(b)



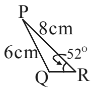
or



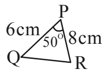
(c)



or



(d)



No, $\because$ they must be congruent Δs (SAS)

24. In ΔACB and ΔECD, BC = CD and $\angle A = \angle E$ (given), $\angle ACB = \angle ECD$ (common),
$\therefore$ $\Delta ACB \cong \Delta ECD$ (A.A.S.), $\therefore$ AC = EC (corr. sides, $\cong\Delta$s); but DC = BC (given),
$\therefore$ AC − DC = EC − BC, i.e. AD = EB

25. $\angle BCE + \angle ECD = \angle A + \angle CBA$ (ext. $\angle$ of Δ), $\therefore$ $90° + \angle ECD = 90° + \angle CBA$,
$\angle ECD = \angle CBA$. In ΔECD and ΔCBA, $\angle ECD = \angle CBA$ (proved), CD = AB (given),
$\angle EDC = \angle CAB = 90°$ (given), $\therefore$ $\Delta ECD \cong \Delta CBA$ (A.A.S.),
$\therefore$ EC = CB (corr. sides, $\cong\Delta$s), i.e. ΔBCE is isosceles

26. In ΔAEB and ΔDEC, AE = ED and BE = EC (given), $\angle AEB = \angle DEC$ (vert. opp. $\angle$s),
$\therefore$ $\Delta AEB \cong \Delta DEC$ (S.A.S.), $\therefore$ $\angle ABE = \angle DCE$ (corr. $\angle$s, $\cong\Delta$s),
$\therefore$ AB // CD (alt. $\angle$s equal)

27. (a) In ΔQAC and ΔQBC, QA = QB (radii), AC = BC (equal radii),
QC = QC (common), $\therefore$ $\Delta QAC \cong \Delta QBC$ (S.S.S.)
(b) $\angle CQA = \angle CQB$ (corr. $\angle$s, $\cong\Delta$s), i.e. QC is the angle bisector of $\angle PQR$.

28. (a) 6 triangles.

(b) ΔABE and ΔCBD are congruent triangles. In ΔABE and ΔCBD, AB = CB (equil. Δ), BE = BD (equil. Δ), $\angle$ABE = 60° = $\angle$CBD (equil. Δs), $\therefore$ $\Delta ABE \cong \Delta CBD$ (S.A.S.)

(c) AE = CD (corr. sides, $\cong\Delta$s); $\angle$BAE = $\angle$BCD and $\angle$AEB = $\angle$CDB (corr. $\angle$s, $\cong\Delta$s)

Unit 12 Similar triangle

[Knowledge about Pythagoras' Theorem (Unit 13) is required in some of the items.]

1. (a) No. ($\because$ $\angle$B and $\angle$P are not the included angles)
   (b) Yes. $\Delta ABC \sim \Delta QRP$ ($\angle$A = 180° – 75° – 40° = 65°, $\therefore$ A.A.A.)
   (c) Yes. $\Delta ABC \sim \Delta RQP$ (ratio of 2 sides, inc. $\angle$)
   (d) No. ($\because$ 3 sides are not proportional.)
2. $\angle$RQP = 180° – 69° – 37° = 74° ($\angle$ sum of Δ); $\angle$EDC = $\angle$RQP = 74° (corr. $\angle$s, ~Δs)
3. (a) YX = QR = 13cm (corr. sides, ~Δs) (b) $\angle$YXZ = $\angle$QRP = 44° (corr. $\angle$s, ~Δs)
   (c) $\angle$PQR = 180° – 47° – 44° = 89° ($\angle$ sum of Δ); $\angle$ZYX = $\angle$ PQR = 89° (corr. $\angle$s, ~Δs)
4. (a) $\frac{BC}{PR} = \frac{AC}{QR} = \frac{AB}{QP}$ (corr. sides, ~Δs), $\frac{x}{6} = \frac{y}{7} = \frac{10}{4}$, $\therefore$ $x = \frac{10}{4} \times 6 = 15$,
   $y = \frac{10}{4} \times 7 = 17.5$; $z = \angle B = 86°$ (corr. $\angle$s, ~Δs)
5. (a) $\Delta ABC \sim \Delta EDF$ (A.A.A.), $\frac{DE}{BA} = \frac{EF}{AC} = \frac{DF}{BC}$ (corr. sides, ~Δs), $\frac{p}{18} = \frac{q}{24} = \frac{9}{12}$,
   $\therefore$ $p = \frac{9}{12} \times 18 = 13.5$, $q = \frac{9}{12} \times 24 = 18$
   (b) $\Delta ABC \sim \Delta QPR$ (A.A.A.), $\frac{AC}{QR} = \frac{BC}{PR} = \frac{AB}{QP}$ (corr. sides, ~Δs), $\frac{x}{6} = \frac{6}{y} = \frac{12}{8}$,
   $\therefore$ $x = \frac{12}{8} \times 6 = 9$, $y = 6 \times \frac{8}{12} = 4$
   (c) $\frac{x}{10} = \frac{y}{6} = \frac{12}{8}$ (corr. sides, ~Δs), $\therefore$ $x = \frac{12}{8} \times 10 = 15$, $y = \frac{12}{8} \times 6 = 9$
   (d) $\frac{7}{4} = \frac{3h+1}{5} = \frac{2k-3}{4}$ (corr. sides, ~Δs), $\therefore$ $35 = 12h + 4$, $h = \frac{31}{12}$;
   $28 = 8k - 12$, $k = \frac{30}{8} = \frac{15}{4}$
6. (a) $\Delta EFG \sim \Delta KFH$ (A.A.A.), $\frac{EF}{KF} = \frac{EG}{KH}$ (corr. sides, ~Δs),
   $\frac{m+6}{6} = \frac{28}{16}$, $m + 6 = \frac{28}{16} \times 6 = 10.5$, $\therefore$ $m = 4.5$
   (b) $\Delta PQR \sim \Delta SQT$ (A.A.A.), $\frac{PQ}{QR} = \frac{SQ}{QT}$ (corr. sides, ~Δs),
   $\frac{z+6}{z} = \frac{3+9}{3} = 4$, $z + 6 = 4z$, $3z = 6$, $\therefore$ $z = 2$
7. (a) $\Delta ABC \sim \Delta AED$ (A.A.A.), $\frac{AB}{AE} = \frac{AC}{AD}$ (corr. sides, ~Δs),
   $\frac{y+6}{8} = \frac{8+7}{6} = \frac{5}{2}$, $2y + 12 = 40$, $2y = 28$, $\therefore$ $y = 14$
   (b) $\Delta PQR \sim \Delta NMR$ (A.A.A.), $\frac{NR}{PR} = \frac{NM}{PQ}$ (corr. sides, ~Δs), $\frac{r}{50} = \frac{8}{20}$,

$\therefore\ r=\frac{8}{20}\times 50=20$

(c) $\Delta PQR \sim \Delta TQS$ (A.A.A.), $\frac{QR}{QS}=\frac{PQ}{TQ}$ (corr. sides, ~Δs),

$\frac{18+x}{28}=\frac{28+8}{18}=2$, $18+x=56$, $\therefore x=38$

8. (a) In ΔEFG and ΔKFH, ∠F = ∠F (common),
∠E = ∠HKF and ∠G = ∠KHF (corr. ∠s, GE//HK),
$\therefore$ ΔEFG ~ ΔKFH (A.A.A.), $\frac{EF}{KF}=\frac{EG}{KH}$ (corr. sides, ~Δs), $\frac{2+r}{r}=\frac{20}{16}=\frac{5}{4}$,
$8+4r=5r$, $\therefore\ r=8$

(b) In ΔADE and ΔABC, ∠A = ∠A (common),
∠ADE=∠ABC and ∠AED = ∠ACB (corr. ∠s, DE//BC),
$\therefore$ ΔADE~ΔABC (A.A.A.), $\frac{AE}{AC}=\frac{DE}{BC}$ (corr. sides, ~Δs), $\frac{y}{y+15}=\frac{8}{20}=\frac{2}{5}$,
$5y=2y+30$, $3y=30$, $\therefore\ y=10$

(c) In ΔPQR and ΔFGR, ∠R = ∠R (common),
∠P=∠RFG and ∠Q = ∠RGF (corr. ∠s, PQ//FG), $\therefore$ ΔPQR~ΔFGR (A.A.A.),
$\frac{FG}{PQ}=\frac{FR}{PR}$ (corr. sides, ~Δs), $\frac{a}{36}=\frac{9}{9+18}=\frac{1}{3}$, $3a=36$, $\therefore\ a=12$

9. (a) ∠ACB = ∠ECD (vert. opp. ∠s), ∠A=∠E and ∠B = ∠D (alt. ∠s, AB//DE),
$\therefore$ ΔABC~ΔEDC (A.A.A.), $\frac{AB}{ED}=\frac{BC}{DC}=\frac{AC}{EC}$ (corr. sides, ~Δs),
$\frac{8}{y}=\frac{x}{6}=\frac{3}{4}$, $\therefore\ x=\frac{3}{4}\times 6=4.5$, $y=8\times\frac{4}{3}=\frac{32}{3}$

(b) ∠PRQ = ∠TRS (vert. opp. ∠s), ∠P=∠T and ∠Q = ∠S (alt. ∠s, PQ//ST),
$\therefore$ΔPQR~ΔTSR (A.A.A.), $\frac{PQ}{TS}=\frac{QR}{SR}=\frac{PR}{TR}$ (corr. sides, ~Δs),
$\frac{5}{k}=\frac{4}{10}=\frac{h}{12}$, $\therefore\ h=\frac{4}{10}\times 12=4.8$, $k=5\times\frac{10}{4}=12.5$

10. Let y m be the actual height. $\frac{y\text{ m}}{6\text{ cm}}=\frac{60\text{ m}}{8\text{ cm}}$, $y=\frac{60}{8}\times 6=45$

Ans. The actual height is 45 m.

11. (a) $\frac{XZ}{XM}=\frac{XY}{XN}$ (corr. sides, ~Δs), $\frac{a}{18}=\frac{6}{12}$, $\therefore\ a=\frac{6}{12}\times 18=9$
(b) ∵∠Z = ∠M (corr. ∠s, ~Δs), ∴ YZ//MN (alt. ∠s eq.)

12. (a) No. Their corr.∠s are equal, but their sides may not be proportional.
(b) Yes. Their corr.∠s are equal, and their sides are proportional.
(c) No. Their sides are proportional, but their corr.∠s may not be equal.
(d) Yes. Their corr. ∠s are equal, and their sides are proportional (1:1).

13. $\frac{\text{Perimeter of the second }\Delta}{\text{Perimeter of the first }\Delta}=\frac{96}{5+3+4}=\frac{96}{12}=8$
$\therefore$ the longest side = 5×8 = 40 cm

14. ΔAEC ~ ΔADB (A.A.A.) , $\therefore\ \frac{AC}{1.8}=\frac{40}{2.6}$, AC = 27.7 m (3 sig. fig.)

Ans. The distance between the tree and the building = 27.7 − 1.8 = 25.9 m.

15. ∠BAC = ∠AED = 77° (corr. ∠s, ~Δs), ∠ADE = ∠BCA = 44° (corr. ∠s, ~Δs),

$\therefore$ $\angle DAE = 180° - 77° - 44° = 59°$ ($\angle$ sum of Δ), $\therefore$ $\angle BAD = 77° - 59° = 18°$

16. (a) $\angle Q = 180° - 80° - 32° = 68° = \angle B$ ($\angle$ sum of Δ),
$\angle A = 180° - 68° - 80° = 32° = \angle R$ ($\angle$ sum of Δ), $\angle P = \angle C = 80°$ (given),
$\therefore \Delta PQR \sim \Delta CBA$ (A.A.A.)

(b) $\frac{AB}{RQ} = \frac{AC}{RP} = \frac{BC}{QP}$ (corr. sides, ~Δs), $\frac{m}{9} = \frac{n}{13} = \frac{3}{6}$,
$\therefore$ $m = \frac{3}{6} \times 9 = 4.5$, $n = \frac{3}{6} \times 13 = 6.5$

17. $\Delta LPN \sim \Delta DEN$ (A.A.A.), $\frac{DN}{LN} = \frac{DE}{LP}$ (corr. sides, ~Δs), $\frac{m}{m+6} = \frac{5}{8}$,
$8m = 5m + 30$, $\therefore$ $m = 10$. $\Delta LCD \sim \Delta LMN$ (A.A.A.),
$\frac{LD}{LN} = \frac{DC}{NM}$ (corr. sides, ~Δs), $\frac{6}{6+m} = \frac{n}{12}$, $\therefore$ $n = \frac{6}{6+10} \times 12 = 4.5$

18. (a) $\Delta MNT \sim \Delta RQT$ (A.A.A.), $\frac{QR}{NM} = \frac{QT}{NT}$ (corr. sides, ~Δs), $\frac{QR}{7} = \frac{6}{4}$,
$\therefore$ $QR = \frac{6}{4} \times 7 = 10.5$

(b) $\Delta PMN \sim \Delta PQR$ (A.A.A.), $\frac{PM}{PQ} = \frac{MN}{QR}$ (corr. sides, ~Δs), $\frac{5}{5+MQ} = \frac{7}{10.5}$,
$52.5 = 35 + 7MQ$, $7MQ = 17.5$, $\therefore$ $MQ = 2.5$

19. $\Delta HEF \sim \Delta HCD$ (A.A.A.), $\frac{HF}{HD} = \frac{EF}{CD} = \frac{HE}{HC}$ (corr. sides, ~Δs), $\frac{y}{y+3} = \frac{z}{5} = \frac{8}{8+4} = \frac{2}{3}$,
$\therefore$ $z = \frac{2}{3} \times 5 = \frac{10}{3}$; $3y = 2y + 6$, $\therefore$ $y = 6$;
$\Delta DEF \sim \Delta DGH$ (A.A.A.), $\frac{DF}{DH} = \frac{EF}{GH}$ (corr. sides, ~Δs), $\frac{3}{3+y} = \frac{z}{x}$, $\frac{3}{3+6} = \frac{\frac{10}{3}}{x}$,
$\therefore$ $x = \frac{10}{3} \times \frac{9}{3} = 10$

20. $\Delta ABC \sim \Delta ECD$ (A.A.A.), $\frac{AC}{ED} = \frac{AB}{EC}$ (corr. sides, ~Δs), $\frac{AC}{20} = \frac{28}{16}$, $AC = \frac{28}{16} \times 20 = 35$,
$\therefore$ $s = \sqrt{35^2 - 28^2} = 21$ (Pyth. Thm.)

21. $\Delta ABC \sim \Delta EBD$ (A.A.A.), $\frac{AC}{ED} = \frac{AB}{EB}$ (corr. sides, ~Δs), $\frac{AC}{8} = \frac{18}{12}$, $AC = \frac{18}{12} \times 8 = 12$;
$BD = \sqrt{12^2 - 8^2} = \sqrt{80}$, $\therefore$ $AD = 18 - \sqrt{80} = 9.06$ (3 sig. fig.); $BC = \sqrt{18^2 - 12^2} = \sqrt{180}$,
$\therefore$ $CE = \sqrt{180} - 12 = 1.42$ (3 sig. fig.)

22. (a) $\angle BCA + \angle BCD = \angle CDE + \angle CED$ (ext. $\angle$ of Δ),
$\angle BCA + 90° = \angle CDE + 90°$, $\therefore$ $\angle BCA = \angle CDE$

(b) $\angle A = \angle E = 90°$ (given), $\angle BCA = \angle CDE$ (proved),
$\angle ABC = \angle ECD$ (3^{rd} $\angle$ of Δ), $\therefore$ $\Delta ABC \sim \Delta ECD$ (A.A.A.)

(c) $\frac{CE}{BA} = \frac{CD}{BC}$ (corr. sides, ~Δs), $\frac{CE}{10} = \frac{20}{26}$, $CE = \frac{20}{26} \times 10 = \frac{100}{13}$,
$\therefore$ $x = \sqrt{20^2 - (\frac{100}{13})^2} = 18.5$ (3 sig. fig.)

23. (a) In ΔECB and ΔDCA, $\angle C = \angle C$ (common), $\angle BEC = \angle ADC = 90°$ (given),
$\angle EBC = \angle DAC$ (3^{rd} $\angle$ of Δ), $\therefore$ $\Delta ECB \sim \Delta DCA$ (A.A.A.)

(b) $\frac{EC}{DC}=\frac{BC}{AC}$ (corr. sides, ~Δs), $\frac{k}{18}=\frac{6+18}{30}$, $\therefore$ $k=\frac{24}{30}\times 18=14.4$

24. (a) In ΔMNX and ΔYZX, $\frac{XM}{XY}=\frac{15}{9}=\frac{5}{3}$, $\frac{XN}{XZ}=\frac{20}{12}=\frac{5}{3}$,
∠MXN = ∠YXZ (vert. opp. ∠s), $\therefore$ ΔMNX~ΔYZX (ratio of 2 sides, inc. ∠)
(b) $\frac{YZ}{MN}=\frac{3}{5}$ (corr. sides, ~Δs), $\frac{y}{18}=\frac{3}{5}$, $y=\frac{3}{5}\times 18=10.8$
(c) ∠Y = ∠M (corr.∠s, ~Δs), but we don't know the relationship between ∠Y and ∠N, $\therefore$ YZ is not parallel to MN (alt.∠s not equal).

25. (a) In ΔBAD and ΔCDB, ∠BAD = ∠BDC (given), $\frac{AD}{DB}=\frac{4}{8}=\frac{1}{2}$, $\frac{AB}{DC}=\frac{6}{12}=\frac{1}{2}$,
$\therefore$ ΔBAD~ΔCDB (ratio of 2 sides, inc. ∠)
(b) $\frac{DB}{BC}=\frac{1}{2}$ (corr. sides, ~Δs), $\frac{8}{BC}=\frac{1}{2}$, $\therefore$ BC = 8×2 = 16

26. In ΔABC and ΔCDB, $\frac{AB}{CD}=\frac{14}{21}=\frac{2}{3}$, $\frac{BC}{DB}=\frac{12}{18}=\frac{2}{3}$, $\frac{AC}{CB}=\frac{8}{12}=\frac{2}{3}$,
$\therefore$ ΔABC~ΔCDB (3 sides prop.), $\therefore$ ∠ACB = ∠CBD (corr. ∠s, ~Δs),
$\therefore$ AC//BD (alt. ∠s, eq.)

27. In ΔADB and ΔABC , $\frac{AD}{AB}=\frac{12}{18}=\frac{2}{3}$, $\frac{AB}{AC}=\frac{18}{15+12}=\frac{2}{3}$,
∠DAB=∠BAC (common), $\therefore$ ΔABC~ΔADB (ratio of 2 sides, inc. ∠),
$\therefore$ $\frac{BD}{CB}=\frac{2}{3}$ (corr. sides, ~Δs), $\frac{10}{x}=\frac{2}{3}$, $\therefore$ $x=10\times\frac{3}{2}=15$

28. (a) In ΔPQR and ΔPRS, $\frac{PR}{PS}=\frac{PQ}{PR}$ (given), ∠QPR=∠RPS (common),
$\therefore$ ΔPQR ~ ΔPRS (ratio of 2 sides, inc. ∠)
(b) ∠PSR = 180° – 116° = 64° (adj.∠s on st. line),
∠PRQ = ∠PSR = 64° (corr.∠s, ~Δs), $\therefore$ ∠PRS =64° – θ
In ΔPSR, 84° + 64° + (64° – θ) = 180° (∠sum of Δ), $\therefore$ θ = 32°

Unit 13 Pythagoras' theorem

1. (a) $x^2=10^2+24^2, x=\sqrt{100+576}=\sqrt{676}=26$
(b) $17^2=y^2+8^2, y^2=17^2-8^2, y=\sqrt{289-64}=\sqrt{225}=15$
(c) $x^2=15^2-10^2=225-100=125, x=\sqrt{125}=11.2$
$y^2=x^2-8^2=125-64=61, y=\sqrt{61}=7.8$
(d) $20^2=h^2+(\frac{24}{2})^2, h=\sqrt{400-144}=\sqrt{256}=16$ cm

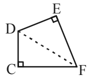
2. (a) $PS=\sqrt{10^2-6^2}=\sqrt{64}=8$; $SR=\sqrt{8^2-6^2}=\sqrt{28}$, $a=PS+SR=8+\sqrt{28}=13.3$
(b) AC = 3 + 6 = 9, $BC^2=10^2-9^2=19$,
$m^2=BC^2+6^2=19+36=55$, $m=\sqrt{55}=7.42$
(c) $GF=\sqrt{13^2-12^2}=\sqrt{169-144}=\sqrt{25}=5$
$EF=\sqrt{15^2-12^2}=\sqrt{225-144}=\sqrt{81}=9$
$x=EF-GF=9-5=4$

(d) $x=\sqrt{39^2-36^2}=\sqrt{225}=15$, $\quad ML=\sqrt{45^2-36^2}=\sqrt{729}=27$

$y = ML - x = 27 - 15 = 12$

(e) In $\triangle PQR$, $PQ^2 + RQ^2 = PR^2$, $\quad (n+8)^2 + 10^2 = 26^2$, $\quad (n+8)^2 = 26^2 - 10^2$,

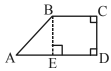
$n+8=\sqrt{26^2-10^2}$, $n+8=\sqrt{576}$, $n+8=24$, $\quad \therefore\ n=16$



(f) Join DF. $\quad DF^2 = 6^2 + 10^2$, and $DF^2 = 7^2 + y^2$,

$\therefore\ y^2 + 49 = 36 + 100$, $y^2 = 87$, $\quad \therefore\ y=\sqrt{87}=9.33$

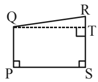
3. (a) Draw $BE \perp AD$. $\quad BE = CD = 8$, $ED = BC = 7$,



$AE = AD - ED = 13 - 7 = 6$

$y=\sqrt{BE^2+AE^2}=\sqrt{8^2+6^2}=\sqrt{100}=10$

(b) Draw $QT \perp RS$. $\quad QT = PS = 40$, $TS = QP = 15$,



$RT=\sqrt{QR^2-QT^2}=\sqrt{41^2+40^2}=\sqrt{81}=9$

$x = RT + TS = 9 + 15 = 24$

4. $(x+2)^2 = x^2 + 12^2$, $\quad x^2 + 4x + 4 = x^2 + 144$, $\quad 4x = 140$, $\quad \therefore\ x = 35$

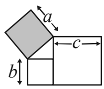
5. Let $BC = x$ cm. $\quad \therefore$ $AB = 2x$ cm, $AB^2 = 4^2 + BC^2$, $(2x)^2 = 16 + x^2$,

$3x^2 = 16, x=\sqrt{\frac{16}{3}}$; $\quad AB = 2x = 2(\sqrt{\frac{16}{3}}) = 4.62$ cm

6. $BD = 14 + 10 = 24$, $AB=\sqrt{25^2-BD^2}=\sqrt{25^2-24^2}=\sqrt{49}=7$

Area of $\triangle ACD = \frac{1}{2}(AB)(CD) = \frac{1}{2}(7)(10) = 35$ sq. units

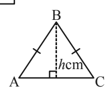
7. $b=\sqrt{49}=7$; $c=\sqrt{169}=13$;



$a^2 = b^2 + (c-b)^2 = 7^2 + (13-7)^2 = 49 + 36 = 85$

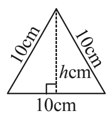
The shaded area $= a^2 = 85\text{ cm}^2$

8. Let the altitude from B to AC be h cm. $\quad \frac{8h}{2}=24, h=6$



$AB^2 = h^2 + (\frac{8}{2})^2 = 6^2 + 4^2 = 52$, $\quad AB=\sqrt{52}=7.21$ cm

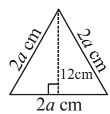
9. Let the height of the equilateral triangle be h cm.



$10^2 = h^2 + (\frac{10}{2})^2$, $\quad h^2 = 100 - 25 = 75$, $\quad h=\sqrt{75}$ cm

$\therefore$ Area $= \frac{1}{2}(10)(\sqrt{75}) = 43.3\text{ cm}^2$

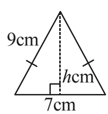
10. Let the side of the equilateral triangle be $2a$ cm.



$(2a)^2 = 12^2 + a^2$, $\quad 4a^2 = 144 + a^2$, $\quad 3a^2 = 144$, $\quad a=\sqrt{\frac{144}{3}}=\sqrt{48}$

$\therefore$ Area $= \frac{1}{2}(2\sqrt{48})(12) = 83.1\text{ cm}^2$

11. Let h cm be the altitude.



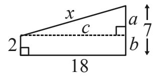
$9^2 = h^2 + (\frac{7}{2})^2$, $\quad h^2 = 81 - \frac{49}{4} = \frac{275}{4}$, $\quad h=\sqrt{\frac{275}{4}}$

$\therefore$ Area $= \frac{1}{2}(7)(\sqrt{\frac{275}{4}}) = 29.0\text{ cm}^2$

12. Let its sides be x m. $\quad x^2 + x^2 = 30$, $2x^2 = 30$, $x^2 = 15$, $x=\sqrt{15}$

$\therefore$ Area $= x^2 = 15\text{ m}^2$; $\quad$ perimeter $= 4x = 4\sqrt{15} = 15.5$ m

13. Let x m be the distance between the tops of the two posts.



$c = 18, \quad b = 2, \quad a = 7 - b = 7 - 2 = 5$

$x = \sqrt{a^2 + c^2} = \sqrt{5^2 + 18^2} = 18.7$

Ans. The distance between the tops is 18.7 m.

14. (a) $AC = \sqrt{15^2 + 20^2} = \sqrt{625} = 25\text{ cm}$

(b) $AS = \sqrt{60^2 + AC^2} = \sqrt{3600 + 625} = \sqrt{4225} = 65\text{ cm}$

15. The diagonal of the rectangle with dimensions $8\text{cm} \times 10\text{cm} = \sqrt{8^2 + 10^2} = \sqrt{164}$ cm.

$\therefore$ The longest pencil = $\sqrt{4^2 + 164} = \sqrt{180} = 13.4\text{ cm}$

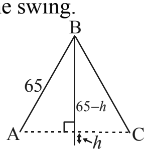
16. $BC = \sqrt{50^2 - 48^2} = \sqrt{196} = 14$ cm. Area = (48)(14) = 672 cm^2

17. Let his speed be x km/h. The distance be traveled in 40 mins $= x \cdot \dfrac{40}{60} = \dfrac{2}{3} x$ km

$(\frac{2}{3}x)^2 + 35^2 = 37^2, \frac{4}{9}x^2 = 37^2 - 35^2 = 144, \quad x = \sqrt{\dfrac{144 \times 9}{4}} = 18$

Ans. His speed was 18 km/h.

18. Let h cm be the distance between the highest and lowest point of the swing.



AC = 25 cm, $\therefore \quad (65 - h)^2 + (\frac{25}{2})^2 = 65^2,$

$(65 - h)^2 = 65^2 - 12.5^2 = 4068.75, \quad 65 - h = \sqrt{4068.75} = 63.79,$

$h = 1.21$ *Ans. The distance is 1.21 cm.*

19. (a) $AB^2 = 52^2 = 2704, AC^2 + BC^2 = 48^2 + 20^2 = 2704$

$\because AB^2 = AC^2 + BC^2, \quad \therefore \triangle ABC$ is right-angled.

(b) $PR^2 = 41^2 = 1681, PQ^2 + QR^2 = 38^2 + 15^2 = 1669$

$\because PR^2 \neq PQ^2 + QR^2, \quad \therefore \triangle PQR$ is not right-angled.

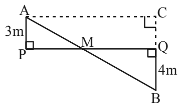
(c) $CE^2 = 137^2 = 18769, CD^2 + DE^2 = 88^2 + 105^2 = 18769$

$\therefore CE^2 = CD^2 + DE^2, \quad \therefore \triangle CDE$ is right-angled.

20. No, the longest side is 175. He should compare: "$49^2 + 168^2$" with 175^2.

$\because 49^2 + 168^2 = 2401 + 28224 = 30625 = 175^2$

$\therefore$ It is a right-angled triangle (converse of Pyth. Thm.)



21. AC = PQ = 24, CB = CQ + QB = AP + QB = 3 + 4 = 7

$\therefore AB = \sqrt{24^2 + 7^2} = \sqrt{625} = 25$ m

22. OQ = OC + CQ = 9 + 6 = 15 (cm), OB = OQ = 15 (radii)

$OC^2 + BC^2 = OB^2, \quad 9^2 + BC^2 = 15^2, \quad BC^2 = 15^2 - 9^2, \quad BC = \sqrt{225 - 81} = 12$ cm

$\therefore$ The area of the rectangle $= OC \times BC = 9 \times 12 = 108\text{ cm}^2$

23. The near vertical distance $= 12 - 4 = 8$ cm

$\therefore$ The distance the foot of the ladder will slide $= \sqrt{20^2 - 8^2} - \sqrt{20^2 - 12^2} = \sqrt{336} - 16 = 2.33\text{ cm}$

24. The horizontal distance between P and Q = 112 + 14 + 14 = 140.

The vertical distance between P and Q = 16 + 16 + 16 = 48

The shortest distance between P and Q $= \sqrt{140^2 + 48^2} = \sqrt{21904} = 148$ (Pyth. Thm.)

25. $AC = \sqrt{AB^2 + BC^2} = \sqrt{7^2 + 24^2} = 25$

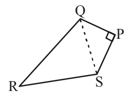
Area of $\triangle ABC = \dfrac{1}{2}(AC)(BM) = \dfrac{1}{2}(AB)(BC)$, 25BM = (7)(24), BM = 6.72 cm

26. Let BC = x cm, $\therefore$ CD = $(24 - x)$cm, $AC^2 = 16^2 + x^2$, and $CE^2 = 12^2 + (24 - x)^2$

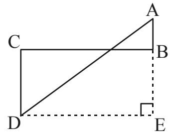
$\because AC = CE, \quad \therefore 16^2 + x^2 = 12^2 + (24 - x)^2 = 12^2 + 24^2 - 48x + x^2$

$48x = 12^2 + 24^2 - 16^2$, $x = \dfrac{464}{48} = 9\dfrac{2}{3}$, $\therefore$ BC $= 9\dfrac{2}{3}$ cm

27. Join QS. QS $= \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm (Pyth. Thm.)
In $\triangle$QSR, $QS^2 + SR^2 = 10^2 + 24^2 = 100 + 576 = 676$
$QR^2 = 26^2 = 676$, $\because$ $QS^2 + SR^2 = QR^2$,
$\therefore$ $\angle QSR = 90^\circ$ (converse of Pyth. Thm.)
$\therefore$ Area of PQRS $= \dfrac{1}{2}$(PQ)(PS) $+ \dfrac{1}{2}$(QS)(SR) $= \dfrac{1}{2}(6)(8) + \dfrac{1}{2}(10)(24) = 144$ cm^2

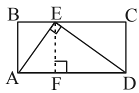


28. AB $= 72 \times \dfrac{6}{60} = 7.2$ km, DE = BC $= 72 \times \dfrac{10}{60} = 12$ km,
BE = CD = 8 km. $\therefore$ AE = 7.2 + 8 = 15.2 km,
AD $= \sqrt{DE^2 + AE^2} = \sqrt{12^2 + 15.2^2} = 19.4$ km
Ans. He is 19.4km from the starting point.

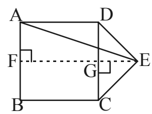


29. DQ $= \sqrt{24^2 + 18^2} = \sqrt{900} = 30$ cm. Let h cm be the distance between PB and DQ.
Area of ABCD $= 2 \times \dfrac{1}{2}(24)(18) + (DQ)(h) = 24^2$
$432 + 30h = 576$, $h = \dfrac{144}{30} = 4.8$. *Ans. The distance between PB and DQ is 4.8 cm.*

30. $ED^2 = \sqrt{25^2 - 7^2} = \sqrt{576} = 24$. Let EF be the height from E to AD.
Area of $\triangle$AED $= \dfrac{1}{2}(AE)(ED) = \dfrac{1}{2}(AD)(EF)$
$\therefore$ 25EF = (7)(24), EF = 6.72; AB = EF = 6.72
$\therefore$ Area of ABCD = (25)(6.72) = 168 sq. units



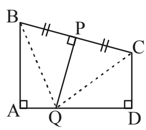
31. From E, draw EF $\perp$ AB. DG = AF $= \dfrac{1}{2}(12) = 6$ cm.
$GE = \sqrt{DE^2 - DG^2} = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$ cm,
FG = BC = 12 cm, FE = FG + GE = 12 + 8 = 20 cm
In $\triangle$AFE, $AE = \sqrt{AF^2 + FE^2} = \sqrt{6^2 + 20^2} = \sqrt{436} = 20.9$ cm



32. (a) DQ = 18 – 16 = 2 cm, BQ $= \sqrt{16^2 + 12^2} = 20$ cm (Pyth. Thm.)
(b) In $\triangle$DPQ, $PQ^2 = PD^2 + DQ^2 = x^2 + 2^2 = x^2 + 4$
In $\triangle$PQB, $y^2 = PQ^2 + BQ^2 = (x^2 + 4) + 20^2 = x^2 + 404$
(c) In $\triangle$APB, AP = $(12 - x)$cm, $\therefore$ $y^2 = AP^2 + 18^2 = (12 - x)^2 + 18^2$
$\therefore$ $(12 - x)^2 + 18^2 = x^2 + 404$, $144 - 24x + x^2 + 324 = x^2 + 404$, $64 = 24x$,
$\therefore$ $x = \dfrac{8}{3}$. From (b), $y^2 = (\dfrac{8}{3})^2 + 404 = \dfrac{3700}{9}$, $y = \sqrt{\dfrac{3700}{9}} = 20.3$

33. Let QD = x cm. QC = AQ = AD – QD = $(16 - x)$ cm; CD = AB = 10 cm
In $\triangle$QCD, $QC^2 = QD^2 + CD^2$, $(16 - x)^2 = x^2 + 10^2$, $256 - 32x + x^2 = x^2 + 100$,
$156 = 32x$, $x = 4.875$, $\therefore$ QD = 4.875 cm

34. Join BQ and CQ. $BQ^2 = QP^2 + BP^2$, and $BQ^2 = AB^2 + 3^2$
$\therefore$ $AB^2 + 9^2 = QP^2 + BP^2$ .... (i)
$CQ^2 = QP^2 + PC^2 = QP^2 + BP^2$, and $CQ^2 = 5^2 + 4^2 = 41$
$\therefore$ $QP^2 + BP^2 = 41$ .... (ii)
From (i) and (ii), $AB^2 + 9 = 41$, $AB^2 = 32$, AB = 5.66



35. $DE^2 = EA^2 - AD^2$, and $DE^2 = EC^2 - DC^2$
$\therefore$ $EA^2 - AD^2 = EC^2 - DC^2$, $EA^2 - EC^2 = AD^2 - DC^2$ .... (i)

$BD^2 = AB^2 - AD^2$, and $BD^2 = BC^2 - DC^2$

$\therefore$ $AB^2 - AD^2 = BC^2 - DC^2$, $AB^2 - BC^2 = AD^2 - DC^2$ ……… (ii)

From (i) and (ii), $EA^2 - EC^2 = AB^2 - BC^2$

36. (a) $PR^2 + QR^2 = (m^2 - n^2)^2 + (2mn)^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 + 2m^2n^2 + n^4$.

$PR^2 = (m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4$

$\because$ $PR^2 + QR^2 = PQ^2$, $\therefore$ PQR is a right-angled triangle.

(b) Let $24 = 2mn$.

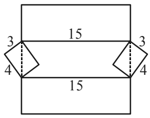
The table below shows the possible values of m, n, m^2-n^2 and m^2+n^2.

$m = 4, n = 3$	$m^2-n^2 = 4^2 - 3^2 = 7$	$m^2+n^2 = 4^2 + 3^2 = 25$
$m = 6, n = 2$	$m^2-n^2 = 6^2 - 2^2 = 32$	$m^2+n^2 = 6^2 + 2^2 = 40$
$m = 12, n = 1$	$m^2-n^2 = 12^2 - 1^2 = 143$	$m^2+n^2 = 12^2 + 1^2 = 145$

Ans. The sides may be: 24 cm, 7 cm, 25 cm; 24 cm, 32 cm, 40 cm; 24 cm, 143 cm, 145 cm.

37. The diagonal of the small rectangular board $= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ cm

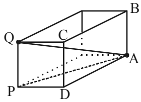
The new enclosed area $= 15 \times 5 - 2 \times (\frac{1}{2} \times 3 \times 4) = 75 - 12 = 63$ cm^2



38. (a) The shortest distance = AQ. $AP^2 = 10^2 + 20^2 = 500$

$AQ = \sqrt{QP^2 + AP^2} = \sqrt{10^2 + 500} = \sqrt{600} = 24.5$ cm

Ans. The shortest distance to fly from Q to A is 24.5 cm.

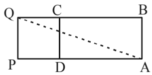


(b) Imagine cutting the box along QP and BA and so that QPDABCQ becomes a rectangle. The shortest distance from A to Q is the diagonal AQ in the rectangle.

AP = AD + DP = 20 + 10 = 30 cm,

$AQ = \sqrt{10^2 + 30^2} = \sqrt{1000} = 31.6$ cm

Ans. The shortest distance to crawl from A to Q is 31.6 cm.



Unit 14 Introduction to trigonometric ratios

1. (a) $\theta = 70.3^o$ (b) $\cos\theta = \frac{2}{3}, \theta = 48.2^o$ (c) $\sin\theta = \frac{4}{5}, \theta = 53.1^o$

2. (a) $\cos\theta = \frac{10}{12}, \theta = 33.6^o$ (b) $\sin x = \frac{21}{25}, x = 57.1^o$ (c) $\tan y = \frac{6}{17}, y = 19.4^o$

3. (a) $a = 13\tan 36^o = 9.45$ (b) $y = \frac{24}{\cos 55^o} = 41.8$ (c) $k = \frac{37}{\tan 61^o} = 20.5$

(d) $x = 8\sin 26^o = 3.51$

4. (a) $\tan\theta = \frac{14}{20}, \theta = 35.0^o$; $x = \sqrt{20^2 + 14^2} = \sqrt{596} = 24.4$

(b) $y = 12\cos 56^o = 6.71$cm; $\sin\theta = \frac{5}{y} = \frac{5}{6.71}$, $\theta = 48.2^o$

(c) In ΔABD, $x = \frac{11}{\sin 27^o} = 24.2$. In ΔABC, $\frac{y}{x} = \tan 27^o$,

$y = x\tan 27^o = 24.2\tan 27^o = 12.3$

(d) In ΔABD, BD $= 16\sin(30^o + 20^o) = 16\sin 50^o$, AD $= 16\cos 50^o$

In ΔACD, CD = AD $\tan 20^o = 16\cos 50^o \tan 20^o$

$x = BD - CD = 16\sin 50^o - 16\cos 50^o \tan 20^o = 8.51$

(e) $x = 8\tan 17^o = 2.45$; $\tan(17^o + \theta) = \dfrac{2x}{8} = \dfrac{2\times 8\tan 17^o}{8} = 0.61146$

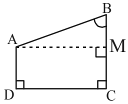
$\therefore\ 17^o + \theta = 31.4^o$, $\theta = 14.4^o$

(f) $a = 60\sin 33^o = 32.7$ cm; $c = \dfrac{a}{\cos 44^o} = \dfrac{60\sin 33^o}{\cos 44^o} = 45.4\,\text{cm}$

$b = a\tan 44^o = 60\sin 33^o \tan 44^o \approx 31.557 \approx 31.6\,\text{cm}$

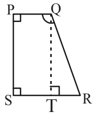
$\text{PR} = 60\cos 33^o, d = \text{PR} - b = 60\cos 33^o - 31.557 = 18.8\,\text{cm}$

5. (a) Draw $\text{AM}\perp\text{BC}$. In ΔABM, $a = \dfrac{\text{BM}}{\cos 50^o} = \dfrac{7-4}{\cos 50^o} = 4.67$



$b = \text{AM} = \text{BM}\tan 50^o = 3\tan 50^o = 3.58$

(b) Draw $\text{QT}\perp\text{SR}$. $\cos\angle\text{RQT} = \dfrac{\text{QT}}{18} = \dfrac{15}{18}$, $\angle\text{RQT} = 33.6^o$



$x = 90^o + \angle\text{RQT} = 90^o + 33.6^o = 123.6^o$

6. (a) $\cos 32^o = \dfrac{0.5y}{17}$, $y = \dfrac{17\cos 32^o}{0.5} = 28.8$

(b) $\sin(\dfrac{50^o}{2}) = \dfrac{0.5x}{8}$, $x = \dfrac{8\sin 25^o}{0.5} = 6.76$

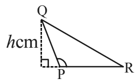
(c) $\sin\dfrac{\theta}{2} = \dfrac{9\div 2}{15} = \dfrac{3}{10}$, $\dfrac{\theta}{2} = 17.4576^o$, $\theta = 34.9^o$

7. (a) $\text{DE} = 8\sin 56^o$, $\text{CE} = 8\cos 56^o$,

$\text{area} = \dfrac{1}{2}(\text{DE})(\text{CE}) = \dfrac{1}{2}(8\sin 56^o)(8\cos 56^o) = 14.8\,\text{cm}^2$

(b) Let h cm be the height from Q to PR.

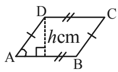
$h = 5\sin(180^o - 110^o) = 5\sin 70^o$



$\text{Area} = \dfrac{1}{2}(\text{PR})(\text{h}) = \dfrac{1}{2}(7)(5\sin 70^o) = 16.4\ \text{cm}^2$

(c) Let h cm be the height from D to AB. $h = 10\sin 60^o$

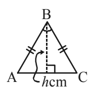
$\text{Area} = 12\cdot h = 12(10\sin 60^o) = 103.9\,\text{cm}^2$



(d) LM = LN = 7 cm, $\angle$ L = 60^o, the height $= 7\sin 60^o$

$\text{Area} = \dfrac{1}{2}(7)(7\sin 60^o) = 21.2\ \text{cm}^2$

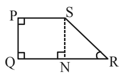
(e) Let h cm be the height from B to AC.



$h = 10\cos\dfrac{46^o}{2} = 10\cos 23^o$, $\dfrac{0.5\text{AC}}{10} = \sin\dfrac{46^o}{2}$, $\text{AC} = 20\sin 23^o$

$\text{Area} = \dfrac{1}{2}(\text{AC})(\text{h}) = \dfrac{1}{2}(20\sin 23^o)(10\cos 23^o) = 36.0\,\text{cm}^2$

(f) Draw $\text{SN}\perp\text{QR}$, SN = PQ = 3 cm, $\tan 58^o = \dfrac{\text{SN}}{\text{NR}}$,

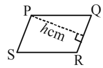


$\text{NR} = \dfrac{3}{\tan 58^o}$, $\therefore\ \text{QR} = 4 + \dfrac{3}{\tan 58^o} = 5.8746$

$\text{Area} = \dfrac{1}{2}(4 + 5.8746)(3) = 14.8\,\text{cm}^2$

8. $h = 3.5\tan 30^o$, $\text{area} = \dfrac{1}{2}(7)(3.5\tan 30^o) = 7.07\ \text{cm}^2$

9. Let h cm be the height from P to QR. $h = 26\sin\angle Q$,



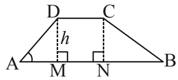
$\therefore\ 14\times 26\sin\angle Q = 160$, $\sin\angle Q = \dfrac{160}{14\times 26} = 0.43956$, $\angle Q = 26.1^o$

10. (a) $BD = 18\tan 43^o$, area of Δ ABC $= \dfrac{1}{2}(18+40)(18\tan 43^o) = 486.8$ sq. units

(b) $a = \dfrac{18}{\cos 43^o} = 24.6$; $b = \sqrt{BD^2 + 40^2} = \sqrt{(18\tan 43^o)^2 + 40^2} = 43.4$

$\tan\theta = \dfrac{40}{BD} = \dfrac{40}{18\tan 43^o} = 2.383$, $\therefore\ \theta = 67.2^o$

11. (a) Let h = DM = CN. $h = 8\sin 54^o = 13\sin\angle B$,

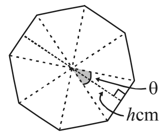


$\therefore\ \sin\angle B = \dfrac{8\sin 54^o}{13} = 0.49786$, $\angle B = 29.86^o$

(b) $AM = 8\cos 54^o$, $NB = 13\cos 29.86^o$, $\therefore$ $AB = 8\cos 54^o + 5 + 13\cos 29.86^o = 21.0\,\text{cm}$

(c) Area of ABCD $= \dfrac{1}{2}(5+20.976)(8\sin 54^o) = 84.1\,\text{cm}^2$

12. Let h cm be the height from the centre to a circle.



$\theta = \dfrac{360^o}{8} = 45^o$, $\dfrac{6\div 2}{h} = \tan\dfrac{45^o}{2}$, $h = \dfrac{3}{\tan 22.5^o}$

Area of the octagon = $8\times\dfrac{1}{2}(6)(\dfrac{3}{\tan 22.5^o}) = 173.8\,\text{cm}^2$

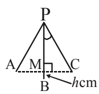
13. Let the length of the rope be x cm. $\cos 72^o = \dfrac{4-2.5}{x}$, $x = \dfrac{1.5}{\cos 72^o} = 4.85$

Ans. The rope is 4.85 m long.

14. $\sin\dfrac{\theta}{2} = \dfrac{8\div 2}{10} = \dfrac{4}{10}$, $\therefore$ $\dfrac{\theta}{2} = 23.578$, $\theta = 47.2^o$

15. Let $\angle POQ = \theta$. $\cos\dfrac{\theta}{2} = \dfrac{2.6}{3}$, $\therefore$ $\dfrac{\theta}{2} \approx 29.9264$, $\theta = 60^o$ (to the nearest degree)

16. Let h cm be the greatest height from its lowest position.



$\cos 28^o = \dfrac{PM}{PC} = \dfrac{24-h}{24}$, $h = 24 - 24\cos 28^o = 2.81\,\text{cm}$

17. The vertical distance he has risen $= 450\sin 11^o + 200\sin 34^o = 198\,\text{m}$

18. The distance between the two ends of the ropes $= \dfrac{19}{\tan 38^o} + \dfrac{19}{\tan 48^o} = 41.4\,\text{m}$

19. Let the distance the plane traveled be x m. $\sin 32^o = \dfrac{350}{x}$, $x = \dfrac{350}{\sin 32^o} = 660.48$

The time taken $= \dfrac{x}{200} = \dfrac{660.48}{200} = 3.3$ minutes

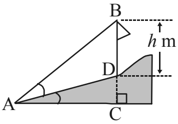
20. Let x m be the length of the post. $\dfrac{x}{3.8} = \tan 40^o$, $x = 3.8\tan 40^o$

Let y m be the length of the new shadow.

$\dfrac{x}{y} = \tan 28^o$, $y = \dfrac{x}{\tan 28^o} = \dfrac{3.8\tan 40^o}{\tan 28^o} = 6.00\,\text{m}$

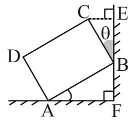
Ans. The new length of the shadow is 6.00 m.

21. Let the length of the pole be h m. $DC = 4\sin 15^o$, $AC = 4\cos 15^o$
In ΔABC, $BC = AC \tan (27^o + 15^o)$,
$h + 4\sin 15^o = 4\cos 15^o \tan 42^o$, $h = 2.44$
Ans. The vertical pole is 2.44 m long.



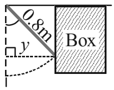
22. (a) $DE = \dfrac{8}{\tan 30^o}$, $\therefore$ area of $\Delta CDE = \dfrac{1}{2}(8)(\dfrac{8}{\tan 30^o}) = 55.4 \text{ cm}^2$
(b) $DC = \dfrac{8}{\sin 30^o} = 16$, $\therefore$ area of $ABCD = 16^2 = 256 \text{ cm}^2$
(c) $\angle ADF = 180^o - 30^o - 90^o = 60^o$, $AD = DC = 16$ cm,
The vertical distance from A to DE = $AD \sin \angle ADF = 16 \sin 60^o = 13.9$ cm

23. Draw $CE \perp EF$. In ΔABF, $BF = AB \sin 30^o = 30 \sin 30^o$
$\angle ABE = 30^o + 90^o$ (ext. $\angle$ of Δ), $\therefore$ $\theta + 90^o = 30^o + 90^o$, $\theta = 30^o$
In ΔCBE, $BE = BC \cos\theta = 20 \cos 30^o$
The vertical distance from C to the horizontal = BF + BE
$= 30\sin 30^o + 20\cos 30^o = 32.3$ cm



24. In ΔBEC, $\dfrac{6}{EC} = \tan\dfrac{50^o}{2}$, $EC = \dfrac{6}{\tan 25^o}$
The distance from the vertex to the bottom of the cylinder = $15 - \dfrac{6}{\tan 25^o} = 2.13$ cm

25. Let y m be the original distance between the box and the closed door. $\dfrac{y}{0.8} = \sin 50^o$, $y = 0.8\sin 50^o = 0.613$,
$x = 0.8 - y = 0.8 - 0.613 = 0.187$



26. (a) $PR = 3 + 4 = 7$, $PQ = \sqrt{12^2 - 7^2} = \sqrt{95}$. $\therefore$ $\cos\angle PQR = \dfrac{\sqrt{95}}{12}$, $\angle PQR = 35.7^o$
$\tan\angle PQS = \dfrac{3}{\sqrt{95}}$, $\angle PQS = 17.1^o$, $\therefore$ $n = 35.7^o - 17.1^o = 18.6^o$
(b) $QS = \sqrt{8^2 + 6^2} = 10$, $RQ = QS = 10$, $\therefore$ $PR = 10 + 8 = 18$
$\tan\angle R = \dfrac{6}{18}$, $\angle R = 18.4^o$, $\theta = \angle R = 18.4^o$ (base $\angle$s, isos.Δ)
(c) $EF = \dfrac{5}{\tan 35^o}$, $CE = \sqrt{13^2 - 5^2} = 12$, $\therefore$ $m = 12 - \dfrac{5}{\tan 35^o} = 4.86$
$\sin\angle DCE = \dfrac{5}{13}$, $\angle DCE = 22.6^o$; $\therefore$ $\theta = 35^o - 22.6^o = 12.4^o$ (ext.$\angle$of Δ)
(d) $\sin\theta = \dfrac{12}{13}$, $\theta = 67.4^o$, $\theta + \angle F + 90^o = 180^o$, $\therefore$ $\angle D = \theta = 67.4^o$
$\cos\angle D = \dfrac{7}{y+12}$, $\cos 67.4^o(y+12) = 7$, $y = \dfrac{7}{\cos 67.4^o} - 12 = 6.22$

27. (a) $3\theta = 64.158^o$, $\theta = 21.4^o$
(b) $\tan(\theta + 18^o) = 1.37374$, $\theta + 18^o = 53.9^o$, $\theta = 53.9^o - 18^o = 35.9^o$
(c) $\cos(2\theta - 15^o) = \dfrac{1}{5}$, $2\theta - 15^o = 78.463^o$, $\theta = \dfrac{78.463^o + 15^o}{2} \approx 46.7^o$

28. No. Because we don't know whether the triangle is right-angled.

29. (a) From the graph, $\sin 20^o = 0.35$, $\sin 38^o = 0.60$,
$\sin 64^o = 0.90$ (correct to the nearest 0.05)

(b) From the graph, $\cos 20^o = 0.95$, $\cos 38^o = 0.80$,
$\cos 64^o = 0.45$ (correct to the nearest 0.05)

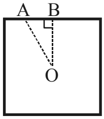
(c) When $\theta = 45^o$; $\cos\theta = \sin\theta$.
When θ increases, $\cos\theta$ decreases but $\sin\theta$ increases.
$\therefore$ If $\cos\theta > \sin\theta$, $\theta < 45^o$. (OR: $0^o < \theta < 45^o$)

30. $\angle B = 180^o - 45^o - 90^o = 45^o$. $\therefore$ ΔABC is isosceles, and CA = CB.
Let CA = CB = 1 unit, $\therefore$ $AB = \sqrt{1^2+1^2} = \sqrt{2}$ units, $\frac{AC}{AB} = \frac{1}{\sqrt{2}}$, $\therefore$ $\cos 45^o = \frac{1}{\sqrt{2}}$

31. PQ = RQ (equil. Δ), $\angle$ QLP = $\angle$ QLR = 90^o (given), QL = QL (common),
$\therefore$ ΔQLP $\cong$ ΔQLR (R.H.S.), $\therefore$ PL = LR (corr. sides, $\cong$ Δs)
Let PQ = 2 units, $PL = \frac{1}{2}PR = \frac{1}{2}(2) = 1$ unit, $\therefore$ $QL = \sqrt{2^2 - 1^2} = \sqrt{3}$,
$\angle P = 60^o$ (equil. Δ), $\angle PQL = 180^o - 90^o - 60^o = 30^o$,
$\tan 60^o = \frac{QL}{PL} = \frac{\sqrt{3}}{1} = \sqrt{3}$, $\sin 30^o = \frac{PL}{PQ} = \frac{1}{2}$

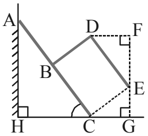
32. In ΔABC, $\sin\angle CBD = \frac{12}{20}$, $\angle CBD = 36.9^o$,
$\angle$DCB = $\angle$CBD (base $\angle$s, isos. Δ), $\therefore$ $\angle DCB = 36.9^o$

33. $\angle AOB = \frac{360^o}{12} = 30^o$, $OB = \frac{15}{2} = 7.5$ cm



The distance between the two marks = AB = OB $\tan 30^o$
$= 7.5\tan 30^o = 4.33$ cm

34. (a) BCED is a rectangle. $\angle ECG = 180^o - 90^o - 57^o = 33^o$,
CE = BD = 28 cm.
The vertical distance from E to the ground = EG = $28\sin 33^o = 15.2$ cm



(b) $\angle DEF + 90° = \angle DEG = 90° + \angle ECG$ (ext.$\angle$ of Δ), $\angle DEF = 33°$.
EF = 40 cos 33°
Vertical distance from D to the ground = FE + EG
= 40 cos 33° + 28 sin 33° = 48.8 cm

(c) DF = 40 sin 33°, HC = 50 cos 57°, CG = 28 cos 33°, AC = 50 + 40 = 90
Horizontal distance from D to the wall = HC + CG – DF
= 90 cos 57° + 28 cos 33° – 40 sin 33° = 50.7 cm

35. (a) PS = 16 cm, $\therefore$ $PB = \frac{16}{\sin 30^o}$. QP = 20 cm, $\angle APQ = 30^o$, $\therefore$ $AP = \frac{20}{\cos 30^o}$
The length of the rod = AP + PB = $\frac{20}{\cos 30^o} + \frac{16}{\sin 30^o} = 55.1$ cm

(b) $SB = \frac{PS}{\tan 30°} = \frac{16}{\tan 30^o}$. Let x cm be the new horizontal distance.
$\because$ the length of the rod = 55.1 cm, $\therefore$ $x = 55.1\cos 20^o$
The distance tip B slides $= x - (PS + SB) = 55.1\cos 20^o - (20 + \frac{16}{\tan 30^o}) = 4.06$ cm

36. Let r cm be the radius of the sphere.
In ΔDAE, DE = r, $\angle DAE = \frac{52^o}{2} = 26^o$, $\therefore$ $DA = \frac{r}{\sin 26^o}$
BD + DA = 8, $\therefore$ $r + \frac{r}{\sin 26^o} = 8$, $r = 8 \div (1 + \frac{1}{\sin 26^o}) = 2.44$

Ans. The radius of the sphere is 2.44 cm.

37. (a) In Δ PQS, PS = QS $\tan 24^o = (3+5)\tan 24^o = 8\tan 24^o$

Area of Δ PQT $= \frac{1}{2}$(QT)(PS) $= \frac{1}{2}(3)(8\tan 24^o) = 5.34$ cm^2

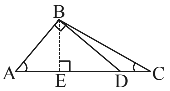
(b) In Δ TSR, SR $= \sqrt{13^2 - 5^2} = \sqrt{144} = 12$. $\tan\angle QRS = \frac{3+5}{12} = \frac{2}{3}$,

$\angle QRS = 33.69^o$. $\sin\angle TRS = \frac{5}{13}$, $\angle TRS = 22.62^o$

$\angle QRT = \angle QRS - \angle TRS = 33.69^o - 22.62^o \approx 11.1^o$

38. (a) In Δ ABD, BD $= 12\sin 48^o = 8.92$

(b) Draw BE $\perp$ AC. $\angle ADB = 180^o - 90^o - 48^o = 42^o$



In Δ BED, BE = BD $\sin 42^o = (12\sin 48^o)(\sin 42^o) = 5.967$

$\therefore$ EC = 12 sin 48° cos 42° + 4 = 10.627

$\tan\theta = \frac{BE}{EC} = \frac{5.967}{10.627} = 0.561494$, $\theta = 29.3^o$

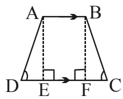
39. In Δ BAC,   AC $= x\cos\theta$. $\because$ $\angle BCD + \angle ADC = 90^o + 90^o = 180^o$,

$\therefore$ BC//AD (int. $\angle$ s supp.). $\angle CAD = \angle ACB = \theta$ (alt. $\angle$ s, BC//AD)

In Δ ACD, $\sin\angle CAD = \frac{CD}{AC}$, $\therefore$ $\sin\theta = \frac{CD}{x\cos\theta}$,   CD $= x\cos\theta\sin\theta$

40. $p^2 = q^2 + 12$, $\therefore$ $p^2 - q^2 = 12$.   Draw AE and BF $\perp$ DC. Let h = AE = BF.

DE = FC $= \frac{p-q}{2}$, AE = DE $\tan\theta = (\frac{p-q}{2})\tan\theta$



Area of the trapezium $= \frac{1}{2}(p+q)\cdot AE = \frac{1}{2}(p+q)\cdot(\frac{p-q}{2})\tan\theta$

$= \frac{1}{4}(p^2 - q^2)\tan\theta = \frac{1}{4}(12)\tan\theta = 3\tan\theta$.

41. (a) $\angle F = \frac{180^o \times 4}{6} = 120^o$,   AF = EF = 6 cm, $\frac{0.5AE}{AF} = \sin\frac{120^o}{2}$,

$\therefore$ AE = 2(6) sin60^o = 10.4 cm

(b) $\angle BAF = \angle F = 120^o$, $\angle EAF = \frac{180^o - 120^o}{2} = 30^o$ (base $\angle$ s, isos. Δ)

$\therefore$ $\angle BAE = \angle BAF - \angle EAF = 120^o - 30^o = 90^o$

$\therefore$ ABE is a right-angled triangle.

(c) Area of Δ ABE $= \frac{1}{2}$(AB)(AE) $= \frac{1}{2}(6)(10.4) = 31.2$ cm^2

42. (a) $\angle ABD = 180^o - 90^o - \theta = 90^o - \theta$, $C = 180^o - 90^o - \theta = 90^o - \theta$,

$\angle DBC = 90^o - \angle ABD = 90^o - (90^o - \theta) = \theta$, $\because$ $\angle A = \angle DBC$ $(=\theta)$,

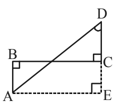
$\angle ABD = \angle DBC$ $(=90^o - \theta)$, $\angle ADB = \angle BDC$ $(=90^o)$, $\therefore$ $\Delta ABD \sim \Delta BCD$ (A.A.A.)

(b) $\frac{BD}{CD} = \frac{AD}{BD}$ (corr. sides, $\sim\Delta$ s), $\therefore$ $\frac{BD}{27} = \frac{48}{BD}$, BD $= \sqrt{48\times 27} = 36$

In Δ ABD, $\tan\theta = \frac{36}{48}$, $\theta = 36.9^o$

43. Produce DC to E, so that AE $\perp$ DE.   AE = BC = 56,
CE = AB = 15.   DE = DC + CE = 18 + 15 = 33.

In Δ ADE, $\tan\theta = \frac{AE}{DE} = \frac{56}{33}$, $\theta = 59.5^o$



44. Let y = AB = CD. $\therefore$ DE $= y\sin 70^o$, BE $= y\sin 40^o$, DE – BE = 4,

$\therefore\ y\sin 70^o - y\sin 40^o = 4$, $y = \dfrac{4}{\sin 70^o - \sin 40^o} = 13.47$.

AE $= y\cos 40^o$, CE $= y = \cos 70^o$

x = AE – CE = $y(\cos 40^o - \cos 70^o) = (13.47)(\cos 40^o - \cos 70^o) = 5.71$

45. Let PB = x. $\therefore$ AP = $2x$, AB = $x + 2x = 3x$

In ΔABC, AC = AB tan 45^o = $3x$ (1) = $3x$. $\tan\angle\text{APC} = \dfrac{\text{AC}}{\text{AP}} = \dfrac{3x}{2x} = \dfrac{3}{2}$, $\therefore$ ∠APC = 56.3°

∠CPQ + 56.3° = 45° + 90° (ext.∠ of Δ), $\therefore$ ∠CPQ = 78.7°

46. (a) Let x = PC = CB. $\tan 56^o = \dfrac{2x}{\text{AB}}$, $\therefore\ \dfrac{x}{\text{AB}} = \dfrac{\tan 56^o}{2}$, but $\tan\theta = \dfrac{x}{\text{AB}}$,

$\therefore\ \tan\theta = \dfrac{\tan 56^o}{2} = 0.74128$, $\theta \approx 36.549^o \approx 36.5^o$

(b) PC $= 10\sin\theta = 10\sin 36.549^o = 5.955$, $\dfrac{2\text{PC}}{\text{PA}} = \sin 56^o$, PA $= \dfrac{2(5.955)}{\sin 56^o} = 14.4$ cm

47. (a) Let a = AE = ED = DC. $\tan 30^o = \dfrac{\text{BC}}{3a}$, $\tan\angle\text{BDC} = \dfrac{\text{BC}}{a}$

$\therefore\ \tan 30^o = \dfrac{1}{3}(\tan\angle\text{BDC})$, $\tan\angle\text{BDC} = 3\tan 30^o = 1.7321$, ∠BDC = 60°

(b) $\angle\text{DBA} = 60^o - 30^o$ (ext.∠ of Δ), $\because\ \angle\text{BAD} = \angle\text{DBA} = 30^o$,

$\therefore$ DA = DB (sides opp. equal ∠ s)

48. (a) In Δ ABD, let ∠ B = x. ∠ BAD = ∠ B = x (base ∠ s, isos. Δ)

In Δ ADC, let ∠ C = y. ∠ CAD = ∠ C = y (base ∠ s, isos. Δ)

In Δ ABC, let ∠ B + ∠ BAD + ∠ CAD + ∠ C = 180^o (∠ sum of Δ)

$x + x + y + y = 180^o$, $x + y = 90^o$, $\angle\text{BAC} = x + y$, $\therefore$ ∠BAC = 90º

(b) $\sin\angle C = \dfrac{12}{10+10} = \dfrac{12}{20}$, $\therefore\ \angle C = 36.9^o$

49. (a) $\text{PQ}^2 + \text{QR}^2 = 10.5^2 + 10^2 = 210.25$, $\text{PR}^2 = 14.5^2 = 210.25$

$\because\ \text{PQ}^2 + \text{QR}^2 = \text{PR}^2$, $\therefore\ \angle\text{Q} = 90^o$ (converse of Pyth. Thm.)

i.e. PQR is a right-angled triangle.

(b) $\angle\text{Q} = 90^o$ (proved); $\cos\angle\text{P} = \dfrac{10.5}{14.5}$, $\angle\text{P} = 43.6^o$, $\angle\text{R} = 180^o - 90^o - 43.6^o = 46.4^o$

50. (a) $\text{AB}^2 + \text{BC}^2 = 13^2 + 15^2 = 394$, $\text{AC}^2 = 18^2 = 324$

$\because\ \text{AB}^2 + \text{BC}^2 \neq \text{AC}^2$, $\therefore$ ABC is not a right-angled triangle.

(b) $\text{BP}^2 = 13^2 - x^2$, and $\text{BP}^2 = 15^2 - \text{PC}^2 = 15^2 - (18-x)^2$

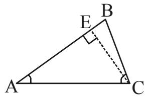
$\therefore\ 169 - x^2 = 225 - (18-x)^2$, $169 - x^2 = 225 - 324 + 36x - x^2$,

$36x = 169 + 324 - 225 = 268$, $x = \dfrac{268}{36} = 7.44$

$\cos\angle\text{A} = \dfrac{x}{13} = \dfrac{268}{36}\cdot\dfrac{1}{13} = \dfrac{67}{117}$, $\therefore\ \angle\text{A} = 55.1^o$

51. (a) $\tan 41^o = \dfrac{h}{\text{AD}}$, $\text{AD} = \dfrac{h}{\tan 41^o}$, $\tan 29^o = \dfrac{h}{\text{CD}}$, $\text{CD} = \dfrac{h}{\tan 29^o}$

(b) AD + CD = 30, $\dfrac{h}{\tan 41^o} + \dfrac{h}{\tan 29^o} = 30$, $h = 30 \div (\dfrac{1}{\tan 41^o} + \dfrac{1}{\tan 29^o}) = 10.2$ cm



52. Draw $CE \perp AB$. $\angle B = 180^o - 30^o - 70^o = 80^o$, $AE = x\cos 30^o$, $BE = y\cos 80^o$. $AB = AE + EB = x\cos 30^o + y\cos 80^o$

53. (a) $PR = \dfrac{QR}{\tan 25^o} = 2.1445QR$, $SR = \dfrac{QR}{\tan 50^o} = 0.8391QR$

$\therefore$ $PS = 2.1445\,QR - 0.8391\,QR = 1.3054\,QR$

$PS : SR = 1.3054QR : 0.8391QR = \dfrac{1.3054}{0.8391} : 1 = 1.556 : 1$, $\therefore$ $n = 1.556$

(b) Area of Δ PQS : Area of Δ QRS $= \frac{1}{2}(PS)(QR) : \frac{1}{2}(SR)(QR) = PS : SR$

$\therefore$ area of Δ PQS $= \dfrac{1.536}{1} \times 30 \approx 46.7\text{ cm}^2$

54. (a) $\because$ Half of the water remained,

$\therefore$ the cross-sectional area of the water is half of ABCD.

The water level is between to the horizontal, $\therefore$ $\angle BAC = x^o$ (alt. $\angle$ s, // lines)

$\tan\angle BAC = \dfrac{BC}{AB}$, $\tan x^o = \dfrac{50}{40}$, $x = 51.3$

(b) $\because$ one-third of the water remained,

$\therefore$ the cross-sectional area of the water $= \frac{1}{3}(40 \times 50) = \dfrac{2000}{3}$

$\therefore$ $\frac{1}{2}(BE)(40) = \dfrac{2000}{3}$, $BE = \dfrac{100}{3}$

The water level is parallel to the horizontal, $\therefore$ $\angle EAB = y^o$ (alt. $\angle$ s, // lines),

$\tan y° = \dfrac{BE}{AB} = \dfrac{\left(\frac{100}{3}\right)}{40} = \dfrac{5}{6}$, $\therefore$ $y = 39.8$

Unit 15 Circles

1. (a) Circumferences $= 2\pi(15) = 94.2$ cm, area $= \pi(15)^2 = 707.9\text{ cm}^2$

(b) Circumferences $= 2\pi(\frac{22}{2}) = 69.1\text{ cm}$, area $= \pi(\frac{22}{2})^2 = 380.1\text{ cm}^2$

2. (a) Radius $= \dfrac{100}{2\pi} = 15.9\text{ cm}$ (b) Radius $= \sqrt{\dfrac{84}{\pi}} = 5.17\text{ cm}$

3. (a) Perimeter $= 6 + 6 + 2\cdot\frac{1}{2}\cdot\pi(6) = 30.8\text{ cm}$. Area $= 6^2 - 2\cdot\frac{1}{2}\cdot\pi(\frac{6}{2})^2 = 7.73\text{ cm}^2$

(b) Perimeter $= 5 + \frac{1}{2}\cdot\pi(5+4) + \frac{1}{2}\cdot\pi(4) = 25.4\text{ cm}$

Area $= \frac{1}{2}\cdot\pi(\frac{5+4}{2})^2 - \frac{1}{2}\cdot\pi(\frac{4}{2})^2 = 25.5\text{ cm}^2$

(c) Perimeter $= \pi(4+4+4) + 2\cdot\frac{1}{2}\cdot\pi(4+4) + 2\cdot\frac{1}{2}\cdot\pi(4) = 75.4\text{ cm}$

Area $= \pi(\frac{4+4+4}{2})^2 - 2\cdot\frac{1}{2}\cdot\pi(\frac{4+4}{2})^2 - 2\cdot\frac{1}{2}\cdot\pi(\frac{4}{2})^2 = 50.3\text{ cm}^2$

(d) Perimeter $= \frac{1}{2}\cdot\pi(6+10) + \frac{1}{2}\cdot\pi(10) + \frac{1}{2}\cdot\pi(6) = 50.3\text{ cm}$

$$\text{Area} = \frac{1}{2}\cdot\pi(\frac{6+10}{2})^2 + \frac{1}{2}\cdot\pi(\frac{10}{2})^2 - \frac{1}{2}\cdot\pi(\frac{6}{2})^2 = 125.7\text{ cm}^2$$

4. (a) The distance travelled = π (24)(6) = 452 m

   (b) The no. of revolutions = $\frac{1200}{24\pi}$ = 16 (to the nearest integer)

5. (a) Radius = $95 \div 5 \div 2\pi = 3.02$ cm

   (b) $\frac{\text{no. of revolutions}}{5} = \frac{4.75\text{ m}}{95\text{ cm}}$, $\therefore$ no. of revolutions $= \frac{475}{95} \times 5 = 25$

6. Let n be the no. of revolutions of the rear wheel. $\pi(24)(36) = \pi(32)n$, $n = \frac{(24)(36)}{32} = 27$.

   Ans. The no. of revolutions of the rear wheel is 27.

7. Let the radius of the small circle be r cm.
   The difference = $2\pi(r+5) - 2\pi r = 2\pi(5) = 10\pi$ cm

8. Area of the path = $\pi(\frac{30}{2}+2)^2 - \pi(\frac{30}{2})^2 = 64\pi\text{ m}^2$

9. Diameter of the small circle = $16 \div 2 = 8$ cm. Area $= \pi(\frac{16}{2})^2 - \pi(\frac{8}{2})^2 = 48\pi\text{ cm}^2$

10. Radius $= \frac{40}{2\pi} = 6.366 = 6.37$ cm. Area = $\pi(6.366)^2 = 127.3\text{ cm}^2$

11. Radius $= \sqrt{\frac{98}{\pi}} = 5.59$ cm. Circumference = $2\pi(5.59) = 35.1$ cm

12. Radius $= \frac{6.28}{2\pi}$ m, $\therefore$ area $= \pi(\frac{6.28}{2\pi})^2 = \frac{6.28^2}{34\pi} = 3.14\text{ m}^2$

13. (a) Arc length $= 2\pi(6)(\frac{40}{360}) = 4.19$ cm. Area of sector = $\pi(6)^2(\frac{40}{360}) = 12.6\text{ cm}^2$

    (b) Angle $= \frac{70}{2\pi(14)} \times 360 = 286.5^\circ$. Area of sector = $\pi(14)^2(\frac{286.5}{360}) = 490\text{ cm}^2$

    (c) Angle $= \frac{100\pi}{\pi 5^2} \times 360 = 1440^\circ$. Arc length $= 2\pi(5)(\frac{1440}{360}) = 40\pi$ cm

    (d) Radius $= \frac{40}{2\pi} \div \frac{120}{360} = 19.1$ cm. Area of sector = $\pi(19.1)^2(\frac{120}{360}) = 382.0\text{ cm}^2$

    (e) Radius $= \sqrt{\frac{85}{\pi} \div \frac{200}{360}} = 6.98$ cm. Arc length $= 2\pi(6.98)(\frac{200}{360}) = 24.4$ cm

14. Angle = $\frac{\pi}{2\pi(5)} \times 360 = 36^\circ$

15. Angle = $38.5 \div [\frac{22}{7}(7)^2] \times 360 = 90^\circ$

16. Distance = $2\pi(7)(\frac{35}{60}) = 25.7$ cm

17. Area = $(15)(7.5) - 2\cdot\pi(7.5)^2(\frac{90}{360}) = 24.1\text{ cm}^2$

18. (a) Area = $(7)(7) - \pi(7)^2(\frac{90}{360}) = 10.5\text{ cm}^2$

    (b) Area = $(2+2)(2+2) - 4\cdot\pi(2)^2(\frac{90}{360}) = 3.43\text{ cm}^2$

    (c) Area = $\pi(5)^2(\frac{90}{360}) - \frac{1}{2}(5)(5) = 7.13\text{ cm}^2$

    (d) Area = $2 \times [\pi(3)^2(\frac{90}{360}) - \frac{1}{2}(3)(3)] = 5.14$ sq. units

19. (a) Perimeter = $2(\frac{22}{7})(\frac{14}{2})(\frac{270}{360})+2\cdot(4+\frac{6}{2}+6)=33+26=59$ units

Area = $[(\frac{22}{7})(\frac{14}{2})^2-(6)(6)](\frac{270}{360})=(154-36)\frac{3}{4}=88.5$ sq. units

(b) Perimeter = $2\times[2(\frac{22}{7})(\frac{28}{2})(\frac{270}{360})+2\cdot(\frac{28}{2})]=2(66+28)=188$ units

Area = $2\times(\frac{22}{7})(\frac{28}{2})^2(\frac{270}{360})=924$ sq. units

20. Shaded area $=\pi[(3+2)^2-3^2]\times\frac{60^o}{360^o}=8.38\,\text{cm}^2$

21. Perimeter $=2\pi(4)\times\frac{300^o}{360^o}+2\pi(10+4)\times\frac{360^o-300^o}{360^o}+10\times 2=55.6\,\text{cm}$

Area $=\pi(4)^2\times\frac{300^o}{360^o}+\pi(10+4)^2\times\frac{360^o-300^o}{360^o}=145\,\text{cm}^2$

22. $2\pi r\times\frac{120^o}{360^o}+2r=36.84$, $3.14r\times\frac{1}{3}+r=18.42$

$3.14r+3r=55.26$, $6.14r=55.26$, $\therefore\ r=9$

23. (a) Let r cm be the radius. $2\pi r\times\frac{1}{2}+2r=514$. $3.14r+2r=514$, $5.14r=514$,

$r=100$ *Ans. The radius of the semi-circle is 100cm.*

(b) Enclosed area $=\pi(100)^2\times\frac{1}{2}=3.14\times 5000=15700\,\text{cm}^2$

24. Shaded area $=\pi(\frac{20}{2})^2\times\frac{1}{4}-\frac{1}{2}\times\frac{20}{2}\times\frac{20}{2}=28.5\,\text{cm}^2$

25. The diagonal $=\sqrt{6^2+8^2}=10$ cm, $\therefore$ radius $=\frac{10}{2}=5$ cm

$\therefore$ Area of shaded region $=\pi(5)^2-6\times 8=30.5\,\text{cm}^2$

26. Area of AED $=\frac{1}{2}\times 6\times 6-\pi(6)^2\times\frac{45^o}{360^o}=3.86\,\text{cm}^2$. BD $=\sqrt{6^2+6^2}=\sqrt{72}$

$\therefore$ Perimeter of AED $=2\pi(6)\times\frac{45^o}{360^o}+(\sqrt{72}-6)+6=13.2\,\text{cm}$

27. (a) Perimeter $=2\pi(\frac{25}{2})+(150-25)\times 2=329\,\text{cm}$

Area $=\pi(\frac{25}{2})^2+(150-25)\times 25=3615.57=3620\,\text{m}^2$

(b) Area of the path $=\pi(\frac{25}{2}+1.5)^2+(150-25)\times(25+1.5\times 2)-3615.87$

$=\pi(14)^2+125\times 28-3615.87=500\,\text{m}^2$

28. $\angle$AOB $=180^o-30^o\times 2=120^o$, height of ΔOAB = 8 × sin 30° = 4cm

AB $=8\times\cos 30^o\times 2=13.8564$

$\therefore$ Area of shaded segment $=\pi(8)^2\times\frac{120^o}{360^o}-13.8564\times 4\times\frac{1}{2}=39.3\,\text{cm}^2$

29. AC $=12\cos 40^o$, BC $=12\sin 40^o$

$\therefore$ Area of shaded region $=\pi(\frac{12}{2})^2\times\frac{1}{2}-(12\cos 40^o)(12\sin 40^o)\times\frac{1}{2}=21.1\,\text{cm}^2$

30. Let r be the original radius, then new radius $= r(1-40\%) = 0.6r$

$\therefore$ Percentage decrease in area $= \dfrac{\pi r^2 - \pi(0.6r)^2}{\pi r^2} \times 100\% = (1-0.36) \times 100\%$
$= 64\%$

31. Let r_1 and r_2 be the radii of the bigger and the smaller circles respectively.

$2\pi r_1 - 2\pi r_2 = 20$, $2\pi(r_1 - r_2) = 20$, $r_1 - r_2 = \dfrac{10}{\pi}$

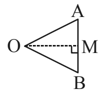
Ans. The difference between the radii is $\dfrac{10}{\pi}$ *m.*

32. Time saved $= [2\pi(\dfrac{34}{2}) \times \dfrac{1}{2} - 29] \div 72 \times 60 = 20.3$ min

33. Total length $= 2\pi(1) \times 3 - 2\pi(1) \times \dfrac{\angle P + \angle Q + \angle R}{360^o} = 6\pi - 2\pi \times \dfrac{180^o}{360^o} = 5\pi$ cm

34. $2\pi(4) \times \dfrac{\angle AOB}{360^o} = 2$, $\dfrac{\angle AOB}{360^o} = \dfrac{1}{4\pi}$, $\therefore$ Area of sector $= \pi(4)^2 \times \dfrac{\angle AOB}{360^o} = 16\pi \times \dfrac{1}{4\pi} = 4\text{ cm}^2$

35. (a) $2\pi(18) \times \dfrac{\angle AOB}{360^o} = 2\pi(5)$, $\dfrac{\angle AOB}{20^o} = 5$, $\therefore$ $\angle AOB = 100^o$



(b) $\angle AOM = 100^o \div 2 = 50^o$, $\therefore$ distance $= 18\sin 50^o \times 2 = 27.6$ cm

36. Let r cm be the radius. $\pi r^2 \times \dfrac{2}{2+5} = 77$, $\dfrac{22}{7} \times r^2 \times \dfrac{2}{7} = 77$,

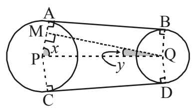
$r = 77 \times \dfrac{49}{44} = \dfrac{343}{4}$, $r = \sqrt{\dfrac{343}{4}} = 9.26$. *Ans. The radius of the circle is 9.26 cm.*

37. Let r cm be the radius, then BD = AE = $2r$ cm and AB = ED $= 2r(\dfrac{3}{2}) = 3r$ cm.

$3r + 3r + 2r + 2\pi r \times \dfrac{1}{2} = 3\pi + 24$, $8r + \pi r = 3\pi + 24$, $r(\pi + 8) = 3(\pi + 8)$, $r = 3$

$\therefore$ Area $= \pi(3)^2 \times \dfrac{1}{2} + (2 \times 3)(3 \times 3) = 4.5\pi + 54 = 68.1\text{ cm}^2$

38. QM $= \sqrt{60^2 - (16-12)^2} = \sqrt{3584}$. $\cos x = \dfrac{4}{60}$, $x = 86.2^o$

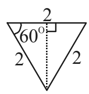


$\therefore$ Reflex $\angle$ APC $= 360^o - 2(86.2^o) = 187.6^o$. $y = 90^o - x = 3.8^o$

$\therefore$ $\angle BQD = 360^o - 2(90^o) - 2(3.8^o) = 172.4^o$

$\therefore$ Length of belt $= 2\pi(16) \times \dfrac{187.6^o}{360^o} + 2\pi(12) \times \dfrac{172.4^o}{360^o} + \sqrt{3584} \times 2 = 208$ cm

39. (a) Height of Δ ABC $= 2\sin 60^o$ cm

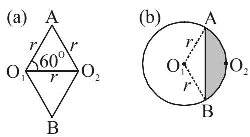


$\therefore$ Area of Δ ABC $= \dfrac{1}{2} \times 2 \times 2\sin 60^\circ = 1.732\text{ cm}^2$

(b) Shaded area $= \pi(2)^2 \times \dfrac{60^o}{360^o} \times 3 - 1.732 \times 2 = 2.82\text{ cm}^2$

40. (a) Let $AO_1 = AO_2 = O_1O_2 = r$ cm.

$r\sin 60^o \times 2 = \sqrt{24}$, $\therefore$ $r \approx 2.82843 \approx 2.83$



(b) $\angle AO_1B = 60^o \times 2 = 120^o$.
Area of segment AO_2B

$$=\pi(2.82843)^2\times\frac{120^o}{360^o}-\frac{1}{2}\times\sqrt{24}\times\frac{2.82843}{2}\quad=4.9135\quad\text{(5 sig. fig.)}$$

$\therefore$ Area of shaded region $=4.9135\times2=9.83\,\text{cm}^2$ (3 sig. fig.)

41. (a) $\because$ EA = AB = EB, $\therefore$ ΔABE is an equilateral triangle and $\angle$EAB = 60^o,

$\therefore$ $\angle$EAD = $90^o - 60^o = 30^o$

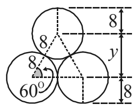
(b) $\overset{\frown}{DE}:\overset{\frown}{EB}=2\pi S\times\frac{30^\circ}{360^\circ}:2\pi S\times\frac{60^o}{360^o}=1:2$

42. Height of ΔADB $=20\sin45^o=20(\frac{\sqrt{2}}{2})=10\sqrt{2}$ cm,

$\therefore$ Area of shaded region $=\left[\pi(20)^2\times\frac{45^o}{360^o}-\frac{1}{2}\times20\times10\sqrt{2}\right]\times2=31.3\,\text{cm}^2$

43. $\because$ OC = OD = 10 cm and $\angle$COE = 45^o, $\therefore$ OE $=10\cos45^o$

$\therefore$ Area of OACE $=(10\cos45^\circ)^2=50\,\text{cm}^2$



44. $y=(8+8)\sin60^o=16\sin60^\circ$ cm

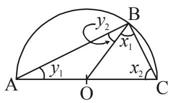
$\therefore$ Height of the stack $=8+16\sin60^\circ+8=29.9\,\text{cm}$

45. (a) $\because$ OA = OB = OC (radii),

$\therefore$ $x_1=x_2$ and $y_1=y_2$ (base $\angle$ s, isos. Δ)

$x_1+x_2+y_1+y_2=180^o$ ($\angle$ sum of Δ),

$2x_1+2y_1=180^o$, $x_1+y_1=90^o$, $\therefore$ $\angle$ABC $=90^o$



(b) AC $=\sqrt{m^2+n^2}$ (Pyth. Thm.), OC $=\frac{1}{2}\sqrt{m^2+n^2}$

$\therefore$ Area of semi-circle $=\pi(\frac{1}{2}\sqrt{m^2+n^2})^2\times\frac{1}{2}=\frac{\pi(m^2+n^2)}{8}$

46. OB = 10 cm. Let OA = y cm and AB = $2y$ cm.

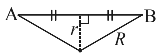
$y^2+(2y)^2=10^2$, $5y^2=100$, $y^2=20$, $y=\sqrt{20}$

$\therefore$ Area of the square $=(\sqrt{20}\times2)^2=80\,\text{cm}^2$

47. Let the radius of the smaller circle and the larger circle be r cm and R cm respectively.

Area of the ring = $\pi R^2-\pi r^2=\pi(R^2-r^2)$

But $R^2=r^2+(\frac{20}{2})^2$ (Pythagoras' Thm.), $\therefore$ $R^2-r^2=100$



$\therefore$ Area of the ring = π (100) = 100π cm²

48. AB $=\sqrt{3^2+3^2}=\sqrt{18}$.

$\therefore$ Area of the crescent = area of ΔOAB + area of semi-circle − area of sector OAB

$$=\frac{1}{2}\times3\times3+\frac{1}{2}\times\pi(\frac{3\sqrt{2}}{2})^2-\pi(3)^2\times\frac{90^o}{360^o}=4.5+2.25\pi-2.25\pi=4.5\,\text{cm}^2$$

49. $AB^2+BC^2=AC^2$ (Pyth. Thm.)

$\because$ Area of the shaded region $=\frac{1}{2}\pi(\frac{AB}{2})^2+\frac{1}{2}\pi(\frac{BC}{2})^2+25-\frac{1}{2}\pi(\frac{AC}{2})^2$

$$=\frac{1}{8}\pi(AB^2+BC^2)+25-\frac{1}{8}\pi AC^2=\frac{1}{8}\pi AC^2+25-\frac{1}{8}\pi AC^2=25\,\text{cm}^2$$

50. Let AB = CD = y. Arc length on AB $=2\pi(\frac{y}{2})\times\frac{1}{2}=\frac{\pi y}{2}$

Sum of arc lengths on CD $=2\pi(\frac{y}{2n})\times\frac{1}{2}\times n=\frac{\pi y}{2}$. *Ans. They are the same.*

Unit 16 Cylinders

1. Volume $= \pi(\frac{14}{2})^2(0.4) = 61.6\,\text{cm}^3$
2. Area $= 2\pi(5)(12) + \pi(5)^2 = 145\pi\ \text{cm}^2$
3. Largest possible volume $= \pi(\frac{12}{2})^2(12) = 432\pi\ \text{cm}^3$

   Total surface area $= 2\pi(\frac{12}{2})(12) + 2\pi(\frac{12}{2})^2 = 216\pi\ \text{cm}^2$
4. Let r cm be the base radius. $\pi r^2(10) = 471$, $r = \sqrt{\frac{471}{10\pi}} = 3.87$

   Ans. The base radius is 3.87 cm.
5. Let h cm be the height. $2\pi(\frac{14}{2})h = 1056$, $h = 1056 \times \frac{1}{14} \times \frac{7}{22} = 24$

   Ans. The height is 24 cm.
6. Let r cm be the base radius. $2\pi r(4) = 704$, $r = 704 \times \frac{1}{8} \times \frac{7}{22} = 28$

   Ans. The base radius is 28 cm.
7. Diameter = 60 cm = 0.6 m, $\therefore$ the area covered = 8 × π (0.6) (1.6) = 24.1 m^2
8. (a) Volume $= \pi(\frac{35}{2})^2(280) \times \frac{1}{2} = 134700\,\text{cm}^3$ = 134.7 L (4 sig. fig.)

   Ans. The trough can hold 134.7 L of water.

   (b) Total internal surface area $= \pi(35)(28) \times \frac{1}{2} + 2 \times \frac{1}{2} \times \pi(\frac{35}{2})^2 = 2501.5\ \text{cm}^2$
9. Total surface area $= 2\pi(\frac{18}{2})(27) \times \frac{1}{2} + \pi(\frac{18}{2})^2 + 6 \times 18 \times 2 + 6 \times 27 \times 2 + 18 \times 27 = 2040\ \text{cm}^2$.

   Volume $= \pi(\frac{18}{2})^2(27) \times \frac{1}{2} + 6 \times 18 \times 27 = 6350\,\text{cm}^3$
10. Total surface area $= 2\pi(9) \times \frac{300}{360} \times 27 + \pi(9)^2 \times \frac{300}{360} \times 2 + 9 \times 27 \times 2 = 2180\ \text{cm}^2$ (3 sig. fig.)
11. (a) External curved surface area $= 2\pi(12)(50) = 1200\pi\ \text{cm}^2$

    (b) Volume $= \pi(12^2 - 10^2)(50) = 2200\pi\ \text{cm}^3$
12. Let y cm be the length. $\pi[(\frac{4}{2})^2 - (\frac{1}{2})^2] \times y = 30 \times 10 \times 6, 3.75\pi \times y = 1800$,

    $y = \frac{480}{\pi}$ = 152.8. *Ans. The length of the pipe is152.8 cm.*
13. Internal radius = $\frac{9}{2} - 0.6 = 3.9$ cm, internal height = 10 – 0.6 = 9.4 cm

    $\therefore$ Internal surface area $= 2\pi(3.9)(9.4) + \pi(3.9)^2 = 278.1\,\text{cm}^2$
14. Let h cm be the depth of water. $\pi(8)^2(h) = 250$, $h = \frac{125}{32\pi}$

    $\therefore$ Required area $= 2\pi(8)(\frac{125}{32\pi}) + \pi(8)^2 = 263.6\,\text{cm}^2$
15. Let h cm be the depth of water. $\pi(\frac{8}{2})^2(h) = \pi(\frac{4}{2})^2(12)$, $h = \frac{4 \times 12}{16} = 3$

Ans. The depth of water is 3 cm.

16. Let h cm be the rise in water level. $\pi(4)^2(h) = (1.5)^3 \times 8$, $h = \dfrac{27}{16\pi}$

$\therefore$ New water level $= 6 + \dfrac{27}{16\pi} = 6.54$ cm

17. Let h cm be the rise in water level. $\pi(\dfrac{16}{2})^2(h) = \pi(\dfrac{8}{2})^2(0.5)\times 50$, $64h = 400$, $h = 6.25$

Ans. The rise in water level is 6.25 cm.

18. Volume of water collected $= \pi(\dfrac{8}{2})^2 \times 6 \times 12 = 1152\pi$ cm^3

19. (a) Total wet surface area $= 2\pi(\dfrac{10}{2})(8) + \pi(\dfrac{10}{2})^2 = 330$ cm^2

(b) Let h cm be the rise in water level. $\pi(\dfrac{10}{2})^2(h) = 2\times 3\times 6$, $h = \dfrac{36}{25\pi}$

$\therefore$ Original water level $= 8 - \dfrac{36}{25\pi} = 7.54$ cm

20. Height of equilateral triangle $= 4\sin 60^o = 3.4641$ cm. Let r cm be the base radius.

$\pi r^2(12) = \dfrac{4\times 3.4641}{2}\times 18$, $\pi r^2 = 10.3923$, $r = \sqrt{\dfrac{10.3923}{\pi}} = 1.82$

Ans. The base radius is 1.82 cm.

21. Let r cm and h cm be the base radius and the height of original cylinder.

$\because$ $2\pi rh = 100$,

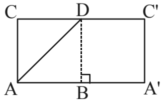
$\therefore$ new curved surface area $= 2\pi(2r)(2h) = 4(2\pi rh) = 4(100) = 400$ cm^2

22. Original surface area $= 2\pi(3r)(2r) + \pi(3r)^2 \times 2 = 30\pi r^2$

New surface area $= 2\pi(3r)(2r) + 2\pi(r)(2r) + \pi[(3r)^2 - r^2]\times 2 = 32\pi r^2$

$\therefore$ Percentage change $= \dfrac{32\pi r^2 - 30\pi r^2}{30\pi r^2}\times 100\% = 6.67\%$

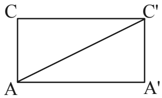
23. (a) The shortest possible length is the diagonal of the rectangle formed by flattening the semi-circular curved surface ABDC.



$\overset{\frown}{AB} = 2\pi(\dfrac{10}{2})\times\dfrac{1}{2} = 5\pi$ cm

$\therefore$ Shortest possible length $= \sqrt{(5\pi)^2 + 12^2} = 19.8$ cm (Pyth. Thm.)

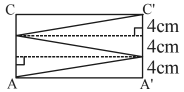
(b) The shortest possible length is the diagonal of the rectangle formed by flattening the curved surface of the cylinder.



Circumference $= 2\pi(\dfrac{10}{2}) = 10\pi$

$\therefore$ Shortest possible length $= \sqrt{(10\pi)^2 + 12^2} = 33.6$ cm (Pyth. Thm.)

(c) Shortest length $= 3\times\sqrt{(10\pi)^2 + (\dfrac{12}{3})^2} = 95.0$ cm (Pyth. Thm.)



24. Time taken $= (15\times 10\times 5)\div(\pi\times 0.05^2\times 10)\div 3600 = 2.65$ hours

25. Capacity of swimming pool $= \dfrac{(1.2+2.4)\times 50}{2}\times 18 = 1620$ m^3

$\therefore$ Time taken $=1620 \div [\pi \times (\dfrac{0.06}{2})^2 \times 15] \div 3600 = 10.6$ hours

26. When the base circumference is AB, volume $= \pi(\dfrac{AB}{2\pi})^2 \cdot BC = \dfrac{AB^2 \cdot BC}{4\pi}$

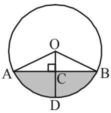
When the base circumference is BC, volume $= \pi(\dfrac{BC}{2\pi})^2 \cdot AB = \dfrac{BC^2 \cdot AB}{4\pi}$

Their ratio of volumes = $\dfrac{AB^2 \cdot BC}{4\pi} : \dfrac{BC^2 \cdot AB}{4\pi}$ = AB : BC = 3 : 2 [Or: 2:3]

27. Let the original water level be x cm. $\pi \cdot 9^2 \cdot x = \pi \cdot 9^2 \cdot 11 - \pi \cdot 4^2 \cdot 11$, $81x = 715$, $x = 8.83$
Ans. The water level was 8.83 cm.

28. Let the rise of water level be h cm.
$\pi (10^2)(14 + h) = \pi (10^2)(14) + \pi (4^2 - 7^2)(14 + h)$,
$1400 + 100h = 1400 + 33(14 + h)$, $100h = 462 + 33h$, $67h = 462$, $h = 6.90$
Ans. The minimum height of the cylindrical vessel = 6.90 + 14 = 20.9 cm

29. (a) OC = 30 – 12 = 18 cm, OA = radius = 30 cm,



$\therefore$ AC $= \sqrt{30^2 - 18^2}$ = 24 cm (Pyth. Thm.)

$\cos\angle AOD = \dfrac{OC}{OA} = \dfrac{18}{30}$, $\angle AOD = 53.13^o$

Area of segment ADB $= \pi(30^2) \cdot \dfrac{53.13 \times 2}{360} - \dfrac{1}{2}(24)(18) \times 2 \approx 402.564 \text{cm}^2$

Volume of water = $\dfrac{402.564 \times 200}{1000} \approx 80.5128 \text{ L} \approx 80.5 \text{ L}$

(b) When the depth is 30 cm, the water will be in the form of a semi-cylinder.

$\therefore$ Water added $= (\dfrac{\pi \cdot 30^2}{2} \cdot 200) \div 1000 - 80.5128$ L ≈ 202.2 L

Unit 17 Histograms & cumulative frequency polygons

1. (a) Lower class limit = \$400, upper class limit = \$490
(b) Lower class boundary = \$95, upper class boundary = \$195
(c) Class mark = $\dfrac{200+290}{2} = \$\,245$
(d) Class width = \$195 – \$95 = \$100
(e) 100 students
(f) The class with most students is \$300–\$390.
Percentage = $\dfrac{30}{100} \times 100\% = 30\%$

2. (a)

Weight (kg)	40 – 44	45 – 49	50 – 54	55 – 59	60 – 64
Tally	𝍸	𝍸 𝍸 𝍸 /	𝍸 𝍸 ////	𝍸 ////	////
Frequency	5	16	14	9	4

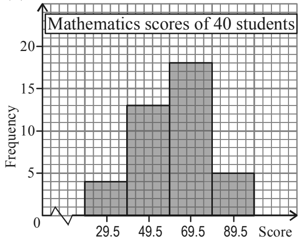
(b) Percentage of girls = $\dfrac{16+14}{48} \times 100\% = \dfrac{30}{48} \times 100\% = 62.5\%$

3. (a)

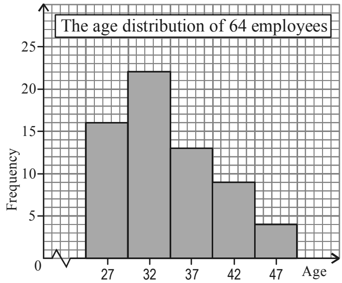
Score	Class boundaries	Class mark	Frequency
20 – 39	19.5 – 39.5	29.5	4
40 – 59	39.5 – 59.5	49.5	13
60 – 79	59.5 – 79.5	69.5	18
80 – 99	79.5 – 99.5	89.5	5

(b) Class width = 39.5 – 19.5 = 20

(c)

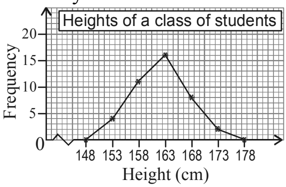


4. (a)



4. (b) None of the employees are younger than 24.5 years old or older than 49.5 years old. Besides, the ages are more concentrated in the first two classes, especially the class of 30–34 years old.

5. (a)



(b) Lower boundary = 168+2.5 = 170.5 cm, upper boundary = 173+2.5 = 175.5 cm

(c) Its lower class boundary = 150.5 cm, upper class boundary = 155.5 cm
$\therefore$ its lower class limit = 151 cm, upper class limit = 155 cm

(d) The class width = 158 – 153 = 5 cm

6. (a)

Pulse rate less than	Cumulative frequency
59.5	0
64.5	2
69.5	13
74.5	46
79.5	71
84.5	76
89.5	80

(c) From the graph, 30 people have pulse rates less than 72.

(b)

7. (a) $60\times\frac{1}{4}=15,\quad 60\times\frac{1}{2}=30,\quad 60\times\frac{3}{4}=45$

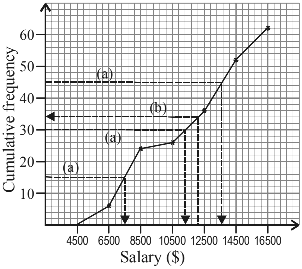
From the graph, the lower quartile
= 6500+1000= \$7500,
the median = 10500+800 = \$11,300,
the upper quartile ≈ 12500+1100 = \$13,600.

(b) From the graph, 34 workers have salaries less than \$12100.

∴ Percentage of experienced workers

$$=\frac{60-34}{60}\times100\%=\frac{26}{60}\times100\%=43.3\%$$

The monthly salaries of 60 workers



8. (a) There is space between the bars of a bar chart, but there is no space between the bars of a histogram. Besides, only one axis of a bar chart is a number line and has a scale, but both axes of a histogram are number lines and have a scale.

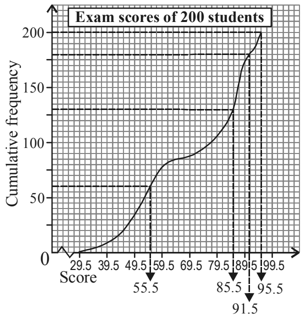
(b) Set A: It should be presented by a histogram. There are 100 data and have to be grouped into classes to show the pattern or characteristics of the set of data.

Set B: It should be presented by a bar chart. There are only 5 data, and they should be treated as discrete data (離散的、不連續的數據).

Set C: A bar chart is better than a histogram. Since there are only 12 data, the pattern or characteristic of the set of data can be observed easily in a bar chart. [A histogram can also be drawn, but it is neither meaningful nor necessary to group 12 data into several classes.]

9. No. of grade D students = 200 × 30% = 60
No. of grades D or C students
= 200 × (30%+35%) = 130
No. of grades D, C or B students
= 200 × (1–10%) = 180
From the graph, 60 students have scores less than 55.5, 130 students have scores less than 85.5, and 180 students have scores less than 91.5. And none of the students have scores less than 29.5 or above 95.5.

∴ Grade D: scores 30–55,
Grade C: scores 56–85,



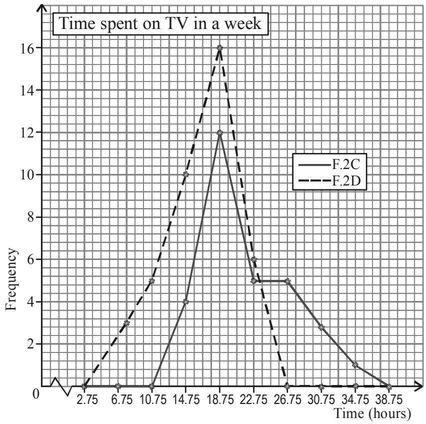
Grade B: scores 86–91, Grade A: scores 92–95

10.

Time spent on TV in a week (hours)	Class boundaries	Class mark	Frequency	
			F.2C	F.2D
5 – 8.5	4.75–8.75	6.75	0	3
9 – 12.5	8.75–12.75	10.75	0	5
13 – 16.5	12.75–16.75	14.75	4	10
17 – 20.5	16.75–20.75	18.75	12	16
21 – 24.5	20.75–24.75	22.75	5	6
25 – 28.5	24.75–28.75	26.75	5	0
29 – 32.5	28.75–32.75	30.75	3	0
33 – 36.5	32.75–36.75	34.75	1	0

(a) No. of students of F.2C
= 4 + 12 + 5 + 5 + 3 + 1 = 30
No. of students of F. 2D
= 3 + 5 + 10 + 16 + 6 = 40

(b)



(c) Both frequency polygons have the greatest frequency for the class mark of 18.75 hours.

(d) The frequency polygon of F.2D lies more to the left, while the frequency polygon of F.2C lies more to the right. This shows that on average students of F.2D spend less time on TV in a week than students of F.2C.

(e) The numbers of students are different in both classes, so we have to compare the percentages.

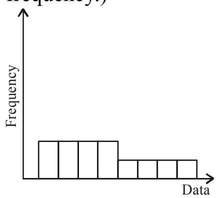
Percentage of F.2C students in this interval = $\dfrac{5}{30} \times 100\% = 16.7\%$

Percentage of F.2D students in this interval = $\dfrac{6}{40} \times 100\% = 15\%$

Ans. *F.2C has a greater proportion of students in the interval of 21 hours to 24.5 hours.*

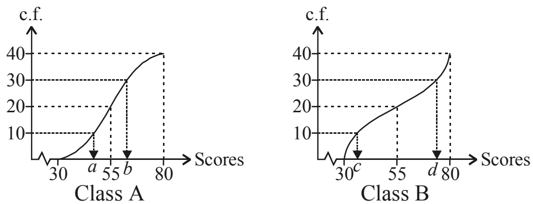
11. Figure 11C is its corresponding cumulative frequency polygon. The horizontal line segment in the middle matches the class of zero frequency in the given histogram. (Figure 11B is wrong because a cumulative frequency polygon must not go downward. Figure 11D is wrong because it does not have any horizontal line segment matching with the class of zero frequency.)

12.

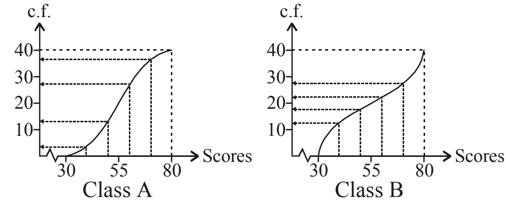


The cumulative frequency polygon is composed of 2 line segments. This shows that the first 4 classes are of equal frequencies and the last 4 classes are of equal frequencies. Besides, the total frequency of the first four classes is about twice of that of the last four classes, so the frequency of each of the first four classes should be twice of that of the last four classes.

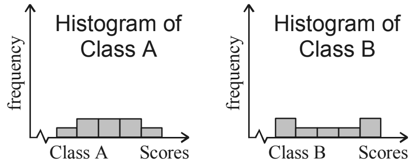
13. (a) In the figures below, a and b are the lower quartile and the upper quartile of Class A, while c and d are the lower quartile and the upper quartile of Class B. Since $c < a$, the average performance of the bottom 25% of Class B is poorer than that of Class A. Since $b < d$, the average performance of the top 25% of Class A is not so good as that of Class B. In sum, there are more students with low scores and high scores in Class B than in Class A.



(b) The figures on the right show that in both Class A and Class B, the frequencies of the three class intervals in the middle are of similar frequencies.



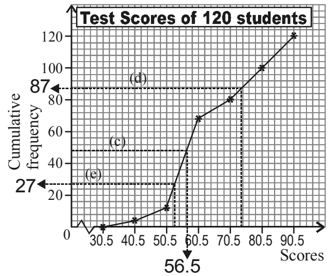
The histograms of the two classes are as follows:



14. (a)

Scores less than	Cumulative frequency
30.5	0
40.5	4
50.5	12
60.5	68
70.5	80
80.5	100
90.5	120

Score interval	Frequency
31 – 40	4
41 – 50	8
51 – 60	56
61 – 70	12
71 – 80	20
81 – 90	20



(b) The class with most students is of scores 51–60.

(c) No. of students failed the test = 120 × (1 – 60%) = 48
From the graph, the passing mark = 56.5.

(d) From the graph, 87 students have scores *less than* 74.

Percentage of students with scores *greater than* 74 = $\dfrac{120-87}{120}\times 100\% = 27.5\%$

(e) From the graph, 27 students have scores less than 52.

Percentage of students who need to take the re-test = $\dfrac{27}{120}\times 100\% = 22.5\%$

Appendix 1 Square roots and irrational numbers

1. (a) 5.20 (b) 3.48 (c) 63.2 (d) $=\sqrt{65}=8.06$
   (e) $=\sqrt{99}=9.95$ (f) $=\sqrt{1600}=40$

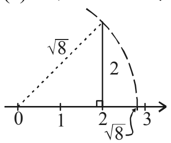
2. (a) $=\sqrt{12^2}=12$ (b) $=\sqrt{13^2}=13$ (c) $=-\sqrt{4\times 144}=-2\times 12=-24$
   (d) $\dfrac{5}{9}$ (e) $=-\sqrt{\dfrac{64}{49}}=-\dfrac{8}{7}$ (f) $=\sqrt{\dfrac{225}{10000}}=\sqrt{\dfrac{9}{400}}=\dfrac{3}{20}$

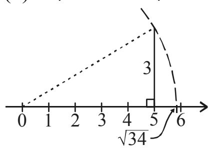
3. (a) $=1-8=-7$, $\therefore$ rational (b) $=\sqrt{3\times 81}=\sqrt{3}\times 9$, $\therefore$ irrational
   (c) recurring decimal, $\therefore$ rational

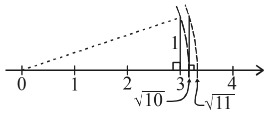
4. (a) $=\sqrt{4\times 10}=2\sqrt{10}$, $\therefore$ surd (b) $=\dfrac{16}{17}$, $\therefore$ not surd

(c) $= \dfrac{5}{\sqrt{3}}$, $\therefore$ surd (d) $= \sqrt{45} = 3\sqrt{5}$, $\therefore$ surd

5. (a) $= \sqrt{4 \times 2} = 2\sqrt{2}$ (b) $= \sqrt{9 \times 2} = 3\sqrt{2}$ (c) $= \sqrt{9 \times 3} = 3\sqrt{3}$
   (d) $= \sqrt{36 \times 2} = 6\sqrt{2}$ (e) $= \sqrt{9 \times 5} = 3\sqrt{5}$ (f) $= \sqrt{16 \times 3} = 4\sqrt{3}$
   (g) $= \sqrt{49 \times 2} = 7\sqrt{2}$ (h) $= \sqrt{25 \times 3} = 5\sqrt{3}$ (i) $= \sqrt{121 \times 3} = 11\sqrt{3}$
   (j) $= \sqrt{81 \times 3} = 9\sqrt{3}$ (k) $= \sqrt{4 \times 3} = 2\sqrt{3}$ (l) $= \sqrt{25 \times 5} = 5\sqrt{5}$
6. (a) $= 2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$ (b) $= 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$
   (c) $= 3\sqrt{2} - 4\sqrt{2} + 5\sqrt{2} = 4\sqrt{2}$ (d) $= 2\sqrt{3} + 5\sqrt{3} - 3\sqrt{2} = 7\sqrt{3} - 3\sqrt{2}$
   (e) $= 24 \times 2 = 48$ (f) $= 10 \times 3 \times \sqrt{3} = 30\sqrt{3}$
   (g) 6 (h) $= \sqrt{10} \times 2 = 2\sqrt{10}$ (i) $= 36 \times 2 = 72$ (j) $= 9 \times 5 = 45$
7. (a) $= \dfrac{2 \times 3}{\sqrt{2}} = 3\sqrt{2}$ (b) $= \dfrac{5 \times 3}{\sqrt{5}} = 3\sqrt{5}$ (c) $= \dfrac{4}{2\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$
   (d) $= \dfrac{24}{4\sqrt{3}} = \dfrac{2 \times 3}{\sqrt{3}} = 2\sqrt{3}$ (e) $\dfrac{4 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \dfrac{4\sqrt{3}}{3}$
   (f) $\dfrac{5 \times 2}{\sqrt{5} \times \sqrt{3}} = \dfrac{\sqrt{5} \times 2}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{15}}{3}$
8. (a) $\sqrt{2^2 + 2^2} = \sqrt{8}$ (b) $\sqrt{5^2 + 3^2} = \sqrt{34}$ (c) $\sqrt{1^2 + 3^2} = \sqrt{10}$
   $\sqrt{1^2 + (\sqrt{10})^2} = \sqrt{11}$







9. (a) Let $a = 0.444\ldots$; $10 \times a - a = (4.444\ldots) - (0.444\ldots) = 4$, $9a = 4$, $\therefore\ a = \dfrac{4}{9}$
   (b) Let $a = 1.6868\ldots$; $100 \times a - a = (168.6868\ldots) - (1.6868\ldots) = 167$, $99a = 167$,
   $\therefore\ a = \dfrac{167}{99}$
   (c) Let $a = 7.01717\ldots$; $100 \times a - a = (701.71717\ldots) - (7.01717\ldots) = 694.7$, $99a = 694.7$,
   $\therefore\ a = \dfrac{6947}{990}$
10. Just like 3.14, $\dfrac{22}{7}$ is only an approximate value of π. Although π is an irrational number, its approximate value can be a rational number. Therefore David was wrong.

Appendix 2 Percentages Interests, growth and decay

1. Simple interest $= \$8{,}800 \times 4\% \times 3.5 = \$1{,}232$
   Amount = \$(8,800 + \$1,232) = \$10,032
2. Let \$$P$ be the principal. $P \times 5\% \times \dfrac{9}{12} = 360$, $P = 9{,}600$.
   Ans. The principal is \$9,600.
3. Let n years be the time taken. $60000 \times 3\% \times n = 4050$, $n = 2.25$.
   Ans. It takes 2.25 years to earn \$4050 simple interest.

4. Let r% p.a. be the interest rate. $72{,}000 \times r\% \times \frac{1}{4} = 1{,}080$, $r\% = 0.06 = 6\%$.
   Ans. The interest rate is 6% p.a.
5. Let \$P be the principal. $P \times (5\% - 3\%) = 230$, $P \times 2\% = 230$, $P = 11{,}500$.
   Ans. His principal is \$11,500.
6. $\text{Amount} = \$65{,}000 \times [1 + 4\% \times 2 + 5\% \times (5.5 - 2)] = \$81{,}575$.
7. (a) $\text{Amount} = \$50{,}000(1 + \frac{6}{2}\%)^{2\times2} = \$56{,}275$
   (b) $\text{Amount} = \$50{,}000(1 + \frac{6}{4}\%)^{2\times4} = \$56{,}325$
   (c) $\text{Amount} = \$50{,}000(1 + \frac{6}{12}\%)^{2\times12} = \$56{,}358$
   (d) $\text{Amount} = \$50{,}000(1 + \frac{6}{365}\%)^{2\times365} = \$56{,}374$
8. (a) $\text{Amount} = \$64{,}000(1 + \frac{12}{12}\%)^{3\frac{3}{4}\times12} = \$64{,}000(1 + 1\%)^{45} = \$100{,}148$
   (b) $\text{Total interest} = \$100{,}148 - \$64{,}000 = \$36{,}148$
9. (a) $\text{Interest} = \$280{,}000\,[(1 + 10\%)^3 - 1] = \$92{,}680$
   (b) $\text{Interest} = \$280{,}000\,[(1 + \frac{10}{12}\%)^{3\times12} - 1] = \$97{,}491$
10. $\text{Compound interest} = \$55{,}000(1 + 5\%)^4 - \$55{,}000 = \$11{,}853$,
    $\text{Simple interest} = \$55{,}000 \times 5\% \times 4 = \$11{,}000$,
    $\therefore$ $\text{Difference} = \$11{,}853 - \$11{,}000 = \$853$
11. (a) $\text{Amount} = \$28{,}000(1 + \frac{4}{4}\%)^{1\times4}(1 + \frac{8}{4}\%)^{2.5\times4} = \$28{,}000(1.01)^4(1.02)^{10} = \$35{,}518$
    (b) $\text{Total interest} = \$35{,}518 - \$28{,}000 = \$7{,}518$
12. (a) Amount = $\$15{,}000\,(1 + 7\%)^5 + \$15{,}000\,(1 + 7\%)^4 + \$15{,}000\,(1 + 7\%)^3 +$
    $\$15{,}000\,(1 + 7\%)^2 + \$15{,}000\,(1 + 7\%)$
    $= \$15{,}000\,(1.07^5 + 1.07^4 + 1.07^3 + 1.07^2 + 1.07) = \$92{,}299$
    (b) $\text{Total interest} = \$92{,}299 - \$15{,}000 \times 5 = \$17{,}299$
13. (a) Let \$P be the principal. $P(1 + \frac{20}{2}\%)^{1.5\times2} = 114{,}466$, $P(1.1)^3 = 114{,}466$,
    $P = 86{,}000$. *Ans. The principal is \$86,000.*
    (b) $\text{Total interest} = \$114{,}466 - \$86{,}000 = \$28{,}466$
14. After 1st payment, amount he owes $= \$20{,}000(1 + 10\%) - \$5{,}000 = \$17{,}000$;
    After 2nd payment, amount he owes $= \$17{,}000(1 + 10\%) - \$5{,}000 = \$13{,}700$;
    After 3rd payment, amount he owes $= \$13{,}700(1 + 10\%) - \$5{,}000 = \$10{,}070$;
    $\therefore$ After 4th payment, amount he owes $= \$10{,}070(1 + 10\%) - \$5{,}000 = \$6{,}077$.
15. (a) $\text{Growth factor} = 1 + 15\% = 1.15$
    (b) $\text{Increase in members} = 3{,}000(1.15)^4 - 3{,}000 = 3{,}000[(1.15)^4 - 1] = 2{,}247$
16. $\text{Decay factor} = \frac{4{,}000}{4{,}500} = \frac{8}{9}$
17. $\text{Value} = \$7{,}500(1 - 9\%)^{3\times2} = \$7{,}500(0.91)^6 = \$4{,}259$
18. (a) $\text{Salary after 3 years} = \$24{,}200(1 + 10\%)^3 = \$32{,}210$
    (b) Let \$$y$ be the salary 2 years ago. $y(1 + 10\%)^2 = 24{,}200$, $y = 20{,}000$.

Ans. Her salary was $20,000 2 years ago.

19. (a) Volume after one day $= 3{,}000(1-6\%)^{24} = 680cm^3$

(b) Let y cm^3 be the volume 3 hours ago. $y(1-6\%)^3 = 3{,}000$, y=3,612.

Ans. The volume of the balloon was 3,612 cm^3 3 hours ago.

20. (a) Value after 1 year $= \$3{,}100(1+2\%)^3 \approx \$3289.7 \approx \$3{,}290$

(b) Value after 2 years $= \$3{,}289.7(1-2\%)^4 = \$3{,}034$

21. (a) At beginning of 2nd month, amount he owed $= \$35{,}000(1+\frac{18}{12}\%) - \$10{,}000 = \$25{,}525$

$\therefore$ At beginning of 3rd month, amount he owed $= \$25{,}525(1+1.5\%) - \$10{,}000 = \$15{,}908$

(b) At beginning of 4th month, amount he owed $= \$15{,}908(1+1.5\%) - \$10{,}000$

$= \$6{,}146$, at end of 4th month, amount he owed $= \$6{,}146(1+1.5\%)$

$= \$6{,}239 < \$10{,}000$. *Ans. 4 payments were needed.*

22. Amount owed after 1st installment $= \$8{,}000 - \$1{,}200 = \$6{,}800$,

Amount he owed after 2nd installment $= \$6{,}800(1+\frac{10}{12}\%) - \$900 = \$5{,}956.67$,

$\therefore$ Amount he owed after 3rd installment $= \$5{,}956.67(1+\frac{10}{12}\%) - \$900 = \$5{,}106$

23. Amount at end of 1st year $= \$20{,}000(1+\frac{5}{2}\%)^2$

Amount at end of 2nd year $= \$20{,}000(1+\frac{5}{2}\%)^4$

$\therefore$ Interest earned in 2nd year $= \$20{,}000\ [(1.025)^4 - (1.025)^2] = \$1{,}064$

24. Let x be the growth factor. $18x^2 = 36$, $x^2 = 2$, $x = \sqrt{2}$

$\therefore$ No. of bacteria in $\frac{1}{2}$ hour $= 18(\sqrt{2})^{30} = 590{,}000$ (corr. to nearest 1000)

25. Let $\$x$ be the amount of each payment. $[90{,}000(1+25\%) - x](1+25\%) - x = 0$,

$(112{,}500 - x)(1.25) - x = 0$, $140{,}625 - 2.25x = 0$, $x = 62{,}500$,

$\therefore$ Total interest $= \$62{,}500 \times 2 - \$90{,}000 = \$35{,}000$

26. Let $\$P$ and $r\%$ be the principal and the minimum interest rate respectively.

$P(1+r\%)^{10} \geq P(1+100\%)$, $(1+r\%)^{10} \geq 2$, $1 + r\% \geq 1.0718$, $r\% \geq 7.18\%$.

Ans. The minimum interest rate is 7.18%.

27. (a) Value of his flat $= \$4{,}000{,}000(1+10\%)^3 = \$5{,}324{,}000$

(b) Amount he owes Peter $= \$2{,}500{,}000(1+\frac{8}{12}\%)^{3\times12} = =\$3{,}175{,}593$

(c) Increase in value of the flat $= \$(5{,}324{,}000 - 4{,}000{,}000) = \$1{,}324{,}000$

Interest he has to pay to Peter $= \$(3{,}175{,}593 - 2{,}500{,}000) = \$675{,}593$

The profit $= \$(1{,}324{,}000 - 675{,}593 - 380{,}000) = \$268{,}407$

28. (a) His debt $= \$30{,}000(1+40\%)^3 = \$82{,}320$

(b) After 1st payment $= \$30{,}000(1+40\%) - \$15{,}000 = \$27{,}000$,

After 2nd payment $= \$27{,}000(1+40\%) - \$15{,}000 = \$22{,}800$,

$\therefore$ Amount owed after 3rd payment $= \$22{,}800(1+40\%) - \$15{,}000 = \$16{,}920$

(c) Amount he owes after 1month $= \$30{,}000(1+40\%) - \$10{,}000 = \$32{,}000$,

$\because$ The amount keeps increasing, $\therefore$ he can never clear his debt.

(d) Amount he owes after 1month $= \$30{,}000(1+40\%) - \$12{,}000 = \$30{,}000 =$ the principal,

$\therefore$ He will still owe the loan shark \$30,000 after 20 years.

Appendix 3 Rate and ratio

1. (a) $=\frac{1200}{25}=48$ (b) $=\frac{128}{32}=4$ (c) $=\frac{48}{30}=1.6$

2. 90 km/day = $\frac{90\times1000\text{ m}}{24\times60\text{ min}}$ = 62.5 m/min, $\therefore$ 70 m/min is the higher rate

3. Speed = $\frac{15}{6}$ = 2.5 m/s, $\therefore$ Time taken = $\frac{100}{2.5}$ = 40 s

4. (a) Time taken = $\frac{100}{80}$ = 1.25 h (b) Distance travelled = $80\times\frac{3}{4}$ = 60 km

5. (a) $=\frac{96}{24}:\frac{360}{24}=4:15$ (b) $=9.8\times10:4.2\times10=\frac{98}{14}:\frac{42}{14}=7:3$

   (c) $=\frac{3}{10}:\frac{32}{15}=\frac{3}{10}\times30:\frac{32}{15}\times30=9:64$

   (d) = $(3\times60\times60)$ seconds : 200 seconds = $\frac{10800}{200}:\frac{200}{200}=54:1$

   (e) = (2.5×100) ¢ : 15 ¢ = $\frac{250}{5}:\frac{15}{5}=50:3$

   (f) = 240 g : (1.2×1000) g = $\frac{240}{240}:\frac{1200}{240}=1:5$

   (g) = (3×1000) m : 800 m = $\frac{3000}{200}:\frac{800}{200}=15:4$

   (h) = $(0.4\times10000)\text{ cm}^2:500\text{ cm}^2=\frac{4000}{500}:\frac{500}{500}=8:1$

6. (a) $\frac{x}{3}=\frac{7}{2}$, $2x=21$, $\therefore\ x=10\frac{1}{2}$ (b) $\frac{6}{y}=\frac{5}{4}$, $24=5y$, $\therefore\ y=4\frac{4}{5}$

7. (a) $\frac{800g}{1000g}=\frac{\$a}{\$120}$, $\frac{4}{5}=\frac{a}{120}$, $480=5a$, $\therefore\ a=96$

   (b) $\frac{(7\times100000)\text{ cm}}{3\text{ cm}}=\frac{(2\times100000)\text{ cm}}{k\text{ cm}}$, $7k=6$, $\therefore\ k=\frac{6}{7}$

8. Let x be the original number, new number : original number

   $=x(1+\frac{1}{8}):x=\frac{9x}{8}:x=\frac{9}{8}\times8:1\times8=9:8$

9. Paul's weight : his brother's weight = 48 : (48 – 16) = $\frac{48}{16}:\frac{32}{16}=3:2$

10. (a) Speed of car = $33\div\frac{1}{3}$ = 99 km/h; speed of train = $\frac{270}{2}$ = 135 km/h

    (b) Speed of car : speed of train = 99 : 135 = $\frac{99}{9}:\frac{135}{9}=11:15$

11. Let x be the number of boys in the group, $\frac{x}{x+10}=\frac{4}{5}$, $5x=4x+40$, $x=40$.

    Ans. Number of boys in the group is 40.

12. Let \$$x$ be his monthly salary, $x(\frac{3}{5+3})=4800$, $3x=4800\times8$, $x=12800$.

    Ans. His monthly salary is \$12800.

13. Let x litres be the volume of water added, $\frac{6}{4+x}=\frac{2}{3}$, $18 = 2(4+x)$, $10 = 2x$, $x = 5$.

Ans. *5 litres of water must be added.*

14. New salary rate = $\$\frac{276}{10}$/h = \$ 27.6 / h,

$\therefore$ New salary : original salary = 27.6 : 24 = $\frac{276}{12}:\frac{240}{12}$ = 23 : 20

15. (a) $x : y = 5\times5 : 3\times5 = 25 : 15$, $y : z = 5\times3 : 2\times3 = 15 : 6$, $\therefore$ $x : y : z = 25 : 15 : 6$

(b) $y : x = 4 : 3$, $x : z = 1\times3 : 2\times3 = 3 : 6$, $\therefore$ $x : y : z = 3 : 4 : 6$

16. (a) $4a = 3b$, $\frac{a}{b}=\frac{3}{4}$, $\therefore$ $a : b = 3 : 4$ (b) $\frac{4}{5}=\frac{a}{b}$, $\therefore$ $a : b = 4 : 5$

(c) $7b = a$, $\frac{7}{1}=\frac{a}{b}$, $\therefore$ $a : b = 7 : 1$

(d) $4a + 2b = 21a - 7b$, $9b = 17a$, $\frac{9}{17}=\frac{a}{b}$, $\therefore$ $a : b = 9 : 17$

17. $x : 12 = 4 : y$, $\frac{x}{12}=\frac{4}{y}$, $\therefore$ $xy = 48$

18. A : B = 1 : 3, B : C = 1 : 2 = $1\times3 : 2\times3$ = 3 : 6, $\therefore$ A : B : C = 1 : 3 : 6

$\therefore$ The amount B gets = $500\times\frac{3}{1+3+6}$ = $500\times\frac{3}{10}$ = \$150

19. The scale = 4 cm : 2 km = 4 cm : 200000 cm = 1 : 50000

20. Map distance of a runway = $2.5\text{ km}\times\frac{1}{100000}$ = $\frac{250000\text{ cm}}{100000}$ = 2.5 cm

21. Actual distance = 3 mm$\times$2500000 = 7500000 mm = 7.5 km

22. Length of the hall = 5 cm$\times$400 = 2000 cm = 20 m

Width of the hall = 4 cm$\times$400 = 1600 cm = 16 m

$\therefore$ Actual area of the hall = $20\times16 = 320\text{ m}^2$

23. Distance travelled = $60\times\frac{90}{60}$ = 90 km,

$\therefore$ Volume of petrol used = $\frac{90}{6}$ = 15 litres

24. No. of \$1 coins = $240\times\frac{8}{8+3+4}$ = 128, no. of 50¢ coins = $240\times\frac{3}{8+3+4}$ = 48,

no. of 20¢ coints = $240\times\frac{4}{8+3+4}$ = 64

$\therefore$ Total amount = 128 × 1 + 48 × 0.5 + 64 × 0.2 = \$164.8

25. Average speed = $\frac{5}{1000}\text{km}\div\frac{3}{3600}\text{h}$ = $\frac{5}{1000}\times\frac{3600}{3}$ km / h = 6 km/h

26. Let $2k$ be the present age of Mary, then the present age of Lily is $3k$,

$\frac{2k+4}{3k+4}=\frac{5}{7}$, $14k + 28 = 15k + 20$, $k = 8$, $\therefore$ $2k = 16$, $3k = 24$.

Ans. *The present ages of Mary and Lily are 16 and 24 respectively.*

27. Let $3k$ be the length of the smaller part, then the length of the larger part is $7k$,

$\therefore$ Ratio of the three parts = $3k : 7k\times\frac{2}{2+3} : 7k\times\frac{3}{2+3}$ = $3k\times5 : \frac{14k}{5}\times5 : \frac{21k}{5}\times5$

= 15 : 14 : 21

28. $\frac{4}{a+1} \div \frac{3}{a} = \frac{7}{6}$, $\frac{4a}{3(a+1)} = \frac{7}{6}$, $24a = 21a + 21$, $3a = 21$, $\therefore\ a = 7$

29. $a : b = 3\times25 : 2\times25 = 75 : 50$, $a : c = \frac{5}{2}\times30 : \frac{2}{5}\times30 = 75 : 12$,
$\therefore\ a : b : c = 75 : 50 : 12$, $b : c = 50 : 12 = 25 : 6$; but $b : d = 6 : 7$,
$\therefore\ b : c : d = 25\times6 : 6\times6 : 7\times25 = 150 : 36 : 175$, $\therefore\ c : d = 36 : 175$

30. $2p = 3q$, $p : q = 3 : 2 = 3\times9 : 2\times9 = 27 : 18$,
$8q = 9r$, $q : r = 9 : 8 = 9\times2 : 8\times2 = 18 : 16$, $\therefore\ p : q : r = 27 : 18 : 16$

31. $\frac{1}{a} : \frac{1}{b} = 3 : 4$, $\frac{1}{a} \div \frac{1}{b} = \frac{3}{4}$, $\frac{b}{a} = \frac{3}{4}$, $\therefore\ a : b = 4 : 3$, $\frac{1}{b} : \frac{1}{c} = 4 : 5$, $\frac{1}{b} \div \frac{1}{c} = \frac{4}{5}$,
$\frac{c}{b} = \frac{4}{5}$, $\therefore\ b : c = 5 : 4$, $\therefore\ a : b : c = 4\times5 : 3\times5 : 4\times3 = 20 : 15 : 12$
(OR: $\frac{1}{a} = 3k$, $\frac{1}{b} = 4k$, $\frac{1}{c} = 5k$,
$\therefore\ a : b : c = \frac{1}{3k} : \frac{1}{4k} : \frac{1}{5k} = \frac{1}{3k}\times 60k : \frac{1}{4k}\times 60k : \frac{1}{5k}\times 60k = 20 : 15 : 12$)

32. Length scale of map = 1 mm : 0.25 m = 0.1 cm : 0.25 m = 1 cm : 2.5 m,
area scale of map $= (1\times1)\text{ cm}^2 : (2.5\times2.5)\text{ m}^2 = 1\text{ cm}^2 : 6.25\text{ m}^2$,
$\therefore$ Map area of the field = $400\times\frac{1}{6.25} = 64\text{ cm}^2$

33. Area scale of map $= (4\times4)\text{ cm}^2 : (1\times1)\text{ km}^2 = 16\text{ cm}^2 : 1\text{ km}^2$,
map area = $(2 \times 3)\text{ cm}^2 = 6\text{ cm}^2$, $\therefore$ Actual area = $6\times\frac{1}{16} = \frac{3}{8}\text{ km}^2$

34. $x = y(1+25\%)$, $\frac{x}{y} = 1.25 = \frac{5}{4}$, $\therefore\ x : y = 5 : 4$,
$x = z(1-20\%)$, $\frac{x}{z} = 0.8 = \frac{4}{5}$, $\therefore\ x : z = 4 : 5$, $\therefore\ x : y : z = 5\times4 : 4\times4 : 5\times5 = 20 : 16 : 25$

35. $2a + 3b = 3a + b$, $a = 2b$,
$\therefore\ \sqrt{4a+b} : \sqrt{3a-2b} = \sqrt{4(2b)+b} : \sqrt{3(2b)-2b} = \sqrt{9b} : \sqrt{4b} = 3 : 2$

36. Let $2a = 3b = 7c = k$, then $a = \frac{k}{2}$, $b = \frac{k}{3}$, $c = \frac{k}{7}$,
$\therefore\ (a-b+c) : (a+b-c) = (\frac{k}{2}-\frac{k}{3}+\frac{k}{7}) : (\frac{k}{2}+\frac{k}{3}-\frac{k}{7}) = \frac{13}{42} : \frac{29}{42} = 13 : 29$

37. Let $\frac{a}{b} = \frac{c}{d} = k$, then $a = bk$, $c = dk$, L.H.S. = $\frac{a+c}{b+d} = \frac{bk+dk}{b+d} = \frac{k(b+d)}{b+d} = k$,
R.H.S. = $\frac{a-c}{b-d} = \frac{bk-dk}{b-d} = \frac{k(b-d)}{b-d} = k$ = L.H.S., $\therefore\ \frac{a+c}{b+d} = \frac{a-c}{b-d}$

38. $\because$ The largest angle = $180^\circ\times\frac{8}{3+8+1} = 180^\circ\times\frac{8}{12} = 120^\circ > 90^\circ$,
$\therefore$ It is an obtuse-angled triangle.

39. Let $3k$ cm and $4k$ cm be the width and the length of the rectangle respectively,
$(3k)(4k) = 432$, $12k^2 = 432$, $k^2 = 36$, $k = 6$, $\therefore\ 3k = 18$, $4k = 24$.
Ans. Perimeter of the rectangle is 2(18+24) = 84 cm.

40. Let $r : 1$ be the ratio of the two types of tea,
$75(\frac{r}{1+r}) + 50(\frac{1}{1+r}) = 60$, $75r + 50 = 60(1+r)$, $15r = 10$, $r = \frac{2}{3}$.

Ans. The ratio of the mixture is $\frac{2}{3}:1 = 2:3$.

41. The cost of the mixture = 60 ÷ (1 + 25%) = \$48. Let the cost price of coffee B be \$$y$.

$15 \times \frac{2}{2+3} + y \times \frac{3}{2+3} = 48$, $30 + 3y = 240$, $y = 70$.

Ans. The cost price of coffee B is \$70/kg.

42. $150 \times \frac{m}{m+n} + 200 \times \frac{n}{m+n} = 150\,(1+10\%) \times \frac{m}{m+n} + 200\,(1-30\%) \times \frac{n}{m+n}$,

$150m + 200n = 165m + 140n$, $60n = 15m$, $\frac{m}{n} = \frac{60}{15} = \frac{4}{1}$, $\therefore\ m : n = 4 : 1$

43. Let \$$2k$ and \$$k$ be the daily wages of a man and a woman respectively,

$\therefore$ Ratio of hourly wages of a man and a woman $= \frac{2k}{10} : \frac{k}{8} = \frac{1}{5} \times 40 : \frac{1}{8} \times 40 = 8:5$

44. Let d m be the distance between home and school,

total time taken = $\frac{d}{x} + \frac{d}{y} = \frac{d(x+y)}{xy}$ seconds,

$\therefore$ His average speed = $2d \div \frac{d(x+y)}{xy} = 2d \times \frac{xy}{d(x+y)} = \frac{2xy}{x+y}$ m/s

45. Let x litres be the amount of water, amount of water 1 man needs per day = $\frac{x}{6\times 8} = \frac{x}{48}$ litres,

amount of water 1 boy needs per day = $\frac{x}{8\times 10} = \frac{x}{80}$ litres,

$\therefore$ Time lasting = $x \div (\frac{x}{48} \times 12 + \frac{x}{80} \times 4) = x \div \frac{3x}{10} = x \times \frac{10}{3x} = 3\frac{1}{3}$ days

46. Work done by A : work done by B : work done by C = $\frac{1}{10} : \frac{1}{20} : \frac{1}{30} = \frac{60}{10} : \frac{60}{20} : \frac{60}{30} = 6:3:2$

$\therefore$ The amount A will receive = $11000 \times \frac{6}{6+3+2} = \$6000$

47. If there are 60 chickens, the remaining food can last for 30 – 10 = 20 days,

$\therefore$ The number of days for 50 chickens = $\frac{60 \times 20}{50} = 24$.

Ans. The remaining food can last for 24 – 20 = 4 days more.

48. Let BQ = x, then QC = 36 – x, $\frac{(18+x)\times \text{AB}}{2} : \frac{[18+(36-x)]\times \text{AB}}{2} = 3:2$,

$\frac{18+x}{54-x} = \frac{3}{2}$, $36 + 2x = 162 - 3x$, $5x = 126$, $x = 25.2$. *Ans. The value of BQ is 25.2.*

49. Let the longer side and the shorter side of the original rectangle be b cm and a cm respectively.

The longer side of the new rectangle will be a cm, and the shorter side will be $\frac{b}{2}$ cm,

$a : \frac{b}{2} = b : a$, $\frac{2a}{b} = \frac{b}{a}$, $2a^2 = b^2$, $\frac{a^2}{b^2} = \frac{1}{2}$, $\frac{a}{b} = \frac{1}{\sqrt{2}}$.

Ans. The ratio of lengths of the original rectangle is $1:\sqrt{2}$. [OR: $\sqrt{2}:1$]

50. Let AB = x, BC = $2x$; let PQ = $3y$, QR = $2y$, $2(x+2x) = 2(3y+2y)$, $3x = 5y$,

$x = \frac{5y}{3}$, $\therefore$ Area of ABCD : area of PQRS = $x(2x):(3y)(2y) = x^2 : 3y^2$

$= (\frac{5y}{3})^2 : 3y^2 = \frac{25}{9} : 3 = \frac{25}{9} \times 9 : 3 \times 9 = 25 : 27$