ANSWERS

Unit 1 Laws of indices

1. (a) $= 4x^{8-2} = 4x^6$ (b) $= \frac{9\times 3}{6} y^{3+6-1} = \frac{9}{2} y^8$ (c) $= -3\times p^{2+2+2}\times q^{1+2} = -3p^6q^3$

(d) $= \frac{4k^2\times 2k^3}{-16k^5} = -\frac{1}{2}k^{2+3-5} = -\frac{1}{2}$ (e) $= \frac{(-5)^2a^4b^2c^2}{(-bc^2)(-5)^3a^9} = \frac{b^{2-1}c^{2-2}}{5a^{9-4}} = \frac{b}{5a^5}$

(f) $= \frac{(-2)(-15)}{-\frac{1}{5}} r^{2-1+3}s^{3-6+12} = -150r^4s^9$

(g) $= \frac{a^6b^8\times 36a^4b^6}{-12a^2b} = -3a^{6+4-2}b^{8+6-1} = -3a^8b^{13}$

(h) $= \frac{x^8y^4\times 9x^2y^4}{-81x^{10}y^2} = -\frac{x^{8+2-10}y^{4+4-2}}{9} = -\frac{y^6}{9}$

(i) $= (\frac{3^2b^2}{2a^2})^{-2}(\frac{3a^4}{2^2b^6})^3 = \frac{3^{-4}b^{-4}}{2^{-2}a^{-4}}\cdot\frac{3^3a^{12}}{2^6b^{18}} = \frac{3^{-1}a^{16}}{2^4b^{22}} = \frac{a^{16}}{48b^{22}}$

(j) $= \frac{-6m^2np^2}{(n^4p^2)(mn^2p)}\times\frac{(-3mn)(25n^4)}{10m^3np} = 45m^{2-1+1-3}n^{1-4-2+1+4-1}p^{2-2-1-1} = \frac{45}{mnp^2}$

2. (a) $= -(1)\times 8 = -8$ (b) $= (1)\times 81 = 81$ (c) $-\frac{1}{7}$

(d) $= -\frac{1}{6^2} = -\frac{1}{36}$ (e) $= (\frac{3}{5})^{-3} = \frac{125}{27}$ (f) $= 2-\frac{1}{8} = \frac{15}{8}$

(g) $= -1\times 13^{-5}\times 13^4 = -\frac{1}{13}$ (h) $= -16-\frac{1}{16}+4 = \frac{-12\times 16-1}{16} = \frac{-193}{16}$

(i) $= (\frac{4}{3})^{-3-(-2)-(-2)} = \frac{4}{3}$ (j) $= \frac{2^{2(11)}}{2^{23}}+\frac{1}{4} = \frac{1}{2}+\frac{1}{4} = \frac{3}{4}$

(k) $= [-(\frac{1}{2})^2+\frac{1}{3}]\div(\frac{1}{5})^2 = (\frac{-3}{12}+\frac{4}{12})\cdot\frac{25}{1} = \frac{25}{12}$

3. (a) $= \frac{2}{x}\div\frac{3x}{y^3} = \frac{2}{x}\times\frac{y^3}{3x} = \frac{2y^3}{3x^2}$ (b) $= \frac{a^2}{6}\times\frac{2}{b^3} = \frac{a^2}{3b^3}$

(c) $= a^4\times\frac{-3}{a^3}\times a = -3a^2$ (d) $= \frac{-36a^{2+5}b^{-2-6}}{6(1)} = \frac{-6a^7}{b^8}$

(e) $= \frac{x^{-9}y^{-6}}{2^3}\div\frac{1}{(-4x^4)^2} = \frac{1}{2^3x^9y^6}\times 2^4x^8 = \frac{2}{xy^6}$

(f) $= 4p^4q^{-6}\times p^{-3}q^{-2}\times 3p^{-1}q^2 = 12p^{4-3-1}q^{-6-2+2} = \frac{12}{q^6}$

(g) $= \frac{36a^{-3}b^2}{-8a^2b}\times(-24^3a^6b^{-12}) = \frac{24^3\times 36}{8}a^{-3-2+6}b^{2-1-12} = \frac{62208a}{b^{11}}$

(h) $= (\frac{8x^{-1}y^{-1}}{3})^{-2}\cdot\frac{2^3x^{-12}y^{12}}{(-15)x^3y^{-3}} = \frac{8^{-2}x^2y^2}{3^{-2}}\cdot\frac{2^3y^{15}}{-15x^{15}} = -\frac{9\cdot 8y^{17}}{8^2\cdot 15x^{13}} = -\frac{3y^{17}}{40x^{13}}$

4. (a) $10^{y-3} = 10^0$, $\therefore\ y-3 = 0, y = 3$ (b) $9^{4x-1} = 9^2$, $\therefore\ 4x-1 = 2, x = \frac{3}{4}$

(c) $4^{5y} = 4^{-3}$, $\therefore$ $5y = -3, y = -\frac{3}{5}$ (d) $5^{2n+1} = 5^{-3}$, $\therefore$ $2n+1 = -3, n = -2$

(e) $2^{-n} = 2^{5}$, $\therefore$ $-n = 5, n = -5$ (f) $y^{-2} = (\frac{6}{7})^{2} = (\frac{7}{6})^{-2}$, $\therefore$ $y = \frac{7}{6}$

(g) $x^{-3} = \frac{27}{8} = (\frac{3}{2})^{3} = (\frac{2}{3})^{-3}$, $\therefore$ $x = \frac{2}{3}$

(h) $3^{3(x+2)} = 3^{2(x+1)}$, $\therefore$ $3(x+2) = 2(x+1)$, $3x+6 = 2x+2$, $x = -4$

(i) $11^{x+3-2x} = 11^{0}$, $\therefore$ $x+3-2x = 0$, $x = 3$

(j) $5\times3^{2+x} = 5\times27$, $5\times3^{2+x} = 5\times3^{3}$, $\therefore$ $2+x = 3$, $x = 1$

(k) $5^{2}\times5^{2x-8} = 1$, $5^{2+2x-8} = 5^{0}$, $\therefore$ $2+2x-8 = 0$, $2x-6 = 0$, $x = 3$

5. (a) $= \frac{2^{3x}\times2^{2(x+1)}}{2^{5x}} = 2^{3x+2x+2-5x} = 2^{2} = 4$

(b) $= \frac{2^{n+1}\times3^{n}}{2^{n-1}\times3^{n-1}} = 2^{n+1-(n-1)}\times3^{n-(n-1)} = 2^{2}\times3^{1} = 12$

(c) $= \frac{5^{n}\times3^{n}}{5^{n+1}\times3^{n-2}} = 5^{n-(n+1)}\times3^{n-(n-2)} = 5^{-1}\times3^{2} = \frac{9}{5}$

(d) $= 7^{-2x}\times\frac{7^{x+1+x-1}}{7^{x(x-1)+x}} = 7^{-2x+2x-[x(x-1)+x]} = \frac{1}{7^{x^{2}}}$

6. (a) 3.48×10^{-5} (b) 2.50×10^{11} (c) -9.42×10^{8}

(d) $= [7.10\times10^{-1}\div(6.29\times10^{7})]^{3} = (\frac{7.10}{6.29})^{3}\times10^{(-1-7)\times3} = 1.44\times10^{-24}$

7. (a) $= \sqrt{49\times10^{-16}} = \sqrt{(7\times10^{-8})^{2}} = 7.00\times10^{-8}$

(b) $= (28\times10^{-11})\div(5\times10^{3}) = 5.60\times10^{-14}$

(c) $= 2.3\times10^{8} + 0.19\times10^{8} = (2.3+0.19)\times10^{8} = 2.49\times10^{8}$

(d) $= 0.4\times10^{-7} - 2\times10^{-7} = (0.4-2)\times10^{-7} = -1.60\times10^{-7}$

8. (a) $2^{k} = \frac{1}{8} = \frac{1}{2^{3}} = 2^{-3}$, $\therefore$ $k = -3$

(b) $x^{-3} = \frac{91}{125} + 1 = \frac{216}{125} = (\frac{6}{5})^{3} = (\frac{5}{6})^{-3}$, $\therefore$ $x = \frac{5}{6}$

(c) $(2^{-2})^{3x-1} = 2^{3(2x+4)}$, $2^{-6x+2} = 2^{6x+12}$, $\therefore$ $-6x+2 = 6x+12$, $-12x = 10$, $x = -\frac{5}{6}$

(d) $5^{n+1}\times5^{4n}\times5^{-9n} = 5^{-4}$, $5^{1-4n} = 5^{-4}$, $\therefore$ $1-4n = -4$, $n = \frac{5}{4}$

(e) $7^{2x}\times7^{x+3} = 7^{-1}$, $7^{3x+3} = 7^{-1}$, $\therefore$ $3x+3 = -1$, $x = -\frac{4}{3}$

(f) $3\times5^{6-x} = 75$, $3\times5^{6-x} = 3\times5^{2}$, $\therefore$ $6-x = 2$, $x = 4$

9. (a) $= 6^{n+1} - 6^{n} = 6^{n}(6-1) = 5\times6^{n}$ (b) $= 7^{n-2}(7^{3}-6) = 337\times7^{n-2}$

(c) $= 2^{3(n+1)} - 2^{3n-1} = 2^{3n+3} - 2^{3n-1} = 2^{3n-1}(2^{4}-1) = 15\times2^{3n-1}$

(d) $=\dfrac{2^{n-2}(1-2^4)}{2^{n-2}\times 2^3}=-\dfrac{15}{8}$ (e) $=\dfrac{2\times 3^{n-1}-4\times 3^{n-1}}{3^{n-1}\cdot 3^2-3^{n-1}\cdot 3}=\dfrac{3^{n-1}(2-4)}{3^{n-1}(3^2-3^1)}=-\dfrac{2}{6}=-\dfrac{1}{3}$

(f) $=\dfrac{3\times 2\times 2^{2(3n-2)}+2\times 2^{3(2n)}}{2^{2(3n)}+2^{3(2n-1)}}=\dfrac{3\times 2^{6n-3}+2^{6n+1}}{2^{6n}+2^{6n-3}}=\dfrac{2^{6n-3}(3+2^4)}{2^{6n-3}(2^3+1)}=\dfrac{19}{9}$

10. (a) $3^x(3-1)=2\times 81$, $3^x=3^4$, $\therefore\ x=4$

(b) $10\times 2^{n-1}+2^{n-1}=88$, $2^{n-1}(10+1)=88$, $2^{n-1}=2^3$, $\therefore\ n-1=3, n=4$

(c) $4^x+4^{x-1}=80$, $4^{x-1}(4+1)=80$, $4^{x-1}=4^2$, $\therefore\ x-1=2, x=3$

(d) $3^{n-1}(3^2-3+1)=7$, $3^{n-1}=1$, $3^{n-1}=3^0$, $\therefore\ n-1=0, n=1$

(e) $5^{2y+1}-5^{2y}=\dfrac{4}{5}$, $5^{2y}(5-1)=4\times 5^{-1}$, $4\times 5^{2y}=4\times 5^{-1}$, $\therefore\ 2y=-1$, $y=-\dfrac{1}{2}$

(f) $2^{2x}-2^{2x+3}+\dfrac{7}{4}=0$, $2^{2x}(2^3-1)=\dfrac{7}{4}$, $2^{2x}=2^{-2}$, $\therefore\ 2x=-2, x=-1$

11. (a) $=\dfrac{1}{2a^{-1}+b^{-1}}\times\dfrac{ab}{ab}=\dfrac{ab}{2b+a}$ (b) $=\dfrac{1}{2x^{-1}-3y^{-1}}\times\dfrac{xy}{xy}=\dfrac{xy}{2y-3x}$

(c) $=(\dfrac{1}{r}+\dfrac{5}{s})^{-2}=(\dfrac{s+5r}{rs})^{-2}=\dfrac{r^2s^2}{(s+5r)^2}$

(d) $=\dfrac{1}{(m-n)^2}\times(\dfrac{1}{m^{-2}-n^{-2}}\times\dfrac{m^2n^2}{m^2n^2})=\dfrac{1}{(m-n)^2}\times\dfrac{m^2n^2}{n^2-m^2}$

$=\dfrac{m^2n^2}{(n-m)^2(n-m)(n+m)}=\dfrac{m^2n^2}{(n-m)^3(n+m)}$

12. $(2^{x+3})^2=10^2$, $2^{x+3}=10$, $2^x\cdot 2^3=10$, $2^x=\dfrac{10}{8}=\dfrac{5}{4}$

13. $(3^{n+2})^2=36=6^2$, $3^{n+2}=6$, $3^{n+1}\times 3=6$, $\therefore\ 3^{n+1}=2$

14. $2^{2(y-2x)}=2^{-2}$, $(2^{y-2x})^2=(2^{-1})^2$, $\therefore\ y-2x=-1$, $\therefore\ y=2x-1\ldots$(i).

Sub (i) into $9^{x+y}-3^{4y-x}=0$, we have $9^{x+(2x-1)}-3^{4(2x-1)-x}=0$, $3^{2(3x-1)}=3^{7x-4}$,

$\therefore\ 6x-2=7x-4$, $x=2$.

Sub $x=2$ into (i), we have $y=2(2)-1=3$. $\therefore\ x=2$ and $y=3$

15. (a) $(x-x^{-1})^2=(\dfrac{9}{20})^2$, $x^2-2x(x^{-1})+x^{-2}=\dfrac{81}{400}$, $\therefore\ x^2+x^{-2}=\dfrac{81}{400}+2=2\dfrac{81}{400}$

(b) $(x+x^{-1})^2=x^2+2(x)(x^{-1})+x^{-2}=x^2+x^{-2}+2=2\dfrac{81}{400}+2=4\dfrac{81}{400}$,

$\therefore\ x+x^{-1}=\sqrt{4\dfrac{81}{400}}=\sqrt{\dfrac{1681}{400}}=\dfrac{41}{20}$, $\dfrac{x}{2}+\dfrac{x^{-1}}{2}=\dfrac{1}{2}\cdot\dfrac{41}{20}=\dfrac{41}{40}$

16. (a) 1cm = 10^{-2} m = $10^{-2}\times 10^9$ nanometer = 10^7 nanometer

$\therefore$ 126 cm = 126×10^7 nanometer = 1.26×10^9 nanometer

(b) 0.80km = 0.80×10^3 m = $0.80\times 10^3\times 10^9$ nanometer

= 8.0×10^{11} nanometer

17. (a) The lower limit = 3.95×10^5 m = 3.95×10^2 km = 395 km.

The upper limit = 4.05×10^5 m = 4.05×10^2 km = 405 km.

(b) The relative error = $\frac{0.05\times10^5}{4.00\times10^5}=\frac{1}{80}$,

the percentage error = $\frac{1}{80}\times 100\% = 1.25\%$

(c) 4.00×10^5 m means that the length is accurate to 3 significant figures. But we don't know the number of significant figures in 400000 m, and without knowing the degree of accuracy, we can't find the maximum percentage error.

18. Let the thickness of the oil layer be t mm. $(40\times10)^2\pi t=4$, $t=\frac{4}{(16\times10^4)\pi}=7.96\times10^{-6}$ mm.

Ans. Thickness of the oil layer is 7.96×10^{-6} *mm.*

19. (a) Speed $=300000000\div1000\times3600=1080000000=1.08\times10^9$ km/hr

(b) Distance $=(1.08\times10^9)\times24\times365\times100=9.46\times10^{14}$ km

20. The no. of years for the light from star A47 to reach the Earth $=\frac{1.32\times10^{15}}{9.46\times10^{12}}$

= 140 years (3 sig. fig.). 2018 – 140 = 1878 (the year); at that time the star A47 still existed, ∴ it is possible for us to see it in the year 2018.

Unit 2 Surds

1. $\sqrt{32}=\sqrt{4^2\times2}=4\sqrt{2},\sqrt{256}=\sqrt{4^4}=4^2,\sqrt{9000}=\sqrt{3^2\times10^2\times10}=30\sqrt{10},\ \frac{\sqrt{25}}{3}=\frac{5}{3}$,

∴ $\sqrt{32}$ and $\sqrt{9000}$ are surds.

2. (a) $5\sqrt{3}-8\sqrt{5}$ (b) $7\sqrt{7}-2\sqrt{11}$ (c) $3\sqrt{w}$ (d) $5\sqrt{y}-2\sqrt{x}$

3. (a) $=\sqrt{2^2\times15}=2\sqrt{15}$ (b) $=\sqrt{7^2\times3}=7\sqrt{3}$

(c) $=\sqrt{2^3\times7^2}=2\sqrt{2}\times7=14\sqrt{2}$ (d) $=\sqrt{3^4\times5}=3^2\times\sqrt{5}=9\sqrt{5}$

(e) $=\sqrt{2}\times\sqrt{5^2}\times\sqrt{m^2}\times\sqrt{n^4}=5mn^2\sqrt{2}$

(f) $=\sqrt{2\times3^2\times ab\times b^4}=3b^2\sqrt{2ab}$

4. (a) $=2\sqrt{3}+6\sqrt{3}-5\sqrt{3}=3\sqrt{3}$ (b) $=2\sqrt{7}-5\sqrt{7}-3\sqrt{7}=-6\sqrt{7}$

(c) $=8\sqrt{x}-7\sqrt{x}=\sqrt{x}$ (d) $=2m\sqrt{2mn}+3m\sqrt{2mn}=5m\sqrt{2mn}$

5. (a) $=(2\times\sqrt{2})\times(\sqrt{2}\times\sqrt{13})=4\sqrt{13}$ (b) $=(\sqrt{6}\times\sqrt{10})\times\sqrt{10}=10\sqrt{6}$

(c) $=(6\times\sqrt{3})\times(5\times\sqrt{3}\times\sqrt{2})=90\sqrt{2}$ (d) $=15\sqrt{6}$

(e) $=2\sqrt{33}\times\sqrt{33}=66$ (f) $=3\sqrt{17}\times2\sqrt{17}=6\times17=102$

(g) $=3\sqrt{7ab}\times(a\sqrt{7ab}\times\sqrt{2})=3a\times7ab\times\sqrt{2}=21a^2b\sqrt{2}$

(h) $=(\sqrt{3n}\times\sqrt{7mn})\times\sqrt{7mn}=7mn\sqrt{3n}$

6. (a) $=\dfrac{\sqrt{2}\times3\sqrt{7}}{3\times3\sqrt{7}}=\dfrac{\sqrt{2}}{3}$ (b) $=\dfrac{5\times\sqrt{3}}{2\times\sqrt{3}}=\dfrac{5}{2}$

(c) $=\dfrac{2\sqrt{2}\times\sqrt{14}}{3\times\sqrt{14}}=\dfrac{2\sqrt{2}}{3}$ (d) $=\dfrac{(4\times\sqrt{3}\times\sqrt{5})\times(2\times3\times\sqrt{3})}{4\times3\times\sqrt{5}}=2\times3=6$

7. (a) $=2-4\sqrt{2}-5=-3-4\sqrt{2}$ (b) $=30-17\sqrt{5}+12=42-17\sqrt{5}$

(c) $=(7-3\sqrt{3})(\sqrt{3}-2)=7\sqrt{3}-14-9+6\sqrt{3}=13\sqrt{3}-23$

(d) $=\sqrt{14}\cdot\sqrt{7}-\sqrt{14}\cdot\sqrt{2}+5\sqrt{7}-5\sqrt{2}=7\sqrt{2}-2\sqrt{7}+5\sqrt{7}-5\sqrt{2}=2\sqrt{2}+3\sqrt{7}$

(e) $=5y-6+7\sqrt{y}$ (f) $=3x-4\sqrt{xy}+6\sqrt{xy}-8y=3x-8y+2\sqrt{xy}$

8. (a) $=7-2\sqrt{35}+5=12-2\sqrt{35}$ (b) $=18+12\sqrt{2}+4=22+12\sqrt{2}$

(c) $=48+8\sqrt{15}+5=53+8\sqrt{15}$ (d) $=24-2\sqrt{24}\sqrt{12}+12=36-24\sqrt{2}$

(e) $=25x+4-20\sqrt{x}$ (f) $=9x+49y+42\sqrt{xy}$

9. (a) $=5-4=1$ (b) $=9-20=-11$ (c) $x-4y^2$

(d) $=(4\sqrt{a}-3\sqrt{b})(4\sqrt{a}+3\sqrt{b})=16a-9b$

10. (a) $=\dfrac{2\sqrt{7}\times\sqrt{7}}{\sqrt{7}}=2\sqrt{7}$ (b) $=\dfrac{8\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}=\dfrac{8\sqrt{5}}{5}$ (c) $=\dfrac{4\sqrt{3}\times\sqrt{3}}{\sqrt{3}}=4\sqrt{3}$

(d) $=\dfrac{6\sqrt{7}\times\sqrt{7}}{2\sqrt{7}}=3\sqrt{7}$ (e) $=\dfrac{\sqrt{4x}\times\sqrt{a}}{\sqrt{a}\times\sqrt{a}}=\dfrac{2\sqrt{ax}}{a}$ (f) $=\dfrac{2m\times\sqrt{n}}{3\sqrt{n}\times\sqrt{n}}=\dfrac{2m\sqrt{n}}{3n}$

11. (a) $\sqrt{12}=2\sqrt{3}\approx2\times1.732=3.464$ (b) $\dfrac{1}{\sqrt{3}}=\dfrac{\sqrt{3}}{3}\approx\dfrac{1.732}{3}=0.577$ (3 sig. fig.)

12. (a) $=\dfrac{1}{\sqrt{2}+\sqrt{5}}\times\dfrac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}}=\dfrac{\sqrt{2}-\sqrt{5}}{2-5}=\dfrac{\sqrt{5}-\sqrt{2}}{3}$

(b) $=\dfrac{3}{3-\sqrt{5}}\times\dfrac{3+\sqrt{5}}{3+\sqrt{5}}=\dfrac{3(3+\sqrt{5})}{9-5}=\dfrac{3(3+\sqrt{5})}{4}$

(c) $=\dfrac{8}{\sqrt{7}+2\sqrt{3}}\times\dfrac{\sqrt{7}-2\sqrt{3}}{\sqrt{7}-2\sqrt{3}}=\dfrac{8(\sqrt{7}-2\sqrt{3})}{7-12}=\dfrac{8(2\sqrt{3}-\sqrt{7})}{5}$

(d) $=\dfrac{4}{3\sqrt{5}-\sqrt{3}}\times\dfrac{3\sqrt{5}+\sqrt{3}}{3\sqrt{5}+\sqrt{3}}=\dfrac{4(3\sqrt{5}+\sqrt{3})}{45-3}=\dfrac{4(3\sqrt{5}+\sqrt{3})}{42}=\dfrac{2(3\sqrt{5}+\sqrt{3})}{21}$

13. (a) $=\dfrac{\sqrt{5}}{\sqrt{5}+2}\times\dfrac{\sqrt{5}-2}{\sqrt{5}-2}=\dfrac{\sqrt{5}(\sqrt{5}-2)}{5-4}=5-2\sqrt{5}$

(b) $=\dfrac{\sqrt{14}}{\sqrt{7}-\sqrt{2}}\times\dfrac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}=\dfrac{\sqrt{14}(\sqrt{7}+\sqrt{2})}{7-2}=\dfrac{7\sqrt{2}+2\sqrt{7}}{5}$

(c) $=\dfrac{2\sqrt{3}}{\sqrt{27}+\sqrt{7}}\times\dfrac{\sqrt{27}-\sqrt{7}}{\sqrt{27}-\sqrt{7}}=\dfrac{2\sqrt{3}(3\sqrt{3}-\sqrt{7})}{27-7}=\dfrac{18-2\sqrt{21}}{20}=\dfrac{9-\sqrt{21}}{10}$

(d) $=\frac{3+4\sqrt{5}}{3(1-\sqrt{5})}\times\frac{1+\sqrt{5}}{1+\sqrt{5}}=\frac{(3+4\sqrt{5})(1+\sqrt{5})}{3(1-5)}=\frac{3+7\sqrt{5}+20}{-12}=-\frac{23+7\sqrt{5}}{12}$

(e) $=\frac{x}{y+\sqrt{x}}\times\frac{y-\sqrt{x}}{y-\sqrt{x}}=\frac{x(y-\sqrt{x})}{y^2-x}$

(f) $=\frac{\sqrt{a}+\sqrt{b}}{\sqrt{b}-\sqrt{a}}\times\frac{\sqrt{b}+\sqrt{a}}{\sqrt{b}+\sqrt{a}}=\frac{(\sqrt{b}+\sqrt{a})^2}{b-a}=\frac{b+a+2\sqrt{ab}}{b-a}$

14. $=\frac{1}{4}(10)\sqrt{3}+\frac{\sqrt{3}}{2}-2\sqrt{3}+\frac{\sqrt{3}}{3}=\sqrt{3}(\frac{5}{2}+\frac{1}{2}-2+\frac{1}{3})=\frac{4\sqrt{3}}{3}$

15. $x=\sqrt{3}, y=\sqrt{27}=\sqrt{3^3}=(\sqrt{3})^3$, $\therefore\ y=x^3$; but $y=\sqrt{27}=\sqrt{3^3}=3\sqrt{3}$. $\therefore\ y=3x$.

Ans. Both of them are correct.

16. (a) $\sqrt{1.8}=\sqrt{\frac{18}{10}}=\sqrt{\frac{3^2}{5}}=\frac{3}{\sqrt{5}}=\frac{3\sqrt{5}}{5}=\frac{3\times 2.236}{5}=1.3416$

(b) $\sqrt{12}-\sqrt{8}=2\sqrt{3}-2\sqrt{2}=2(\sqrt{3}-\sqrt{2})=2\ (1.732-1.414)=0.636$

17. (a) $=(10)-1=9$ (b) $9=(\sqrt{10}-1)(\sqrt{10}+1)$, $\therefore\ 3=\frac{(\sqrt{10}-1)(\sqrt{10}+1)}{3}, \frac{3}{\sqrt{10}+1}=\frac{\sqrt{10}-1}{3}$

(c) L.H.S. $=\frac{3}{\sqrt{10}+1}\cdot\frac{\sqrt{10}-1}{\sqrt{10}-1}=\frac{3(\sqrt{10}-1)}{10-1}=\frac{3(\sqrt{10}-1)}{9}=\frac{\sqrt{10}-1}{3}=$ R.H.S.

18. $=\frac{(\sqrt{a})^2}{\sqrt{a}\sqrt{a+1}}-\frac{(\sqrt{a+1})^2}{\sqrt{a}\sqrt{a+1}}-\frac{1}{\sqrt{a}\sqrt{a+1}}=\frac{a-a-1-1}{\sqrt{a}\sqrt{a+1}}=\frac{-2}{\sqrt{a(a+1)}}$

19. $=[\frac{\sqrt{x}(\sqrt{x}-\sqrt{y})+(\sqrt{y}(\sqrt{x}+\sqrt{y})}{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}]^{-1}=(\frac{x-\sqrt{xy}+\sqrt{xy}+y}{x-y})^{-1}=(\frac{x+y}{x-y})^{-1}=\frac{x-y}{x+y}$

20. $m=\sqrt{7}-\sqrt{6}, \frac{1}{m}=\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\sqrt{7}+\sqrt{6}$

$\therefore\ m-\frac{1}{m}=(\sqrt{7}-\sqrt{6})-(\sqrt{7}+\sqrt{6})=-2\sqrt{6}$

21. $x-1=\frac{6}{\sqrt{2}+1}=\frac{6(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}=\frac{6\sqrt{2}-6}{2-1}, x-1=6\sqrt{2}-6, x=6\sqrt{2}-5$

22. $(\sqrt{3}+1)\sqrt{x}=2, \sqrt{x}=\frac{2}{\sqrt{3}+1}=\frac{2(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{2(\sqrt{3}-1)}{3-1}=\sqrt{3}-1$

$\therefore\ x = (\sqrt{3}-1)^2 = 3-2\sqrt{3}+1 = 4-2\sqrt{3}$. $\therefore\ a = 4, b = -2$.

23. $x(\sqrt{3}-2) < 1$, $\because \sqrt{3} < \sqrt{4}$, $\therefore\ \sqrt{3}-2 < 0$

$\therefore\ x > \frac{1}{\sqrt{3}-2},\ x > \frac{\sqrt{3}+2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})},\ x > \frac{\sqrt{3}+2}{3-4},\quad x > -(\sqrt{3}+2)$

24. $(x-\frac{1}{x})^2 = (3\sqrt{5}-2)^2, x^2-2+\frac{1}{x^2} = 45-12\sqrt{5}+4.\quad \therefore\ x^2+\frac{1}{x^2} = 51-12\sqrt{5}$

25. $AB = \sqrt{27}cm = 3\sqrt{3}cm, BC = \sqrt{75}cm = 5\sqrt{3}cm, AC = 12cm = 2\sqrt{3}cm$

$AB + AC = 3\sqrt{3}+2\sqrt{3} = 5\sqrt{3} = BC$. $\therefore$ A, B, C form a straight line (line segment).

26. $\sqrt{32}+\sqrt{18} = 4\sqrt{2}+3\sqrt{2} = 7\sqrt{2}$, but $\sqrt{147} = \sqrt{7\times7\times3} = 7\sqrt{3}$, $\therefore \sqrt{32}+\sqrt{18} < \sqrt{147}$

Ans. Since the sum of 2 sides is smaller than the third side, they cannot form a triangle.

27. (a) $PQ^2 = 93$, $PR^2+QR^2 = (3\sqrt{5})^2+(4\sqrt{3})^2 = 45+48 = 93$

$\because\ PR^2+QR^2 = PQ^2$, $\therefore$ $\triangle$PQR is a right-angled triangle.

(b) $\because$ PR$\perp$QR, $\therefore$ area $= \frac{1}{2}(3\sqrt{5})(4\sqrt{3}) = 6\sqrt{15}cm^2$

28. $(2\sqrt[3]{3})^3 = (\sqrt{x})^3$, $2^3\cdot3 = \sqrt{x^3}$, $\therefore\ 2^3\cdot3 = \sqrt{y}$, $\sqrt{y} = 24, y = 24^2 = 576$

29. $\frac{1}{\sqrt{1}+\sqrt{2}} = \frac{\sqrt{2}-\sqrt{1}}{(\sqrt{2}+\sqrt{1})(\sqrt{2}-\sqrt{1})} = \frac{\sqrt{2}-\sqrt{1}}{2-1} = \sqrt{2}-\sqrt{1}$,

$\frac{1}{\sqrt{2}+\sqrt{3}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \sqrt{3}-\sqrt{2}$,

$\frac{1}{\sqrt{3}+\sqrt{4}} = \frac{\sqrt{4}-\sqrt{3}}{(\sqrt{4}+\sqrt{3})(\sqrt{4}-\sqrt{3})} = \sqrt{4}-\sqrt{3}$, ……….

$\frac{1}{\sqrt{99}+\sqrt{100}} = \frac{\sqrt{100}-\sqrt{99}}{(\sqrt{100}+\sqrt{99})(\sqrt{100}-\sqrt{99})} = \sqrt{100}-\sqrt{99}$

$\therefore$ The given expression $= (\sqrt{2}-\sqrt{1})+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\ldots..+(\sqrt{100}-\sqrt{99})$

$= -\sqrt{1}+\sqrt{100} = 10-1 = 9$

30. (a) $\frac{m}{n} = (\sqrt{5}-2)\div\frac{1}{\sqrt{5}+2} = (\sqrt{5}-2)(\sqrt{5}+2) = 5-4 = 1$

(b) $\dfrac{m^{2005}}{n^{2003}}=(\dfrac{m}{n})^{2003}\cdot m^2=\ (1)^{2003}\cdot(\sqrt{5}-2)^2\ =\ 5-4\sqrt{5}+4=\ 9-4\sqrt{5}$

31. $=\left[\ \left(\sqrt{3}-\sqrt{2}+1\right)\ \left(\sqrt{3}+\sqrt{2}-1\right)\ \right]^5=\left\{\ \left[\sqrt{3}-\left(\sqrt{2}-1\right)\ \right]\ \left[\sqrt{3}+\left(\sqrt{2}-1\right)\ \right]\ \right\}^5$

$=\left[(\sqrt{3})^2-\left(\sqrt{2}-1\right)^2\right]^5=\left[\ 3-\left(2-2\sqrt{2}+1\right)\ \right]^5=\left(2\sqrt{2}\right)^5$

$=2^5\cdot\left(\sqrt{2}\right)^5=32\cdot\left(\sqrt{2}\right)^4\sqrt{2}=32\cdot 2^2\cdot\sqrt{2}=128\sqrt{2}$

32. Let $x=0.\dot{4}=0.4444...,\ \therefore\ 10x=4.444\ldots\ \therefore\ 10x-x=(4.444\ldots)-(0.444\ldots)=4$

$9x=4,\ \therefore\ x=\dfrac{4}{9},\quad \sqrt{0.\dot{4}}=\sqrt{\dfrac{4}{9}}=\dfrac{2}{3}\quad \therefore\ \sqrt{\sqrt{0.\dot{4}}}\ =\ \sqrt{\dfrac{2}{3}}=\dfrac{\sqrt{2}}{\sqrt{3}}=\dfrac{a}{b}$

Unit 3 Percentages – Interests, growth and decay

1. Simple interest $=\$8{,}800\times 4\%\times 3.5=\$1{,}232$. Amount = \$(8,800 + \$1,232) = \$10,032
2. Let $\$P$ be the principal. $P\times 5\%\times\dfrac{9}{12}=360$, $P=9{,}600$. *Ans. The principal is \$9,600.*
3. Let n years be the time taken. $60000\times 3\%\times n=4050$, $n=2.25$.
   Ans. It takes 2.25 years to earn \$4050 simple interest.
4. Let r% p.a. be the interest rate. $72{,}000\times r\%\times\dfrac{1}{4}=1{,}080$, $r\%=0.06=6\%$.
   Ans. The interest rate is 6% p.a.
5. Let \$P be the principal. $P\times(5\%-3\%)=230$, $P\times 2\%=230$, $P=11{,}500$.
   Ans. His principal is \$11,500.
6. Amount $=\$65{,}000\times[1+4\%\times 2+5\%\times(5.5-2)]=\$81{,}575$.
7. (a) Amount $=\$50{,}000(1+\dfrac{6}{2}\%)^{2\times 2}=\$56{,}275$

   (b) Amount $=\$50{,}000(1+\dfrac{6}{4}\%)^{2\times 4}=\$56{,}325$

   (c) Amount $=\$50{,}000(1+\dfrac{6}{12}\%)^{2\times 12}=\$56{,}358$

   (d) Amount $=\$50{,}000(1+\dfrac{6}{365}\%)^{2\times 365}=\$56{,}374$
8. (a) Amount $=\$64{,}000(1+\dfrac{12}{12}\%)^{3\frac{3}{4}\times 12}=\$64{,}000(1+1\%)^{45}=\$100{,}148$

   (b) Total interest $=\$100{,}148-\$64{,}000=\$36{,}148$
9. (a) Interest $=\$280{,}000[(1+10\%)^3-1]=\$92{,}680$

(b) Interest $= \$280{,}000[(1+\frac{10}{12}\%)^{3\times 12}-1] = \$97{,}491$

10. Compound interest $= \$55{,}000(1+5\%)^4 - \$55{,}000 = \$11{,}853$,
Simple interest $= \$55{,}000 \times 5\% \times 4 = \$11{,}000$, $\therefore$ Difference $= \$11{,}853 - \$11{,}000 = \$853$

11. (a) Amount $= \$28{,}000(1+\frac{4}{4}\%)^{1\times 4}(1+\frac{8}{4}\%)^{2.5\times 4} = \$28{,}000(1.01)^4(1.02)^{10} = \$35{,}518$
(b) Total interest $= \$35{,}518 - \$28{,}000 = \$7{,}518$

12. (a) Amount $= \$15{,}000\ (1 + 7\%)^5 + \$15{,}000\ (1 + 7\%)^4 + \$15{,}000\ (1 + 7\%)^3 +$
$\$15{,}000\ (1 + 7\%)^2 + \$15{,}000\ (1 + 7\%)$
$= \$15{,}000\ (1.07^5 + 1.07^4 + 1.07^3 + 1.07^2 + 1.07) = \$92{,}299$
(b) Total interest $= \$92{,}299 - \$15{,}000 \times 5 = \$17{,}299$

13. (a) Let \$P be the principal. $P(1+\frac{20}{2}\%)^{1.5\times 2} = 114{,}466$, $P(1.1)^3 = 114{,}466$,
$P = 86{,}000$. *Ans. The principal is \$86,000.*
(b) Total interest $= \$114{,}466 - \$86{,}000 = \$28{,}466$

14. After 1st payment, amount he owes $= \$20{,}000(1+10\%) - \$5{,}000 = \$17{,}000$;
After 2nd payment, amount he owes $= \$17{,}000(1+10\%) - \$5{,}000 = \$13{,}700$;
After 3rd payment, amount he owes $= \$13{,}700(1+10\%) - \$5{,}000 = \$10{,}070$;
$\therefore$ After 4th payment, amount he owes $= \$10{,}070(1+10\%) - \$5{,}000 = \$6{,}077$.

15. (a) Growth factor $= 1 + 15\% = 1.15$
(b) Increase in members $= 3{,}000(1.15)^4 - 3{,}000 = 3{,}000[(1.15)^4 - 1] = 2{,}247$

16. Decay factor $= \frac{4{,}000}{4{,}500} = \frac{8}{9}$

17. Value $= \$7{,}500(1-9\%)^{3\times 2} = \$7{,}500(0.91)^6 = \$4{,}259$

18. (a) Salary after 3 years $= \$24{,}200(1+10\%)^3 = \$32{,}210$
(b) Let \$$y$ be the salary 2 years ago. $y(1+10\%)^2 = 24{,}200$, $y = 20{,}000$.
Ans. Her salary was \$20,000 2 years ago.

19. (a) Volume after one day $= 3{,}000(1-6\%)^{24} = 680cm^3$
(b) Let $y\ cm^3$ be the volume 3 hours ago. $y(1-6\%)^3 = 3{,}000$, $y = 3{,}612$.
Ans. The volume of the balloon was 3,612 cm^3 3 hours ago.

20. (a) Value after 1 year $= \$3{,}100(1+2\%)^3 \approx \$3289.7 \approx \$3{,}290$
(b) Value after 2 years $= \$3{,}289.7(1-2\%)^4 = \$3{,}034$

21. (a) At beginning of 2nd month, amount he owed $= \$35{,}000(1+\frac{18}{12}\%) - \$10{,}000 = \$25{,}525$,
$\therefore$ At beginning of 3rd month, amount he owed $= \$25{,}525(1+1.5\%) - \$10{,}000 = \$15{,}908$
(b) At beginning of 4th month, amount he owed $= \$15{,}908(1+1.5\%) - \$10{,}000$
$= \$6{,}146$, at end of 4th month, amount he owed $= \$6{,}146(1+1.5\%)$
$= \$6{,}239 < \$10{,}000$. *Ans. 4 payments were needed.*

22. Amount owed after 1st installment $= \$8{,}000 - \$1{,}200 = \$6{,}800$,
Amount he owed after 2nd installment $= \$6{,}800(1+\frac{10}{12}\%) - \$900 = \$5{,}956.67$,

$\therefore$ Amount he owed after 3rd installment $= \$5{,}956.67(1+\frac{10}{12}\%) - \$900 = \$5{,}106$

23. Amount at end of 1st year $= \$20{,}000(1+\frac{5}{2}\%)^2$

Amount at end of 2nd year $= \$20{,}000(1+\frac{5}{2}\%)^4$

$\therefore$ Interest earned in 2nd year $= \$20{,}000\ [(1.025)^4 - (1.025)^2] = \$1{,}064$

24. Let x be the growth factor. $18x^2 = 36,\ x^2 = 2,\ x = \sqrt{2}$

$\therefore$ No. of bacteria in $\frac{1}{2}$ hour $= 18(\sqrt{2})^{30} = 590{,}000$ (corr. to nearest 1000)

25. Let $\$x$ be the amount of each payment. $[90{,}000(1+25\%) - x](1+25\%) - x = 0$,

$(112{,}500 - x)(1.25) - x = 0$, $140{,}625 - 2.25x = 0$, $x = 62{,}500$,

$\therefore$ Total interest $= \$62{,}500 \times 2 - \$90{,}000 = \$35{,}000$

26. Let $\$P$ and $r\%$ be the principal and the minimum interest rate respectively.

$P(1+r\%)^{10} \geq P(1+100\%)$, $(1+r\%)^{10} \geq 2$, $1 + r\% \geq 1.0718$, $r\% \geq 7.18\%$.

Ans. The minimum interest rate is 7.18%.

27. (a) Value of his flat $= \$4{,}000{,}000(1+10\%)^3 = \$5{,}324{,}000$

(b) Amount he owes Peter $= \$2{,}500{,}000(1+\frac{8}{12}\%)^{3\times 12} = = \$3{,}175{,}593$

(c) Increase in value of the flat $= \$(5{,}324{,}000 - 4{,}000{,}000) = \$1{,}324{,}000$

Interest he has to pay to Peter $= \$(3{,}175{,}593 - 2{,}500{,}000) = \$675{,}593$

The profit $= \$(1{,}324{,}000 - 675{,}593 - 380{,}000) = \$268{,}407$

28. (a) His debt $= \$30{,}000(1+40\%)^3 = \$82{,}320$

(b) After 1st payment $= \$30{,}000(1+40\%) - \$15{,}000 = \$27{,}000$,

After 2nd payment $= \$27{,}000(1+40\%) - \$15{,}000 = \$22{,}800$,

$\therefore$ Amount owed after 3rd payment $= \$22{,}800(1+40\%) - \$15{,}000 = \$16{,}920$

(c) Amount he owes after 1month $= \$30{,}000(1+40\%) - \$10{,}000 = \$32{,}000$,

$\because$ The amount keeps increasing, $\therefore$ he can never clear his debt.

(d) Amount he owes after 1month $= \$30{,}000(1+40\%) - \$12{,}000 = \$30{,}000$

= the principal, $\therefore$ He will still owe the loan shark \$30,000 after 20 years.

Unit 4 Taxes & multiple percentage changes

1. Rates payable each year $= 85000 \times 5\% = \$4{,}250$
2. Quarterly rates payable $= 314000 \times 5\% \div 4 = \$3{,}925$
3. Rates payable $= 85600 \times 5\% \times 1\frac{1}{4} = \$5{,}350$
4. Rateable value $= 585 \div 5\% = \$11{,}700$
5. Rateable value $= 1120 \times 4 \div 5\% = \$89{,}600$

6. Rateable value = $3570 \div \dfrac{9}{12} \div 5\% = \$95{,}200$
7. Property tax payable = $659000 \times (1-20\%) \times 15\% = \$79{,}080$
8. Property tax = $(7200 \times 12)(1-20\%) \times 15\% = 69120 \times 15\% = \$10{,}368$
9. Annual rental income = $6300 \div 15\% \div (1-20\%) = 42000 \div 0.8 = \$52{,}500$
10. Monthly rental income = $9648 \div 15\% \div (1-20\%) \div 12 = 80400 \div 12 = \$6{,}700$
11. Difference in property tax payable = difference in monthly rent $\times 12 \times (1-20\%) \times 15\%$
    $= (8500 \times 10\%) \times 12 \times 80\% \times 15\% = \$1{,}224$
12. (a) $\because$ Net chargeable income $= 98000 < 100000$, i.e. the basic allowance,
    $\therefore$ salaries tax = 0.
    (b) Net chargeable income $= 202000 - 100000 = 102000 = \$30000 \times 3 + \$12000$
    Salaries tax $= 30000 \times (2\% + 8\% + 14\%) + 12000 \times 20\% = 7200 + 2400 = \$9{,}600$
    (c) Net chargeable income $= 180000 - 100000 - 30000 = 50000 = \$30000 + \$20000$
    Salaries tax $= 30000 \times 2\% + 20000 \times 8\% = \$2{,}200$
    (d) Net chargeable income $= 282000 - 100000 - 30000 \times 2 = 122000 = \$30000 \times 3 + \$32000$
    Salaries tax $= 30000 \times (2\% + 8\% + 14\%) + 32000 \times 20\% = 7200 + 6400 = \$13{,}600$
13. (a) Net chargeable income $= 415000 - 200000 - 30000 \times 3 = 125000 = \$30000 \times 3 + \$35000$
    Salaries tax $= 30000 \times (2\% + 8\% + 14\%) + 35000 \times 20\% = 7200 + 7000 = \$14{,}200$
    (b) Net chargeable income $= 370000 - 200000 - 30000 \times 3 = 80000 = \$30000 \times 2 + \$20000$
    Salaries tax $= 30000 \times (2\% + 8\%) + 20000 \times 14\% = 3000 + 2800 = \$5{,}800$
14. Net chargeable income $= 18000 \times 12 - 100000 - 30000 \times 2 = 56000 = \$30000 + \$26000$
    Salaries tax $= 30000 \times 2\% + 26000 \times 8\% = \$2{,}680$
15. Max. total income = total allowance $= 100000 + 30000 \times 2 = \$160{,}000$
16. Salaries tax on the first \$90000 = 30000 × (2% + 8% + 14%) = \$7,200
    $\therefore$ Income = $90000 + (8000 - 7200) \div 20\% + 100000 = \$194{,}000$
17. Net chargeable income $= 2400000 - 100000 - 30000 \times 4 = 2180000 = \$30000 \times 3 + \$2090000$
    Progressive salaries tax $= 30000 \times (2\% + 8\% + 14\%) + 2090000 \times 20\%$
    $= 7200 + 418000 = \$425200$
    Upper limit of salaries tax = $2400000 \times 16\% = \$384000 < \$425200$,
    $\therefore$ salaries tax $= \$384{,}000$
18. Let his total income be \$$x$. The standard rate is 16%.
    $x \times 16\% = 30000 \times (2\% + 8\% + 14\%) + [(x - 100000) - 30000 \times 3] \times 20\%$,
    $7200 + (x - 190000) \times 0.2 = 0.16x$, $0.04x = 30800$, $x = 770000$
    Ans. His total income was \$770,000
19. The amount he spent $= 1440 \div 15\% = \$9600$
20. His original budget $= 88000 \div (1 + 20\%) = \$73300$ (3 sig. fig.)
21. His present monthly income $= 15000(1 - 30\%)(1 + 20\%) = \$12600$
22. Let the original length and width be x and y respectively.
    Original area = xy. New area = $x(1-10\%) \times y(1-12\%) = 0.792xy$

Percentage decrease in area = $\dfrac{xy-0.792xy}{xy}\times 100\% = (1-0.792)\times 100\% = 20.8\%$

23. Let the side of a small cube be x, $\therefore$ the side of the large cube $=\sqrt[3]{8x^3} = 2x$.

Percentage increase in total surface area = $\dfrac{8\times 6x^2 - 6(2x)^2}{6(2x)^2}\times 100\%$

$= \dfrac{48-24}{24}\times 100\% = 100\%$

24. Her weight before diet = $90 \div (1+25\%) \div (1-10\%) = 80\text{kg}$
25. Let the original base and height be x and y respectively.

Original area $=\dfrac{1}{2}xy$. New area $=\dfrac{1}{2}\times x(1+28\%)\times y(1-12\%) = 0.5632xy$

Percentage change in area = $\dfrac{0.5632xy-0.5xy}{0.5xy}\times 100\% = \dfrac{0.0632}{0.5}\times 100\% = 12.64\%$ (increase)

26. Length of the fish two months ago = $40 \div (1+15\%)^2 = 30.2\text{cm}$ (3 sig. fig.)
27. Total pass percentage = $60\% + (1-60\%)(1-45\%) = 60\% + 22\% = 82\%$
28. Amount paid by Miss Ng $= 60000(1+5\%)(1-8\%) = \$57960$
29. The cost paid by Andrew $= 282 \div (1+50\%) \div (1-6\%) = \$200$
30. Percentage change in the running cost

$= [45\%(1+20\%) + 35\%(1-5\%) + 20\%(1+10\%)] - 100\%$

$= 54\% + 33.25\% + 22\% - 100\% = 9.25\%$ (increase)

31. $A = C(1-10\%)(1+20\%) = 1.08C$, $\therefore$ $A:C = 1.08:1 = 108:100 = 27:25$
32. (a) Let x cm be the height of Helen.

Height of May $= x(1-15\%)(1+10\%) = 0.935x$

Percentage difference = $\dfrac{0.935x - x}{x}\times 100\% = -0.065\times 100\% = -6.5\%$

Ans. May is shorter than Helen by 6.5%.

(b) Percentage difference = $\dfrac{x-0.935x}{0.935x}\times 100\% = 6.95\%$

Ans. Helen is taller than May by about 6.95%.

33. Let A be the original area, r_1 be the original radius and r_2 be the new radius,

$\pi r_1^2 = A$ ……(1) and $\pi r_2^2 = A(1-11.64\%) = 0.8836A$ ……(2)

$\dfrac{(2)}{(1)}: 0.8836 = \dfrac{r_2^2}{r_1^2}$, $\sqrt{r_2^2} = \sqrt{0.8836r_1^2}$, $r_2 = 0.94r_1$

Percentage decrease in radius = $\dfrac{(r_1 - 0.94r_1)}{r_1}\times 100\% = 6\%$

34. Let $n\%$ be the percentage change in height.

$15\,(1 + 20\%) \times 18\,(1 + 15\%) \times 18\,(1 + n\%) = 15 \times 8 \times 18 \times (1 - 10\%)$, $n\% = \dfrac{0.9}{(1.2)(1.15)} - 1$,

$n = -34.8$ *Ans. The percentage decrease in height is 34.8%*

35. Let the length be ℓ cm and the width be w cm. $\ell \times (1 + 12\%) = w \times (1 - 20\%)$,

$\ell = \dfrac{80}{112} w = \dfrac{5}{7} w$. The percentage = $(1 - \dfrac{5}{7}) \times 100\% = 28.6\%$

Ans. The length is shorter than the width by 28.6%.

36. Let the percentage decrease be r%, and the original weight be W kg.

$W\,(1 + 20\%)\,(1 - r\%) = W$, $1 - r\% = \dfrac{100}{120} = \dfrac{5}{6}$, $r\% = \dfrac{1}{6} = \dfrac{1}{6} \times 100\% = 16\dfrac{2}{3}\%$

Ans. The percentage decrease should be $16\dfrac{2}{3}\%$.

37. Let the number be N. $N(1 + x\%)\,(1 - y\%) = N$, $(1 + x\%)\,(\dfrac{100 - y}{100}) = 1$,

$x\% = \dfrac{100}{100 - y} - 1 = \dfrac{100 - (100 - y)}{100 - y} = \dfrac{y}{100 - y}$, $\therefore\ x = \dfrac{100y}{100 - y}$

38. Let his original hourly income be $\$x$.

$\therefore$ Percentage change = $\dfrac{9x \times (1 - 15\%) - 8x}{8x} \times 100\% = -4.375\%$ (decrease)

39. Let the original total cost be $\$C$.

	Material A	Material B	Material C
Original cost	$C \times \dfrac{1}{1+5+4} = 0.1C$	$C \times \dfrac{5}{1+5+4} = 0.5C$	$C \times \dfrac{4}{1+5+4} = 0.4C$
New cost	$0.1C \times (1 + 30\%) = 0.13C$	$0.5C \times (1 + 2\%) = 0.51C$	$0.4C \times (1 + 25\%) = 0.5C$

New total cost = (0.13 + 0.51 + 0.5)C = 1.14C

Overall percentage increase = $(1.14 - 1) \times 100\% = 14\%$.

40. Percentage change = $(1 + 15\%)\,(1 - 15\%) - 100\% = -2.25\%$ (decrease)

41. Let the selling price of one pen be $\$S$ and the cost price be \$C. $15S = 18C$,

$S = 1.2C$. Profit percent = $\dfrac{S - C}{C} \times 100\% = \dfrac{1.2C - C}{C} \times 100\% = 20\%$.

42. (a) Profit percentage = $(1 + 40\%)\,(1 - 25\%) - 100\% = 5\%$

(b) Marked price = $8820 \div (1 - 25\%) = \$11760$. Cost price $= 11760 \div (1 + 40\%) = \$8400$

43. The cost paid by B $= 2100 \div 15\% = \$14000$.

$\therefore$ The amount paid by C $= 14000 + 2100 = \$16100$,

and profit gained by A = $14000 - 14000 \div (1 + 28\%) = \$3062.5$

44. Total cost price = $3300 \div (1 + 10\%) + 2100 \div (1 - 20\%) = 3000 + 2625 = \$5625$

Selling price of wardrobe $= 5625 - 2500 = \$3125$

45. The total cost price of the watches $= x \div (1 + 15\%) + x \div (1 - 20\%) = 2.1196x$ (5 sig. fig.)

$\therefore$ The profit percentage = $\dfrac{2.1196x - 2x}{2.1196x} \times 100\% = 5.64\%$

Ans. The percentage profit on the whole is 5.64%

46. Let the total number of monitors be N, and the cost price of each monitor be \$$y$,

$50y(1+40\%)+(N-50)\,y(1-40\%)=Ny(1+10\%)$,

$70+0.6N-30=1.1N,\quad 40=0.5N,\quad N=80$

Ans. The merchant had 80 monitors at the beginning.

47. (a) Let the number be N, $N(1-10\%)(1+50\%)=810,\quad 1.35N=810,\quad \text{N}=600$

Ans. The original number is 600.

(b) $600\times(1+25\%)\times(1-y\%)=600+75,\ 1-y\%=\dfrac{675}{600\times1.25}=0.9,\quad y\%=0.1=10\%$,

$\therefore\ y=10$

48. Let P kg be Paul 's weight. $D=P(1+20\%),\quad D=\dfrac{120}{100}P,\quad \therefore\ P=\dfrac{5}{6}D$

Weight of Mr. Lau = $D+\dfrac{5}{6}D=\dfrac{11}{6}D=D\times\dfrac{11}{6}\times100\%=D\times183\dfrac{1}{3}\%$.

49. Let his monthly income be \$$I$. Original expenditure $=I(1-30\%)=0.7I$;

new expenditure = new income – new savings

$=I(1+5\%)-I\times30\%\times(1+20\%)\ =\ 1.05I-0.36I=0.69I$,

Percentage change in his monthly expenditure $=\dfrac{0.69I-0.7I}{0.7I}\times100\%\ =-1.43\%$ (3 sig. fig.)

50. Let his original income be \$$I$. New expenditure = new income × (1–20%),

$\therefore$ new income = $I(1-10\%)(1+20\%)\div(1-20\%)=1.35I$.

$\therefore$ Percentage increase in his income = $(1.35-1)\times100\%=35\%$

51. Ratio of liquid A to liquid B in the mixture = $1000:250=4:1$.

Amount of liquid B remained = $(1000+250)\times(1-40\%)\times\dfrac{1}{4+1}=150\ \ \text{cm}^3$.

52. Let y cm^3 be the amount of water to be added.

Amount of alcohol = 1000 × 60% = (1000 + y) × 20%, 1000 + y = 3000,

$y=2000$. *Ans. 2000 cm^3 of water should be added.*

53. Let the amount of salt to be added be k kg.

$x\cdot x\%=(x+k)\cdot\dfrac{x}{2}\%,\quad x=(x+k)\cdot\dfrac{1}{2},\quad 2x=x+k,\quad \therefore\ k=x$

Ans. x kg of water should be added to the salt water.

54. Let t_1, v_1 and v_2, and be the original walking time, original speed, and the new speed respectively. The distance = $v_1t_1=v_2t_1(1–20\%)$, $v_1=0.8v_2$, $v_2=1.25v_1$,

$\therefore$ The percentage change in speed = $(1.25-1)\times100\%=25\%$

Ans. Miss Chan should increase her walking speed by 25%.

55. Let his original speed be v_1 m/min, new speed be v_2 m/min. $45v_1=(45-5)v_2$,

$v_2=\dfrac{9}{8}v_1$, $\therefore$ Percentage increase in his speed = $(\dfrac{9}{8}-1)\times100\%=12.5\%$

56. Let the distance between P and Q be d m, and his speed in the return trip be x km/h. The total

time take = $\frac{d}{60}+\frac{d}{x}=\frac{2d}{72}$, $\therefore$ $\frac{1}{60}+\frac{1}{x}=\frac{1}{36}$, $\frac{1}{x}=\frac{1}{36}-\frac{1}{60}=\frac{1}{90}$,

$\therefore$ $x = 90$. Percentage increase in speed $=\frac{90-60}{60}=\times 100\% = 50\%$

Unit 5 Factorization: Cross-method

1\. (a)
$y \quad +13$
$y \quad +4$
$+13y+4y=+17y$
Ans. $(y+13)(y+4)$

(b)
$a \quad -9$
$a \quad -6$
$-9a-6a=-15a$
Ans. $(a-9)(a-6)$

(c)
$x \quad +6$
$x \quad -2$
$+6x-2x=+4x$
Ans. $(x+6)(x-2)$

(d)
$y \quad -7$
$y \quad +4$
$-7y+4y=-3y$
Ans. $(y-7)(y+4)$

(e) $(c+16)(c-2)$ (f) $(a+8)(a-11)$ (g) $(m-8)(m+9)$
(h) $(a+6)(a-10)$ (i) $(m-3)(m-15)$ (j) $(x+3)(x-14)$
(k) $(t+2)(t+10)$ (l) $(y+12)(y-4)$ (m) $(k-3)(k-24)$
(n) $(a-2)(a+12)$

2\. (a)
$9 \quad -x$
$7 \quad -x$
$-7x-9x=-16x$
Ans. $(9-x)(7-x)$

(b) $=y^2+7y-44$
$y \quad +11$
$y \quad -4$
$+11y-4y=+7y$
Ans. $(y-4)(y+11)$

(c) $=-(m^2-18m+65)=-(m-13)(m-5)$ (d) $(18-a)(3+a)$
(e) $=-(x^2+19x+48)=-(x+3)(x+16)$ (f) $(20+y)(4-y)$
(g) $=-(n^2-21n+90)=-(n-6)(n-15)$ (h) $=50+23x-x^2=(25-x)(2+x)$

3\. (a)
$3x \quad +2$
$x \quad +3$
$2x+9x=11x$
Ans. $(3x+2)(x+3)$

(b)
$2y \quad -1$
$7y \quad -1$
$-7y-2y=-9y$
Ans. $(2y-1)(7y-1)$

(c)
$3x \quad -1$
$5x \quad +4$
$-5x+12x=+7x$
Ans. $(3x-1)(5x+4)$

(d)
$2y \quad -3$
$7y \quad +3$
$-21y+6y=-15y$
Ans. $(2y-3)(7y+3)$

(e) $(y+3)(12y-7)$ (f) $=40n^2+78n-27=(4n+9)(10n-3)$
(g) $(5-2x)(1-6x)$ (h) $(5m+3)(3m+5)$ (i) $(7y+2)(5y+2)$
(j) $(9-4p)(2-3p)$ (k) $(6a-5)(2a+1)$ (l) $(8m-3)(3m+4)$

(m) $(4b + 5)(2b + 3)$ (n) $(5a - 1)(a - 4)$ (o) $(9x + 4)(2x - 7)$

(p) $(11b - 5)(3b + 5)$

4. (a) $= 3(x^2 - 11x + 24) = 3(x - 3)(x - 8)$ (b) $= 2(y^2 + 5y - 36) = 2(y + 9)(y - 4)$

   (c) $= 6(2a^2 + 13a + 15) = 6(2a + 3)(a + 5)$ (d) $= 7(3b^2 + 5b - 12) = 7(b + 3)(3b - 4)$

   (e) $= 5(4 - 13m + 10m^2) = 5(4 - 5m)(1 - 2m)$

   (f) $= 6(14n^2 + 31n - 10) = 6(7n - 2)(2n + 5)$

   (g) $= -4(15x^2 + 8x - 12) = -4(3x - 2)(5x + 6)$ (h) $= 3(9 + 30y + 16y^2) = 3(3 + 8y)(3 + 2y)$

5. (a)

   $xy \quad -7$

   $xy \quad -6$

   $-7xy - 6xy = -13xy$

   Ans. $(xy - 6)(xy - 7)$

   (b)

   $m \quad +8n$

   $m \quad -3n$

   $+8mn - 3mn = +5mn$

   Ans. $(m - 3n)(m + 8n)$

   (c) $(x + 16y)(x + 4y)$ (d) $(ab + 12)(ab - 8)$ (e) $(x^2 - 5)(x^2 + 9)$

   (f) $(2 - m^2)(14 - m^2)$ (g) $(x^3 + 10)(x^3 + 5)$ (h) $(y^3 - 11)(y^3 + 6)$

   (i) $(a^2 - 15b^2)(a^2 + 3b^2)$ (j) $(x^2 - 5y^2)(x^2 - 14y^2)$

   (k) $= x^2 + 14x + 48 = (x + 6)(x + 8)$ (l) $= a^2 - 17a + 60 = (a - 5)(a - 12)$

6. (a) $= (5y - 3)(y - 2)$ (b) $= 23x - 12x^2 - 10 = -(12x^2 - 23x + 10) = -(4x - 5)(3x - 2)$

   (c) $= 2n + 8n^2 + 3 + 12n - 18 = 8n^2 + 14n - 15 = (4n - 3)(2n + 5)$

   (d) $= 24x^2 - 54x + 4x - 9 - 48x - 8 = 24x^2 - 98x - 17 = (4x - 17)(6x + 1)$

   (e) $= 8y^2 - y - 15 + 15y = 8y^2 + 14y - 15 = (4y - 3)(2y + 5)$

7. (a) $(y - 2)(5y - 3)$ (b) $(5y + 2)(y - 3)$ (c) can't be factorized.

   (d) can't be factorized. (e) $(a - \frac{1}{a})(a + \frac{3}{a})$

8. $= \frac{1}{9}(2x^2 - 9x - 18) = \frac{1}{9}(x - 6)(2x + 3)$

9. $= (x^2 + 3)(x^2 - 9) = (x^2 + 3)(x + 3)(x - 3)$

10. $= (x^2 - 4y^2)(4x^2 - 9y^2) = (x + 2y)(x - 2y)(2x + 3y)(2x - 3y)$

11. $= 2a(x^2 + 3x + 2) + 3b(x^2 + 3x + 2) = (2a + 3b)(x^2 + 3x + 2) = (2a + 3b)(x + 2)(x + 1)$

12. (a) $(6a - 1)(2a + 5)$

    (b) Put $2x + 1 = a$ into part (a), we get: $[6(2x + 1) - 1][2(2x + 1) + 5] = (12x + 5)(4x + 7)$

13. (a) $(x - 6y)(x + 2y)$

    (b) $= 3[b^2 - 4b(a + 1) - 12(a + 1)^2]$. Put $b = x, a + 1 = y$ into part (a), we get:
    $3[b - 6(a + 1)][b + 2(a + 1)] = 3(b - 6a - 6)(b + 2a + 2)$

14. (a) $(x + 12)(x + 2)$

    (b) $= (a + 2b)^2 + 14(a + 2b) + 24$. Put $a + 2b = x$ into part (a),
    we get: $[(a + 2b) + 12][(a + 2b) + 2] = (a + 2b + 12)(a + 2b + 2)$

15. (a) $= \frac{1}{4}(3y^2 - 8y + 4) = \frac{1}{4}(3y - 2)(y - 2)$

    (b) $= 3m - n + \frac{3}{4}(m + n)^2 - 5m - n + 1 = \frac{3}{4}(m + n)^2 - 2m - 2n + 1 = \frac{3}{4}(m + n)^2 - 2(m + n) + 1$

    Put $m + n = y$ into part (a), we get:

$\frac{1}{4}[(3(m+n)-2][(m+n)-2]=\frac{1}{4}(3m+3n-2)(m+n-2)$

16. (a) $= x(y + 8) - 2(y + 8) = (x - 2)(y + 8)$

(b) $a^2 - b^2 = (a + b)(a - b)$, let $x = a - b$, $y = a + b$,

$\therefore$ $8x - 2y = 8(a - b) - 2(a + b) = 8a - 8b - 2a - 2b = 6a - 10b$

$\therefore$ From (a), $a^2 - b^2 + 6a - 10b - 16 = [(a - b) - 2][(a + b) + 8] = (a - b - 2)(a + b + 8)$

17. (a) $(7)(-12) = -84$, $7 + (-12) = -5$, $\because$ $m > n$, $\therefore$ $m = 7$, $n = -12$.

(b) $x^3 + 8x^2 - 5x - 84 = x^3 + 7x^2 + (x^2 - 5x - 84) = x^2(x + 7) + (x + 7)(x - 12)$

$= (x + 7)[x^2 + (x - 12)] = (x + 7)(x + 4)(x - 3)$

Unit 6 Factorization : Cubes

1. (a) $= a^3 + 2^3 = (a + 2)(a^2 - 2a + 4)$ (b) $= b^3 - 3^3 = (b - 3)(b^2 + 3b + 9)$

(c) $= (10y)^3 - 1^3 = (10y - 1)[(10y)^2 + 10y + 1] = (10y - 1)(100y^2 + 10y + 1)$

(d) $= 5^3 + (3m)^3 = (5 + 3m)(25 - 15m + 9m^2)$

(e) $= (2x)^3 + (5y)^3 = (2x + 5y)(4x^2 - 10xy + 25y^2)$

(f) $= 6^3 - (ab)^3 = (6 - ab)(36 + 6ab + a^2b^2)$

2. (a) $= 3(x^3 + 64) = 3(x^3 + 4^3) = 3(x + 4)(x^2 - 4x + 16)$

(b) $= 4(y^3 - 125) = 4(y^3 - 5^3) = 4(y - 5)(y^2 + 5y + 25)$

(c) $= 2(64 - 27b^3) = 2[4^3 - (3b)^3] = 2(4 - 3b)(16 + 12b + 9b^2)$

(d) $= 3(125 - x^3y^3) = 3[5^3 - (xy)^3] = 3(5 - xy)(25 + 5xy + x^2y^2)$

(e) $= 2(216a^3 + 125b^3) = 2[(6a)^3 + (5b)^3)] = 2(6a + 5b)(36a^2 - 30ab + 25b^2)$

(f) $= 4(64p^3q^3 - 27) = 4[(4pq)^3 - 3^3] = 4(4pq - 3)(16p^2q^2 + 12pq + 9)$

3. (a) $= [(2x + 3) - 1][(2x + 3)^2 + (2x + 3)(1) + 1^2]$

$= (2x + 2)(4x^2 + 12x + 9 + 2x + 3 + 1) = 2(x + 1)(4x^2 + 14x + 13)$

(b) $= [4 + (3x - 1)][4^2 - (4)(3x - 1) + (3x - 1)^2]$

$= (3x + 3)(16 - 12x + 4 + 9x^2 - 6x + 1)$

$= 3(x + 1)(9x^2 - 18x + 21) = 9(x + 1)(3x^2 - 6x + 7)$

(c) $= [3 - (4x + 1)][3^2 + (3)(4x + 1) + (4x + 1)^2]$

$= (2 - 4x)(9 + 12x + 3 + 16x^2 + 8x + 1) = 2(1 - 2x)(16x^2 + 20x + 13)$

(d) $= 5^3 + 2^3(x + 2)^3 = 5^3 + (2x + 4)^3$

$= [5 + (2x + 4)][5^2 - (5)(2x + 4) + (2x + 4)^2]$

$= (2x + 9)(25 - 10x - 20 + 4x^2 + 16x + 16) = (2x + 9)(4x^2 + 6x + 21)$

(e) $= 125[8 - (x - 2)^3] = 125[2 - (x - 2)][2^2 + 2(x - 2) + (x - 2)^2]$

$= 125(4 - x)(4 + 2x - 4 + x^2 - 4x + 4) = 125(4 - x)(x^2 - 2x + 4)$

(f) $= [(x + 4) + (2x - 1)][(x + 4)^2 - (x + 4)(2x - 1) + (2x - 1)^2]$

$= (3x + 3)[(x^2 + 8x + 16) - (2x^2 + 7x - 4) + (4x^2 - 4x + 1)]$

$= 3(x + 1)(3x^2 - 3x + 21) = 9(x + 1)(x^2 - x + 7)$

(g) $= [(3x - 1) - (x + 3)][(3x - 1)^2 + (3x - 1)(x + 3) + (x + 3)^2]$

$= (2x - 4)(9x^2 - 6x + 1 + 3x^2 + 8x - 3 + x^2 + 6x + 9)$

$= 2(x - 2)(13x^2 + 8x + 7)$

(h) $= 3[2^3(2x + 1)^3 - 3^3(x + 1)^3] = 3[(4x + 2)^3 - (3x + 3)^3]$
$= 3[(4x + 2) - (3x + 3)][(4x + 2)^2 + (4x + 2)(3x + 3) + (3x + 3)^2]$
$= 3(x - 1)(16x^2 + 16x + 4 + 12x^2 + 18x + 6 + 9x^2 + 18x + 9)$
$= 3(x - 1)(37x^2 + 52x + 19)$

4. (a) $= 1^3 + (x^2)^3 = (1 + x^2)(1 - x^2 + x^4)$
(b) $= (y^3)^2 - 1^2 = (y^3 + 1)(y^3 - 1) = (y + 1)(y - 1)(y^2 + y + 1)(y^2 - y + 1)$
(c) $= (a^2)^3 + (b^2)^3 = (a^2 + b^2)(a^4 - a^2b^2 + b^4)$
(d) $= (m^2)^3 - 2^3 = (m^2 - 2)(m^4 + 2m^2 + 4)$
(e) $= 3^3 - (x^2)^3 = (3 - x^2)(9 + 3x^2 + x^4)$
(f) $= 1^2 - [(2y)^3]^2 = [1 - (2y)^3][1 + (2y)^3] = (1 - 2y)(1 + 2y)(1 + 2y + 4y^2)(1 - 2y + 4y^2)$
(g) $= 2[(3m^2)^3 + (4n^2)^3] \quad = 2(3m^2 + 4n^2)[(3m^2)^2 - (3m^2)(4n^2) + (4n^2)^2]$
$= 2(3m^3 + 4n^2)(9m^4 - 12m^2n^2 + 16n^4)$

5. $= (m^3 + n^3)^2 = (m + n)^2(m^2 - mn + n^2)^2$

6. $= (y^3 - 1)(27y^3 + 8) = (y - 1)(y^2 + y + 1)(3y + 2)[(3y)^2 - (3y)(2) + 2^2]$
$= (y - 1)(3y + 2)(y^2 + y + 1)(9y^2 - 6y + 4)$

7. $= (a^6 - 1)(a^6 + 1) = (a^3 - 1)(a^3 + 1)[(a^2)^3 + 1]$
$= (a - 1)(a^2 + a + 1)(a + 1)(a^2 - a + 1)(a^2 + 1)(a^4 - a^2 + 1)$
$= (a - 1)(a + 1)(a^2 + 1)(a^2 + a + 1)(a^2 - a + 1)(a^4 - a^2 + 1)$

8. $= (16x^4 + 2x) - (24x^3y + 3y) = 2x(8x^3 + 1) - 3y(8x^3 + 1)$
$= (2x - 3y)(8x^3 + 1) = (2x - 3y)(2x + 1)(4x^2 - 2x + 1)$

9. (a) $(x - 4)(x^2 + 4x + 16)$
(b) $= (x^3 - 64) + (x - 4) = (x - 4)(x^2 + 4x + 16) + (x - 4)$
$= (x - 4)(x^2 + 4x + 16 + 1) = (x - 4)(x^2 + 4x + 17)$

10. (a) $(3y + 1)(9y^2 - 3y + 1)$
(b) $= (54y^3 + 2) - 1 - 3y = 2(27y^3 + 1) - (3y + 1)$
$= 2(3y + 1)(9y^2 - 3y + 1) - (3y + 1) = (3y + 1)[2(9y^2 - 3y + 1) - 1]$
$= (3y + 1)(18y^2 - 6y + 1)$

11. $= 19(a + 2) - (a + 2)(a^2 - 2a + 4) = (a + 2)[19 - (a^2 - 2a + 4)]$
$= (a + 2)(15 + 2a - a^2) = (a + 2)(5 - a)(3 + a)$

12. $243 = 9\times27$, the given expression $= 9(r^3 - 27s^3) - 9s^2 - r^2 - 3rs$
$= 9(r - 3s)(r^2 + 3rs + 9s^2) - (r^2 + 3rs + 9s^2) = (r^2 + 3rs + 9s^2)[9(r - 3s) - 1]$
$= (r^2 + 3rs + 9s^2)(9r - 27s - 1)$

Unit 7 Algebraic fractions (2)

1. (a) $\dfrac{3}{5y^4}$ (b) $\dfrac{3b^2}{7}$ (c) $\dfrac{3(x+2y)}{x-2y}$ (d) $\dfrac{4}{3(2a+b)}$

(e) $= \dfrac{8(1-k)}{-k^2(1-k)} = -\dfrac{8}{k^2}$ (f) $= \dfrac{9(x-y)^2}{12(x-y)} = \dfrac{3}{4}(x-y)$

(g) $= \dfrac{-3x^2(5y-2x)}{7(5y-2x)} = -\dfrac{3x^2}{7}$ (h) $= \dfrac{9x^2(6x-1)}{9x} = x(6x-1)$

2. (a) $= \dfrac{-(a-b)}{(a-b)(a+b)} = \dfrac{-1}{a+b}$ (b) $= \dfrac{4(x^2-2x+1)}{x(x-a)-(x-a)} = \dfrac{4(x-1)^2}{(x-a)(x-1)} = \dfrac{4(x-1)}{x-a}$

(c) $= \dfrac{3x(x^2-a)+(x^2-a)}{7x(3x+1)} = \dfrac{(3x+1)(x^2-a)}{7x(3x+1)} = \dfrac{x^2-a}{7x}$

3. (a) $= \dfrac{x(y-1)+3(y-1)}{(x-8)(x+3)} = \dfrac{(y-1)(x+3)}{(x-8)(x+3)} = \dfrac{y-1}{x-8}$

(b) $= \dfrac{(y-2)(y+3)}{y(x-2)+3(x-2)} = \dfrac{(y-2)(y+3)}{(x-2)(y+3)} = \dfrac{y-2}{x-2}$

(c) $= \dfrac{3(x^3-2^3)}{9(x^2+2x+4)} = \dfrac{3(x-2)(x^2+2x+4)}{9(x^2+2x+4)} = \dfrac{1}{3}(x-2)$

(d) $= \dfrac{(x-1)^2+(x+8)(x-1)}{x-1} = \dfrac{(x-1)(x-1+x+8)}{x-1} = 2x+7$

(e) $= \dfrac{(x+3)(x+4)}{x^3+3^3} = \dfrac{(x+3)(x+4)}{(x+3)(x^2-3x+9)} = \dfrac{x+4}{x^2-3x+9}$

(f) $= \dfrac{x^2(x-y)+y^2(x-y)}{(x^2)^2-(y^2)^2} = \dfrac{(x^2+y^2)(x-y)}{(x^2+y^2)(x^2-y^2)} = \dfrac{x-y}{(x+y)(x-y)} = \dfrac{1}{x+y}$

4. (a) $= \dfrac{x^4}{3(x-2)} \times \dfrac{3}{x^3} = \dfrac{x}{x-2}$ (b) $= \dfrac{1}{5}ab^2 \times \dfrac{b^2}{5a} \times a^2 = \dfrac{a^2b^4}{25}$

(c) $= \dfrac{3(b+2)}{6(b-2)} \times \dfrac{4(b-2)}{5(2+b)} = \dfrac{2}{5}$ (d) $= \dfrac{m(1-a)}{b(x-1)} \div \dfrac{c(1-a)}{n(1-x)} = \dfrac{m(1-a)}{b(x-1)} \times \dfrac{-n(x-1)}{c(1-a)} = -\dfrac{mn}{bc}$

5. (a) $= \dfrac{12r-(2r+3)}{9} = \dfrac{10r-3}{9}$ (b) $= \dfrac{5(a-3)+2(a+2)}{30} = \dfrac{5a-15+2a+4}{30} = \dfrac{7a-11}{30}$

(c) $= \dfrac{4x-(x+2)}{4} = \dfrac{4x-x-2}{4} = \dfrac{3x-2}{4}$

6. (a) $= \dfrac{3-(4+b)}{3a} = \dfrac{3-4-b}{3a} = -\dfrac{b+1}{3a}$ (b) $= \dfrac{8x-(1-6x)}{4x} = \dfrac{8x-1+6x}{4x} = \dfrac{14x-1}{4x}$

(c) $= \dfrac{3(2x+a)-2(a-3x)-(x+a)}{6x} = \dfrac{6x+3a-2a+6x-x-a}{6x} = \dfrac{11x}{6x} = \dfrac{11}{6}$

7. (a) $= \dfrac{4}{y+5} - \dfrac{y}{3(y+5)} = \dfrac{12-y}{3(y+5)}$

(b) $= \dfrac{2}{3(x-y)} + \dfrac{4}{-7(x-y)} = \dfrac{14-12}{21(x-y)} = \dfrac{2}{21(x-y)}$

(c) $= \dfrac{10y+9y-6y}{12(a-b)} = \dfrac{13y}{12(a-b)}$

8. (a) $= \dfrac{x(x-1)-2x^2}{x-1} = \dfrac{x^2-x-2x^2}{x-1} = \dfrac{x(1+x)}{1-x}$

(b) $= \dfrac{2(y+2)-3(y-3)}{(y-3)(y+2)} = \dfrac{2y+4-3y+9}{(y-3)(y+2)} = \dfrac{13-y}{(y-3)(y+2)}$

(c) $=\dfrac{y(2y-x)-x(2x-y)}{(2x-y)(2y-x)}=\dfrac{2y^2-xy-2x^2+xy}{(2x-y)(2y-x)}=\dfrac{2(y+x)(y-x)}{(2x-y)(2y-x)}$

9. (a) $=\dfrac{4(b+1)}{(a-3)}\times\dfrac{18(a-3)}{4(a-1)}\times\dfrac{1}{6(b+1)}=\dfrac{3}{a-1}$

(b) $=\dfrac{(x-y)(x+y)}{(x^2+y^2)}\times\dfrac{1}{(x+y)^2}\times(x^2+y^2)(x-y)(x+y)=(x-y)(x-y)=(x-y)^2$

(c) $=\dfrac{b(a+b)}{a(2a-b)}\times\dfrac{5(2a-b)}{b^2(3a+1)}\times\dfrac{1}{(a+b)}=\dfrac{5}{ab(3a+1)}$

(d) $=\dfrac{x(y+1)-(y+1)}{x^2(x-1)-(x-1)}\times\dfrac{-12(x-1)}{8(y+1)}=\dfrac{(x-1)(y+1)}{(x-1)(x+1)(x-1)}\times\dfrac{-3(x-1)}{2(y+1)}=\dfrac{-3}{2(x+1)}$

(e) $=(5x-3)(x+4)\times\dfrac{4}{9}\times\dfrac{1}{2(5x-3)}=\dfrac{2}{9}(x+4)$

(f) $=(2a+3b)^2(2a-3b)^2\cdot\dfrac{(4a^2+6ab+9b^2)}{(2a-3b)(4a^2+6ab+9b^2)}$

$=(2a-3b)(2a+3b)^2$

10. (a) $=\dfrac{5}{2(a-4b)}+\dfrac{a-b}{b(a-4b)}=\dfrac{5b+2(a-b)}{2b(a-4b)}=\dfrac{3b+2a}{2b(a-4b)}$

(b) $=\dfrac{4(x-2)+3(x+2)-7x}{(x+2)(x-2)}=\dfrac{4x-8+3x+6-7x}{(x+2)(x-2)}=\dfrac{-2}{(x+2)(x-2)}$

(c) $=\dfrac{2(1+k)+2(1-k)-4k}{(1+k)(1-k)}=\dfrac{2+2k+2-2k-4k}{(1+k)(1-k)}=\dfrac{4-4k}{(1+k)(1-k)}=\dfrac{4(1-k)}{(1+k)(1-k)}=\dfrac{4}{1+k}$

(d) $=\dfrac{y}{(y+3)^2}-\dfrac{2}{(y+3)}=\dfrac{y-2(y+3)}{(y+3)^2}=\dfrac{y-2y-6}{(y+3)^2}=-\dfrac{y+6}{(y+3)^2}$

11. (a) $=\dfrac{1}{x(x-1)}+\dfrac{3}{(x-1)(x-4)}=\dfrac{x-4+3x}{x(x-1)(x-4)}=\dfrac{4(x-1)}{x(x-1)(x-4)}=\dfrac{4}{x(x-4)}$

(b) $=\dfrac{5}{(a+1)(a+6)}-\dfrac{4}{(a+2)(6+a)}=\dfrac{5(a+2)-4(a+1)}{(a+1)(a+6)(a+2)}=\dfrac{a+6}{(a+1)(a+6)(a+2)}$

$=\dfrac{1}{(a+1)(a+2)}$

(c) $=\dfrac{2}{(x+3)(x-4)}-\dfrac{1-x}{-(x-4)(4+x)}=\dfrac{2(x+4)+(1-x)(x+3)}{(x+3)(x-4)(x+4)}$

$=\dfrac{2x+8+3-2x-x^2}{(x+3)(x-4)(x+4)}=\dfrac{11-x^2}{(x+3)(x-4)(x+4)}$

(d) $=\dfrac{1}{3x(3x+1)}+\dfrac{4}{(3x+1)(3x+5)}=\dfrac{(3x+5)+4(3x)}{3x(3x+1)(3x+5)}=\dfrac{15x+5}{3x(3x+1)(3x+5)}$

$=\dfrac{5(3x+1)}{3x(3x+1)(3x+5)}=\dfrac{5}{3x(3x+5)}$

(e) $=\dfrac{4}{(3x-4)(x-2)}-\dfrac{2}{(3x-4)(1-x)}=\dfrac{4(1-x)-2(x-2)}{(3x-4)(x-2)(1-x)}$

$$=\frac{8-6x}{(3x-4)(x-2)(1-x)}=\frac{-2(3x-4)}{(3x-4)(x-2)(1-x)}=\frac{2}{(x-2)(x-1)}$$

(f) $=\frac{2}{x(x+2)}+\frac{4}{(x+2)(x+6)}=\frac{2(x+6)+4x}{x(x+2)(x+6)}=\frac{6(x+2)}{x(x+2)(x+6)}=\frac{6}{x(x+6)}$

12. (a) $=2+\frac{12x}{(x-2)^2}-\frac{x}{x-2}=\frac{2(x-2)^2+12x-x(x-2)}{(x-2)^2}$

$$=\frac{2x^2-8x+8+12x-x^2+2x}{(x-2)^2}=\frac{x^2+6x+8}{(x-2)^2}=\frac{(x+4)(x+2)}{(x-2)^2}$$

(b) $=\frac{3x+2y}{(2x-3y)(4x^2+6xy+9y^2)}-\frac{1}{(4x^2+6xy+9y^2)}=\frac{3x+2y-(2x-3y)}{(2x-3y)(4x^2+6xy+9y^2)}$

$$=\frac{x+5y}{(2x-3y)(4x^2+6xy+9y^2)}$$

13. $=(\frac{n^2-m^2}{mn})\div(\frac{m-n}{mn})=\frac{(n+m)(n-m)}{mn}\times\frac{mn}{-(n-m)}=-(n+m)$

14. $=\frac{p^2-25q^2}{5}\div\frac{5(5q-p)}{3}=\frac{(p+5q)(p-5q)}{5}\cdot\frac{3}{-5(p-5q)}=-\frac{3}{25}(p+5q)$

15. $=(\frac{2ab+a^2+b^2}{ab}\div\frac{a^2-b^2}{ab})^2=[\frac{(a+b)^2}{ab}\cdot\frac{ab}{(a+b)(a-b)}]^2=\frac{(a+b)^2}{(a-b)^2}$

16. $x-1=\frac{1}{a+1}-1=\frac{1-(a+1)}{a+1}=\frac{-a}{a+1}$, $y+1=\frac{1}{a-1}+1=\frac{1+(a-1)}{a-1}=\frac{a}{a-1}$,

$\therefore\ \frac{x-1}{y+1}=-\frac{a}{(a+1)}\cdot\frac{(a-1)}{a}=\frac{1-a}{a+1}$

17. $=1+\frac{(2y)\cdot y}{(\frac{1}{y}-y)\cdot y}+\frac{2y}{1-y^2}=1+\frac{2y^2}{1-y^2}+\frac{2y}{1-y^2}=\frac{1-y^2+2y^2+2y}{1-y^2}$

$$=\frac{1+2y+y^2}{1-y^2}=\frac{(1+y)^2}{(1+y)(1-y)}=\frac{1+y}{1-y}$$

18. $=(\frac{q-p}{q})(\frac{q-p}{p})\div(\frac{p^2-q^2}{pq})=\frac{[-(p-q)]^2}{pq}\cdot\frac{pq}{(p+q)(p-q)}=\frac{p-q}{p+q}$

19. $=\frac{a}{(a-b)(a-c)}-\frac{b}{(b-c)(a-b)}+\frac{c}{(a-c)(b-c)}$

$$=\frac{a(b-c)-b(a-c)+c(a-b)}{(a-b)(a-c)(b-c)}=\frac{ab-ac-ab+bc+ac-bc}{(a-b)(a-c)(b-c)}=0$$

20. The other adjacent side = Area × 2 ÷ $(2x+1)$

$$=\frac{6x^2-5x-4}{2}\cdot\frac{2}{2x+1}=\frac{(2x+1)(3x-4)}{2x+1}=3x-4$$

Ans. The other adjacent side is $(3x-4)$ cm.

21. (a) Area of ΔACD = area of ΔABD – area of ΔABC

$= (9x^2+7x-2)-(6x^2-10x+4) = 3x^2+17x-6$

(b) $\frac{1}{2} \times CD \times (3x-1) = 3x^2 + 17x - 6$,

$\therefore$ CD = $\frac{2(3x^2+17x-6)}{3x-1} = \frac{2(3x-1)(x+6)}{3x-1} = 2(x+6)$ cm

22. Let the time taken be y hours. In 1 hour, pipe A fills $\frac{1}{2x+3}$ of the tank, pipe B fills $\frac{1}{5-2x}$, while both pipes together fill $\frac{1}{y}$ of the tank. $\therefore \frac{1}{y} = \frac{1}{2x+3} + \frac{1}{5-2x} = \frac{5-2x+2x+3}{(2x+3)(5-2x)} = \frac{8}{(2x+3)(5-2x)}$, $\therefore y = \frac{1}{8}(2x+3)(5-2x)$

Ans. It will take $\frac{1}{8}(2x+3)(5-2x)$ *hours for them to fill the tank together.*

Unit 8 Congruent triangles (3)

1. (a) Yes. $\Delta ABC \cong \Delta EFD$ (R.H.S.) (b) No ($\because$ it's not the included angle).
   (c) Yes. $\Delta XYZ \cong \Delta LMN$ (A.A.S.), $\because \angle L = 180^\circ - 32^\circ - 105^\circ = 43^\circ$. (d) No.
2. (a) $\Delta PQR \cong \Delta LMN$ (S.A.S.) (b) $\Delta ABC \cong \Delta YXZ$ (R.H.S.)
   (c) $\Delta PQR \cong \Delta FED$ (A.A.S.)
3. (a) $\Delta ABC \cong \Delta QRP$ (R.H.S.) (b) $\Delta PQR \cong \Delta ZYX$ (A.A.S.)
4. $\angle EGF = 180^\circ - 60^\circ - 70^\circ = 50^\circ$ ($\angle$ sum of Δ)
   In ΔEFG and ΔHGF, FG = GF (common), $\angle EFG = \angle HGF$ (given),
   $\angle EGF = \angle HFG$ (proved), $\therefore \Delta EFG \cong \Delta HGF$ (A.S.A.)
5. (a) In ΔABE and ΔDCE, AE = DE (given), BE = CE (given),
   $\angle AEB = \angle DEC$ (vert. opp. $\angle$s), $\therefore \Delta ABE \cong \Delta DCE$ (S.A.S.),
   $\therefore$ AB = DC (corr. sides, $\cong \Delta$s)
   (b) $\because \Delta ABE \cong \Delta DCE$ (proved), $\therefore \angle ABE = \angle DCE$ (corr. $\angle$s, $\cong \Delta$s),
   $\therefore$ AB // CD (alt. $\angle$s eq.)
6. In ΔQLM and ΔQNM, QL = QN (given), QM = QM (common),
   $\angle QLM = \angle QNM = 90^\circ$ (given), $\therefore \Delta QLM \cong \Delta QNM$ (R.H.S.),
   $\therefore$ LM = NM (corr. sides, $\cong \Delta$s).
   In ΔLPM and ΔNRM, LM = NM (proved), $\angle MLP = \angle MNR = 90^\circ$(given),
   PM = RM (given), $\therefore \Delta LPM \cong \Delta NRM$ (R.H.S.)
7. (a) In ΔABC and ΔEDC, $\angle A = \angle E$ (given), $\angle C = \angle C$ (common),
   BC = DC (given), $\therefore \Delta ABC \cong \Delta EDC$ (A.A.S.), $\therefore$ AB = ED (corr. sides, $\cong \Delta$s)
   (b) $\because \Delta ABC \cong \Delta EDC$ (proved), $\therefore$ AC = EC (corr. sides, $\cong \Delta$s), but BC = DC (given),
   $\therefore$ AC – DC = EC – BC, i.e. AD = EB
8. In ΔABC and ΔDCE, AB = DC (given), $\angle A = \angle D = 90^\circ$ (given),
   $\angle ABC + 90^\circ = \angle BCD = 90^\circ + \angle DCE$ (ext. $\angle$ of Δ), $\therefore \angle ABC = \angle DCE$,
   $\therefore \Delta ABC \cong \Delta DCE$ (A.S.A.), $\therefore$ BC = CE (corr. sides, $\cong \Delta$s), $\therefore \Delta BCE$ is isosceles.

9. (a) In ΔBAE and ΔCDE, BA = CD (given), BE = CE (given), $AE = DE = \frac{1}{2}AD$ (given),
$\therefore \Delta BAE \cong \Delta CDE$ (S.S.S.), $\therefore$ $\angle BAD = \angle CDA$ (corr. $\angle$s, $\cong\Delta$s)
(b) In ΔBAD and ΔCDA, AB = DC (given), AD = DA (common),
$\angle BAD = \angle CDA$ (proved), $\therefore$ $\Delta BAD \cong \Delta CDA$ (S.A.S.),
$\therefore$ BD = CA (corr. sides, $\cong\Delta$s)

10. (a) In ΔCEF and ΔCDH, $\angle C = \angle C$ (common), CF = CH (given),
$\angle CFE = 90^{\circ} = \angle CHD$ (given), $\therefore$ $\Delta CEF \cong \Delta CDH$ (A.S.A.),
$\therefore$ EF = DH (corr. sides, $\cong\Delta$s)
(b) $\because$ $\Delta CEF \cong \Delta CDH$ (proved), $\therefore$ CE = CD (corr. sides, $\cong\Delta$s), $\therefore$ ΔCDE is isosceles.
(c) In ΔFGD and ΔHGE, $\angle DFG = 90^{\circ} = \angle EHG$ (given),
$\angle FDG = \angle HEG$ (corr. $\angle$s, $\cong\Delta$s), DF = DC – FC = EC – HC = EH,
$\therefore$ $\Delta FGD \cong \Delta HGE$ (A.S.A.)

11. In ΔABE and ΔCBD, AB = CB and BE = BD (equilateral Δs),
$\angle ABE = \angle CBD = 60^{\circ}$ (equilateral Δs), $\therefore$ $\Delta ABE \cong \Delta CBD$ (S.A.S.),
$\therefore$ $\angle BAE = \angle BCD$ (corr. $\angle$s, $\cong\Delta$s)

12. In ΔABD and ΔCBE, AB = CB and BD = BE (equilateral Δs),
$\angle ABD = \angle ABC - \angle DBF = 60^{\circ} - \angle DBF$ (equilateral Δ),
$\angle CBE = \angle DBE - \angle DBF = 60^{\circ} - \angle DBF$ (equilateral Δ), $\therefore$ $\angle ABD = \angle CBE$,
$\therefore \Delta ABD \cong \Delta CBE$ (S.A.S.), $\therefore$ AD = CE (corr. sides, $\cong\Delta$s)

13. (a) In ΔABF and ΔCBE, AB = CB and BF = BE (equilateral Δs),
$\angle ABF = \angle ABC - \angle FBC = 60^{\circ} - \angle FBC$ (equilateral Δ),
$\angle CBE = \angle FBE - \angle FBC = 60^{\circ} - \angle FBC$ (equilateral Δ), $\therefore$ $\angle ABF = \angle CBE$,
$\therefore$ $\Delta ABF \cong \Delta CBE$ (S.A.S.), $\therefore$ AF = CE (corr. sides, $\cong\Delta$s)
(b) ΔBCE. In ΔFDE and ΔBCE, DE = CE and FE = BE (equilateral Δs),
$\angle DEF = \angle DEC - \angle FEC = 60^{\circ} - \angle FEC$ (equilateral Δ),
$\angle CEB = \angle FEB - \angle FEC = 60^{\circ} - \angle FEC$ (equilateral Δ),
$\therefore$ $\angle DEF = \angle CEB$, $\therefore$ $\Delta FDE \cong \Delta BCE$ (S.A.S.). $\because$ $\Delta FDE \cong \Delta BCE$ (proved),
$\therefore$ FD = BC (corr. sides, $\cong\Delta$s), but AB = BC (corr. sides, $\cong\Delta$s), $\therefore$ FD = AB

14. (a) In ΔABD and ΔACE, AB = AC and AD = AE (equilateral Δs),
$\angle BAD = \angle CAE = 60^{\circ}$ (equilateral Δs), $\therefore$ $\Delta ABD \cong \Delta ACE$ (S.A.S.),
$\therefore$ $\angle ABD = \angle ACE$ (corr. $\angle$s, $\cong\Delta$s)
(b) In ΔCDF and ΔBDA, $\angle ACE = \angle ABD$ (proved),
$\angle CDF = \angle BDA$ (vert. opp. $\angle$s), $\therefore$ $\angle DFC = \angle DAB$ (the 3rd $\angle$ of Δ).
$\angle DAB = 60^{\circ}$ (equilateral Δ), but $\angle DFC = x + y$ (ext. $\angle$ of Δ),
$\therefore$ $x + y = 60^{\circ}$

15. (a) In ΔACE and ΔDBF, AE = DF and CE = BF (given),
AC = AB + BC and DB = DC + BC, but AB = DC (given), $\therefore$ AC = DB,
$\therefore$ $\Delta ACE \cong \Delta DBF$ (S.S.S.), $\therefore$ $\angle A = \angle D$ (corr. $\angle$s, $\cong\Delta$s),
$\therefore$ AE//DF (alt. $\angle$s eq.)

(b) In ΔBAE and ΔCDF, $\angle A = \angle D$ (proved), AE = DF and
AB = DC (given), $\therefore \Delta BAE \cong \Delta CDF$ (S.A.S.),
$\therefore$ EB = FC (corr. sides, $\cong \Delta$s)

16. (a) $\angle BAE = \angle BAC - \angle CAE$, $\angle ACD = \angle FDC - \angle CAE$ (ext. $\angle$ of Δ),
but $\angle BAC = \angle FDC$ (given), $\therefore$ $\angle BAE = \angle ACD$
(b) In ΔABE and ΔCAD, AB = CA (given), $\angle BAE = \angle ACD$ (proved),
$\angle AEB = 180^o - \angle BEF$ and $\angle CDA = 180^o - \angle FDC$ (adj. $\angle$s on st. line),
but $\angle BEF = \angle FDC$ (given), $\therefore$ $\angle AEB = \angle CDA$,
$\therefore$ $\Delta ABE \cong \Delta CAD$ (A.A.S.), $\therefore$ BE = AD (corr. sides, $\cong \Delta$s)

17. In ΔBCD and ΔACE, BC = AC and CD = CE (equilateral Δs),
$\angle ACB = \angle ECD = 60^o$ (equilateral Δs), $\angle ACD = 60^o - 50^o = 10^o$,
$\therefore$ $\angle ECA = 60^o - 10^o = 50^o = \angle DCB$, $\therefore$ $\Delta BCD \cong \Delta ACE$ (S.A.S.),
$\therefore$ $\theta = \angle CBD$ (corr. $\angle$s, $\cong$ Δs), $\therefore$ $\theta = 180^o - 100^o - 50^o = 30^o$ ($\angle$ sum of Δ)

18. (a) In ΔABP and ΔCBS, PB = SB (given), $\angle APB = \angle CSB = 90^o$ (square),
$\angle ABP = \angle CBS$ (vert. opp. $\angle$s), $\therefore$ $\Delta ABP \cong \Delta CBS$ (A.S.A.),
$\therefore$ AB = CB (corr. sides, $\cong \Delta$s)
(b) In ΔABR and ΔCBR, BR = BR (common), $\angle ABR = \angle CBR$ (given),
AB = CB (proved), $\therefore$ $\Delta ABR \cong \Delta CBR$ (S.A.S.), $\therefore$ AR = CR (corr. sides, $\cong \Delta$s).
$\because$ $\Delta ABR \cong \Delta CBR$ (proved),
$\therefore$ the $\perp$ distance from B to AR = the $\perp$ distance from B to RC = BS $= \frac{1}{2} \times 9$ = 4.5 cm

19. (a) In ΔBEC and ΔDFC, BC = DC (square), $\angle CBE = \angle CDF = 90^o$ (square),
$\angle BCE = 90^o - \angle ECD = \angle DCF$, $\therefore$ $\Delta BEC \cong \Delta DFC$ (A.S.A.)
(b) Let EC = FC = x cm (corr. sides, $\cong \Delta$s), $\frac{1}{2}x^2 = 12.5$, $x^2 = 25$, $x = 5$,
$\therefore$ Area of ABCD = $BC^2 = EC^2 - BE^2 = 5^2 - 1.4^2 = 23.04$ cm^2

20. In ΔQBP and ΔRCQ, $a_1 = a_2$ (given), QB = RC = 6 cm (given),
$\angle BQP + a_3 = a_2 + \angle CRQ$ (ext. $\angle$ of Δ), $\therefore$ $\angle BQP = \angle CRQ$ ($a_3 = a_2$),
$\therefore$ $\Delta QBP \cong \Delta RCQ$ (A.S.A.), $\therefore$ BP = CQ = 3 cm (corr. sides, $\cong \Delta$s)

21. (a) Let $\angle OKH = y$. $\because$ OK = OH (radii), $\therefore$ $\angle OHK = \angle OKH = y$ (base $\angle$s, isos. Δ),
$\therefore$ $\angle HOM = 45^o + y$ (ext. $\angle$ of Δ). $\angle KLN = 45^o$ (vert. opp. $\angle$s);
$\angle LKN = 180^o - 90^o - 45^o = 45^o$ ($\angle$ sum of Δ),
$\therefore$ $\angle OKN = \angle OKH + \angle LKN = y + 45^o$, $\therefore$ $\angle OKN = \angle HOM$
(b) In ΔHOM and ΔOKN, $\angle HMO = \angle ONK = 90^o$ (given),
OH = KO (radii), $\angle HOM = \angle OKN$ (proved),
$\therefore$ $\Delta HOM \cong \Delta OKN$ (A.A.S.), $\therefore$ HM = ON = 4 + 3 = 7 cm (corr. sides, $\cong \Delta$s)

Unit 9 Similar triangles (3)

1. (a) Yes. $\Delta ABC \sim \Delta QRP$ (A.A.A.), $\because$ $\angle A = 180^o - 75^o - 40^o = 65^o$
(b) No. (c) Yes. $\Delta ABC \sim \Delta PRQ$ (3 sides prop.)

(d) Yes. $\Delta ABC \sim \Delta RQP$ (ratio of 2 sides, inc. $\angle$)

(e) No. ($\because$ sides not prop.) (f) Yes. $\Delta ABC \sim \Delta PRQ$ (A.A.A.)

2. (a) $\because$ $\Delta ABC \sim \Delta FED$ (A.A.A.), $\therefore$ $\frac{a}{12} = \frac{b}{5} = \frac{19.5}{13}$ (corr. sides, $\sim\Delta$s),

$\therefore$ $a = \frac{19.5}{13} \times 12 = 18$ and $b = \frac{19.5}{13} \times 5 = 7.5$

(b) $\because$ $\Delta ABC \sim \Delta EDF$ (A.A.A.), $\therefore$ $\frac{p}{18} = \frac{q}{24} = \frac{9}{12} = \frac{3}{4}$ (corr. sides, $\sim\Delta$s),

$\therefore$ $p = \frac{3}{4} \times 18 = 13.5$ and $q = \frac{3}{4} \times 24 = 18$

3. (a) $\because$ $\Delta ABC \sim \Delta DEC$ (A.A.A.), $\therefore$ $\frac{x}{5} = \frac{8+4}{8} = \frac{3}{2}$ (corr. sides, $\sim\Delta$s),

$\therefore$ $x = \frac{3}{2} \times 5 = 7.5$

(b) $\because$ $\Delta FHK \sim \Delta FGE$ (A.A.A.), $\therefore$ $\frac{6}{m+6} = \frac{16}{28} = \frac{4}{7}$ (corr. sides, $\sim\Delta$s),

$42 = 4m + 24$, $18 = 4m$, $\therefore$ $m = 4.5$

(c) $\because$ $\Delta ADE \sim \Delta ABC$ (A.A.A.), $\therefore$ $\frac{y}{y+4} = \frac{10}{15} = \frac{2}{3}$ (corr. sides, $\sim\Delta$s),

$3y = 2y + 8$, $\therefore$ $y = 8$

(d) $\because$ $\Delta QST \sim \Delta QPR$ (A.A.A.), $\therefore$ $\frac{a}{a+14} = \frac{9}{9+18} = \frac{1}{3}$ (corr. sides, $\sim\Delta$s),

$3a = a + 14$, $\therefore$ $a = 7$

4. (a) $\because$ $\Delta ABC \sim \Delta EDC$ (A.A.A.), $\therefore$ $\frac{x}{6} = \frac{6}{y} = \frac{3}{4}$ (corr. sides, $\sim\Delta$s),

$\therefore$ $x = \frac{3}{4} \times 6 = 4.5$ and $y = 6 \times \frac{4}{3} = 8$

(b) $\because$ $\Delta PQR \sim \Delta TSR$ (A.A.A.), $\therefore$ $\frac{m}{12} = \frac{n}{10} = \frac{3}{15} = \frac{1}{5}$ (corr. sides, $\sim\Delta$s),

$\therefore$ $m = \frac{1}{5} \times 12 = 2.4$ and $n = \frac{1}{5} \times 10 = 2$

5. (a) $\because$ $\Delta BDE \sim \Delta BAC$ (A.A.A.), $\therefore$ $\frac{10}{10+k} = \frac{8}{12} = \frac{2}{3}$ (corr. sides, $\sim\Delta$s),

$30 = 20 + 2k$, $10 = 2k$, $\therefore$ $k = 5$

(b) $\because$ $\Delta FPQ \sim \Delta FGH$ (A.A.A.), $\therefore$ $\frac{m}{27} = \frac{n}{20+n} = \frac{3}{3+6} = \frac{1}{3}$ (corr. sides, $\sim\Delta$s),

$\therefore$ $m = \frac{1}{3} \times 27 = 9$; $\frac{n}{20+n} = \frac{1}{3}$, $3n = 20 + n$, $\therefore$ $n = 10$

6. (a) $\because$ $\Delta ADE \sim \Delta ACB$ (A.A.A.), $\therefore$ $\frac{3}{4+m} = \frac{4}{3+13} = \frac{1}{4}$ (corr. sides, $\sim\Delta$s),

$12 = 4 + m, \quad \therefore\ m = 8$

(b) $\because\ \Delta MNR \sim \Delta QPR$ (A.A.A.), $\quad \therefore\ \dfrac{n}{35} = \dfrac{6}{15} = \dfrac{2}{5}$ (corr. sides, $\sim\Delta$s),

$5n = 70, \quad \therefore\ n = 14$

(c) $\because\ \Delta ABC \sim \Delta EBD$ (A.A.A.), $\quad \therefore\ \dfrac{x}{12} = \dfrac{27}{18} = \dfrac{3}{2}$ (corr. sides, $\sim\Delta$s),

$2x = 36, \quad \therefore\ x = 18$

(d) $\because\ \Delta PQR \sim \Delta TQS$ (A.A.A.), $\quad \therefore\ \dfrac{x+4}{20} = \dfrac{20+y}{x} = \dfrac{17}{10}$ (corr. sides, $\sim\Delta$s),

$10x + 40 = 340, \ \therefore\ x = 30;\ 200 + 10y = 17(30), \quad 10y = 310, \quad \therefore\ y = 31$

7. (a) $\because\ \Delta ABC \sim \Delta APQ$ (A.A.A.), $\quad \therefore\ \dfrac{6}{6+x} = \dfrac{8}{10} = \dfrac{4}{5}$ (corr. sides, $\sim\Delta$s),

$30 = 24 + 4x, \quad 6 = 4x, \quad \therefore\ x = 1.5$

$\because\ \Delta ACD \sim \Delta AQR$ (A.A.A.), $\quad \therefore\ \dfrac{3}{y} = \dfrac{6}{6+x} = \dfrac{6}{6+1.5}$ (corr. sides, $\sim\Delta$s),

$\dfrac{3}{y} = \dfrac{6}{7.5},\ 22.5 = 6y, \ \therefore\ y = 3.75$

(b) $\because\ \Delta NDE \sim \Delta NLP$ (A.A.A.), $\therefore\ \dfrac{m}{m+6} = \dfrac{5}{8}$ (corr. sides, $\sim\Delta$s),

$8m = 5m + 30, \ \therefore\ m = 10.$

$\because\ \Delta LCD \sim \Delta LMN$ (A.A.A.), $\therefore\ \dfrac{n}{12} = \dfrac{6}{6+m} = \dfrac{6}{6+10}$ (corr. sides, $\sim\Delta$s),

$\dfrac{n}{12} = \dfrac{6}{16} = \dfrac{3}{8}, \quad \therefore\ n = \dfrac{3}{8}\times 12 = 4.5$

(c) $\because\ \Delta HEF \sim \Delta HCD$ (A.A.A.), $\therefore\ \dfrac{y}{y+3} = \dfrac{z}{5} = \dfrac{8}{8+4} = \dfrac{2}{3}$ (corr. sides, $\sim\Delta$s),

$\dfrac{y}{y+3} = \dfrac{2}{3},\ 3y = 2y + 6, \quad \therefore\ y = 6;\ \dfrac{z}{5} = \dfrac{2}{3}, \quad \therefore\ z = \dfrac{2}{3}\times 5 = \dfrac{10}{3}$

$\because \Delta DEF \sim \Delta DGH$ (A.A.A.), $\quad \therefore\ \dfrac{z}{x} = \dfrac{3}{3+y} = \dfrac{3}{3+6}$ (corr. sides, $\sim\Delta$s),

$\dfrac{z}{x} = \dfrac{1}{3}, \quad \therefore\ x = 3z = 3\times\dfrac{10}{3} = 10$

8. (a) In ΔABC and ΔECD, $\angle A = \angle E = 90^\circ$ (given),
$\angle B + 90^\circ = \angle BCE = 90^\circ + \angle ECD$ (ext. $\angle$ of Δ), $\quad \therefore\ \angle B = \angle ECD$,
$\angle ACB = \angle D$ ($3^{rd}\angle$ of Δ), $\quad \therefore\ \Delta ABC \sim \Delta ECD$ (A.A.A.)

(b) $\because\ \Delta ABC \sim \Delta ECD$ (proved), $\quad \therefore\ \dfrac{a}{10} = \dfrac{b}{24} = \dfrac{20}{26} = \dfrac{10}{13}$ (corr. sides, $\sim\Delta$s),

$\dfrac{a}{10} = \dfrac{10}{13}, \quad \therefore\ a = \dfrac{10}{13}\times 10 = \dfrac{100}{13} = 7\dfrac{9}{13};\quad \dfrac{b}{24} = \dfrac{10}{13}, \quad \therefore\ b = \dfrac{10}{13}\times 24 = \dfrac{240}{13} = 18\dfrac{6}{13}$

9. (a) In ΔMNX and ΔYZX, $\angle MXN = \angle YXZ$ (vert. opp. $\angle$s),

$\frac{MX}{YX}=\frac{10}{6}=\frac{5}{3}$, $\frac{NX}{ZX}=\frac{15}{9}=\frac{5}{3}$, $\therefore$ $\frac{MX}{YX}=\frac{NX}{ZX}$,

$\therefore$ ΔMNX～ΔYZX (ratio of 2 sides, inc.∠)

(b) $\because$ ΔMNX～ΔYZX (proved), $\therefore$ $\frac{y}{8}=\frac{5}{3}$ (corr. sides, ～Δs),

$\therefore$ $y=\frac{5}{3}\times 8=\frac{40}{3}=13\frac{1}{3}$

10. (a) In ΔABC and ΔBDC, ∠A = ∠DBC (given), ∠C = ∠C (common),
∠ABC = ∠BDC (3^{rd} ∠ of Δ), $\therefore$ ΔABC～ΔBDC (A.A.A.),

$\therefore$ $\frac{x+6}{14}=\frac{14}{6}=\frac{7}{3}$ (corr. sides, ～Δs), $3x + 18 = 98$, $3x = 80$, $\therefore$ $x=\frac{80}{3}=26\frac{2}{3}$

(b) In ΔPQR and ΔPSQ, ∠PQR = ∠S (given), ∠P = ∠P (common),
∠PRQ = ∠PQS (3^{rd} ∠ of Δ), $\therefore$ ΔPQR～ΔPSQ (A.A.A.)

$\therefore$ $\frac{r}{6}=\frac{6}{r+k}=\frac{14}{21}=\frac{2}{3}$ (corr. sides, ～Δs), $\therefore$ $r=\frac{2}{3}\times 6=4$;

$\frac{6}{r+k}=\frac{6}{4+k}=\frac{2}{3}$, $18 = 8 + 2k$, $\therefore$ $k = 5$

11. $\because$ ΔMNT～ΔRQT (A.A.A.), $\therefore$ $\frac{QR}{7}=\frac{6}{4}=\frac{3}{2}$ (corr. sides, ～Δs), $QR=\frac{3}{2}\times 7=\frac{21}{2}$

$\because$ ΔPMN～ΔPQR (A.A.A.), $\therefore$ $\frac{5}{5+MQ}=\frac{7}{QR}$ (corr. sides, ～Δs),

$\frac{5}{5+MQ}=7\div\frac{21}{2}=\frac{2}{3}$, $15=10+2MQ$, $\therefore$ $MQ = 2.5$

12. (a) In ΔBAD and ΔCDB, ∠A = ∠BDC (given), $\frac{BA}{CD}=\frac{6}{12}=\frac{1}{2}$, $\frac{AD}{DB}=\frac{4}{8}=\frac{1}{2}$,

$\therefore$ $\frac{BA}{CD}=\frac{AD}{DB}$, $\therefore$ ΔBAD～ΔCDB (ratio of 2 sides, inc.∠)

(b) $\because$ ΔBAD～ΔCDB (proved), $\therefore$ $\frac{8}{BC}=\frac{1}{2}$ (corr. sides, ～Δs), $\therefore$ $BC = 16$

13. In ΔABC and ΔCDB, $\frac{AB}{CD}=\frac{14}{21}=\frac{2}{3}$, $\frac{BC}{DB}=\frac{12}{18}=\frac{2}{3}$, $\frac{AC}{CB}=\frac{8}{12}=\frac{2}{3}$,

$\therefore$ $\frac{AC}{CD}=\frac{BC}{DB}=\frac{AC}{CB}$, $\therefore$ ΔABC～ΔCDB (3 sides prop.),

$\therefore$ ∠ACB = ∠CBD (corr. ∠s, ～Δs), $\therefore$ AC//BD (alt. ∠s eq.)

14. (a) In ΔADB and ΔBDC, ∠ADB = ∠BDC = 90° (given); ∠ABD = 90° − ∠CBD,
∠BCD = 180° − 90° − ∠CBD = 90° − ∠CBD (∠ sum of Δ), $\therefore$ ∠ABD = ∠BCD;
∠A = ∠CBD (3^{rd} ∠ of Δ); $\therefore$ ΔADB～ΔBDC (A.A.A.)

(b) $\because$ ΔADB～ΔBDC (proved), $\therefore$ $\frac{x}{9}=\frac{4}{x}$ (corr. sides, ～Δs), $x^2=36$, $\therefore x=\sqrt{36}=6$

15. (a) ΔABC～ΔACD～ΔCBD (A.A.A.)

(b) $AD = \sqrt{15^2 - 12^2} = \sqrt{81} = 9$ cm (Pyth. Thm.)

$\because \Delta ACD \sim \Delta CBD$, $\therefore \dfrac{BD}{12} = \dfrac{12}{9} = \dfrac{4}{3}$ (corr. sides, $\sim\Delta$s),

$\therefore BD = \dfrac{4}{3} \times 12 = 16$ cm, $BC = \sqrt{16^2 + 12^2} = \sqrt{400} = 20$ cm (Pyth. Thm.)

16. (a) In ΔECB and ΔDCA, $\angle BEC = \angle ADC = 90^o$ (given),
$\angle C = \angle C$ (common), $\angle EBC = \angle DAC$ ($3^{rd} \angle$ of Δ),
$\therefore \Delta ECB \sim \Delta DCA$ (A.A.A.)

(b) $\because \Delta ECB \sim \Delta DCA$ (proved), $\therefore \dfrac{k}{18} = \dfrac{6+18}{30} = \dfrac{4}{5}$ (corr. sides, $\sim\Delta$s),

$\therefore k = \dfrac{4}{5} \times 18 = 14.4$

17. Join BE. $\because \Delta ECG \sim \Delta EAB$ (A.A.A.), $\therefore \dfrac{CG}{8} = \dfrac{3}{3+6} = \dfrac{1}{3}$ (corr. sides, $\sim\Delta$s),

$\therefore CG = \dfrac{1}{3} \times 8 = \dfrac{8}{3}$. $\dfrac{EG}{EG+GB} = \dfrac{1}{3}$, 3EG = EG + GB, 2EG = GB,

$\because \Delta BDG \sim \Delta BFE$ (A.A.A.), $\therefore \dfrac{DG}{6} = \dfrac{BG}{BG+EG} = \dfrac{2EG}{3EG} = \dfrac{2}{3}$ (corr. sides, $\sim\Delta$s),

$\therefore DG = \dfrac{2}{3} \times 6 = 4$, $\therefore CD = \dfrac{8}{3} + 4 = 6\dfrac{2}{3}$

18. $AC = \sqrt{(24+5)^2 - 20^2} = \sqrt{441} = 21$ (Pyth. Thm.)

In ΔBDE and ΔBCA, $\angle B = \angle B$ (common), $\angle BDE = \angle BCA = 90^o$ (given),
$\angle BED = \angle A$ ($3^{rd} \angle$ of Δ), $\therefore \Delta BDE \sim \Delta BCA$ (A.A.A.),
$\therefore \dfrac{DE}{21} = \dfrac{5}{20} = \dfrac{1}{4}$ (corr. sides, $\sim\Delta$s), $\therefore DE = \dfrac{1}{4} \times 21 = 5.25$

19. In ΔADE and ΔACB, $\angle A = \angle A$ (common), $\angle ADE = \angle ACB = 90^o$ (given),
$\angle AED = \angle ABC$ ($3^{rd} \angle$ of Δ), $\therefore \Delta ADE \sim \Delta ACB$ (A.A.A.),
$\therefore \dfrac{AE}{25} = \dfrac{DE}{7}$ (corr. sides, $\sim\Delta$s), $\therefore DE = \dfrac{7}{25} AE$;

$\because AC = \sqrt{25^2 - 7^2} = \sqrt{576} = 24$ (Pyth. Thm.), $\therefore$ AE + EC = 24,

AE + DE = 24, $AE + \dfrac{7}{25} AE = 24$, $\dfrac{32}{25} AE = 24$, $\therefore AE = 24 \times \dfrac{25}{32} = 18.75$

20. In ΔABC and ΔBDC, $\angle A = \angle DBC$ (given), $\angle C = \angle C$ (common),
$\angle ABC = \angle BDC$ ($3^{rd} \angle$ of Δ), $\therefore \Delta ABC \sim \Delta BDC$ (A.A.A.),
$\therefore \dfrac{BC}{DC} = \dfrac{AC}{BC}$ (corr. sides, $\sim\Delta$s), $\dfrac{15}{12} = \dfrac{12+AD}{15}$, 225 = 144 + 12AD, $\therefore$ AD = 6.75

21. In ΔBDC and ΔAEC, $\angle C = \angle C$ (common), $\angle BDC = \angle AEC = 90^o$ (given),

$\angle DBC = \angle EAC$ ($3^{rd}\angle$ of Δ), $\therefore$ $\Delta BDC \sim \Delta AEC$ (A.A.A.),

$\therefore$ $\frac{DC}{18} = \frac{6+18}{36} = \frac{2}{3}$ (corr. sides, $\sim\Delta$s), $\therefore$ $DC = \frac{2}{3} \times 18 = 12$ cm,

$\therefore$ $AD = 36 - 12 = 24$ cm. $BD^2 = BC^2 - DC^2 = (18+6)^2 - 12^2 = 432$,

$\therefore$ $AB = \sqrt{BD^2 + AD^2} = \sqrt{432 + 24^2} = \sqrt{1008} = 12\sqrt{7}$ cm

22. (a) $\because$ $\Delta ADE \sim \Delta AFG$ (A.A.A.), $\therefore$ $\frac{DE}{FG} = \frac{5+5}{5+5+5} = \frac{2}{3}$ (corr. sides, $\sim\Delta$s),

$\because$ $\Delta BDH \sim \Delta BFG$ (A.A.A.), $\frac{DH}{FG} = \frac{5}{5+5} = \frac{1}{2}$ (corr. sides, $\sim\Delta$s),

$\therefore$ $\frac{DH}{DE} = \frac{DH}{FG} \div \frac{DE}{FG} = \frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$, $\therefore$ $DH : DE = 3 : 4$

(b) $\frac{DH}{DE} = \frac{3}{4}$ (proved), $\frac{DH}{24} = \frac{3}{4}$, $DH = \frac{3}{4} \times 24 = 18$, $\therefore$ $HE = 24 - 18 = 6$

23. $\because$ $\Delta BFG \sim \Delta BAC$ (A.A.A.), $\therefore$ $\frac{42}{BA} = \frac{4+3}{4+3+1} = \frac{7}{8}$ (corr. sides, $\sim\Delta$s), $336 = 7BA$,

$BA = 48$; $\because$ $\Delta CDE \sim \Delta CBA$ (A.A.A.),

$\therefore$ $\frac{DE}{48} = \frac{3+1}{4+3+1} = \frac{1}{2}$ (corr. sides, $\sim\Delta$s), $\therefore$ $DE = \frac{1}{2} \times 48 = 24$

24. $\because$ $\Delta ABE \sim \Delta ACF \sim \Delta ADG$ (A.A.A.), $\therefore$ $\frac{6}{6+EF+3} = \frac{3}{5}$ (corr. sides, $\sim\Delta$s),

$30 = 27 + 3EF$, $3 = 3EF$, $EF = 1$, also $\frac{6}{6+EF} = \frac{3}{CF}$ (corr. sides, $\sim\Delta$s),

$\frac{3}{CF} = \frac{6}{6+1} = \frac{6}{7}$, $21 = 6CF$, $\therefore$ $CF = \frac{21}{6} = 3.5$

25. (a) $\because$ $\Delta PQR \sim \Delta PST$ (A.A.A.), $\therefore$ $\frac{QR}{ST} = \frac{5}{5+3} = \frac{5}{8}$ (corr. sides, $\sim\Delta$s),

$\therefore$ $QR : ST = 5 : 8$

(b) $\because$ $\Delta PQR \sim \Delta PST$, $\therefore$ $\frac{\text{Height of } \Delta PQR}{\text{Height of } \Delta PST} = \frac{h_1}{h_2} = \frac{5}{8}$,

$\therefore$ $\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta PST} = \frac{5^2}{8^2} = \frac{25}{64}$, Area of $\Delta PST = 100 \times \frac{64}{25} = 256$,

$\therefore$ Area of QRTS $= 256 - 100 = 156$ cm^2

26. In ΔCDF and ΔGEF, $\angle CFD = \angle GFE$ (vert. opp. $\angle$s), $\angle CDF = \angle GEF$ and $\angle DCF = \angle EGF$ (alt. $\angle$s, AD//EG), $\therefore$ $\Delta CDF \sim \Delta GEF$ (A.A.A.),

$\therefore$ $\frac{EG}{CD} = \frac{3}{4}$ (corr. sides, $\sim\Delta$s), $EG = \frac{3}{4}CD$; $\because$ $\Delta BEG \sim \Delta BAC$ (A.A.A.),

$\therefore$ $\frac{BE}{AB} = \frac{EG}{AC}$ (corr. sides, $\sim\Delta$s), but $\frac{AC}{CD} = \frac{3}{2}$, $AC = \frac{3}{2}CD$,

$\therefore\ \frac{BE}{AB}=\frac{3}{4}CD\div\frac{3}{2}CD=\frac{1}{2},\ \ \frac{BE}{AE+BE}=\frac{1}{2},\ \ 2BE=AE+BE,\ \ BE=AE,$

$\frac{AE}{BE}=1,\ \ \therefore\ AE:BE=1:1$

27. In ΔBCE and ΔCHF, $\angle BEC=\angle CFH=90^o$ (square),
$\angle BCE=\angle CHF$ (corr. sides, CD//HG), $\angle CBE=\angle HCF$ ($3^{rd}\angle$ of Δ),
$\therefore\ \Delta BCE\sim\Delta CHF$ (A.A.A.), $\therefore\ \frac{HF}{3-1}=\frac{3}{1}=3$ (corr. sides, $\sim\Delta$s), HF = 6,
$\therefore$ GJ = HG = 6 + 3 = 9 cm

28. In ΔABC and ΔADB, $\angle A=\angle A$ (common), $\frac{AB}{AD}=\frac{18}{12}=\frac{3}{2}$,
$\frac{AC}{AB}=\frac{15+12}{18}=\frac{3}{2},\ \ \therefore\ \frac{AB}{AD}=\frac{AC}{AB}$, $\therefore\ \Delta ABC\sim\Delta ADB$ (ratio of 2 sides, inc. $\angle$),
$\therefore\ \frac{x}{10}=\frac{3}{2}$ (corr. sides, $\sim\Delta$s), $\therefore\ x=\frac{3}{2}\times 10=15$

29. $AC=EG=\sqrt{1^2+1^2}=\sqrt{2}$ (Pyth. Thm.)
In ΔDEG and ΔACG, $\angle DEG=\angle ACG=90^o+45^o=135^o$ (square),
$\frac{DE}{AC}=\frac{1}{\sqrt{2}},\ \ \frac{EG}{CG}=\frac{\sqrt{2}}{1+1}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}},\ \ \frac{DE}{AC}=\frac{EG}{CG},$
$\therefore\ \Delta DEG\sim\Delta ACG$ (ratio of 2 sides, inc. $\angle$)

30. $\because\ \Delta FCD\sim\Delta FAB$ (A.A.A.), $\therefore\ \frac{FD}{FB}=\frac{CD}{4}$ (corr. sides, $\sim\Delta$s) … (1)
$\because\ \Delta BCD\sim\Delta BEF$ (A.A.A.), $\therefore\ \frac{DB}{FB}=\frac{CD}{9}$ (corr. sides, $\sim\Delta$s) … (2)
(1) + (2), $\frac{FD+DB}{FB}=\frac{CD}{4}+\frac{CD}{9},\ \ \frac{FB}{FB}=\frac{13CD}{36},\ \ \therefore\ CD=\frac{36}{13}=2\frac{10}{13}$

31. $\because\ \Delta ECD\sim\Delta EAB$ (A.A.A.), $\therefore\ \frac{r}{p}=\frac{CE}{AE}$ (corr. sides, $\sim\Delta$s),
$\because\ \Delta ACD\sim\Delta AEF$ (A.A.A.), $\therefore\ \frac{r}{q}=\frac{AC}{AE}$ (corr. sides, $\sim\Delta$s),
$\frac{r}{q}=\frac{AE-CE}{AE}=1-\frac{CE}{AE}=1-\frac{r}{p},\ \ \frac{r}{p}+\frac{r}{q}=1,\ \ \therefore\ \frac{1}{p}+\frac{1}{q}=\frac{1}{r}$

Unit 10 Isosceles triangles

1. (a) $y=\frac{180^\circ-90^\circ}{2}=45^\circ$ (b) $2(2y-8)^\circ+3y^\circ=180^\circ,\ 7y^\circ=196^\circ,\ \therefore\ y=28$
(c) $(180^\circ-2y^\circ)\times 2+y^\circ=180^\circ,\ 360^\circ-4y^\circ+y^\circ=180^\circ,\ \therefore\ y=60$
(d) $(y^\circ+y^\circ)\times 2+4y^\circ=180^\circ,\ 8y^\circ=180^\circ,\ \therefore\ y=22.5$
(e) $2(y+5)^\circ=(3y-18)^\circ,\ 2y+10=3y-18,\ \therefore\ y=28$

(f) $180^\circ - 2(2y^\circ) = 5y^\circ$, $9y^\circ = 180^\circ$, $\therefore\ y = 20$

2. (a) The third angle = $180^o - 32^o - 74^o = 74^o$ ($\angle$sum of Δ),
$\therefore\ 4y - 21 = y + 12$ (sides opp. eq.$\angle$s), $3y = 33$, $\therefore\ y = 11$
(b) $\angle R = \angle S = y$ (base$\angle$s, isos.Δ); $y + y = 72^o$ (ext.$\angle$of Δ), $2y = 72^o$, $\therefore\ y = 36^o$
(c) $\angle EGF = 180^o - 120^o = 60^o$ (adj.$\angle$s on st. line);
$m + 60^o + 60^o = 180^o$ ($\angle$ sum of isos. Δ), $\therefore\ m = 60^o$
(d) $\angle A = \angle ABD = 35^o$ (base$\angle$s, isos. Δ); $\angle DBC = n$ (base$\angle$s, isos. Δ);
$n + n + 35^o + 35^o = 180^o$ ($\angle$ sum of Δ), $2n = 110^o$, $\therefore\ n = 55^o$
(e) $\angle CDE = 76^o$ (alt.$\angle$s, AB // CD); $\angle ABE = 115^o - 76^o = 39^o$ (ext.$\angle$of Δ);
$\angle C = \angle ABE = 39^o$ (alt.$\angle$s,, AB//CD), $\therefore\ x = \angle C = 39^o$ (base$\angle$s, isos. Δ)
(f) $\angle LMN = 180^o - 90^o - 63^o = 27^o$ ($\angle$ sum of Δ);
$\angle KLN = 27^o$ (alt.$\angle$s, LK//NM); $\angle KML = \angle KLN = 27^o$ (base$\angle$s, isos. Δ);
$b + 27^o + 27^o = 180^o$ ($\angle$ sum of Δ), $\therefore\ b = 126^o$
(g) $\angle BCA = \angle A = 50^o$ (base$\angle$s, isos. Δ); $\angle CBD = x$ (base$\angle$s, isos. Δ);
$x + x = 50^o$ (ext.$\angle$of Δ), $2x = 50^o$, $\therefore\ x = 25^o$
(h) $\angle CEB = 180^o - 25^o - 85^o = 70^o$ (adj.$\angle$s on st. line);
$\angle CBE = \angle CEB = 70^o$ (base$\angle$s, isos. Δ); $y + 70^o = 85^o$ (alt.$\angle$s, AB // DF),
$\therefore\ y = 15^o$
(i) $\angle S = \angle SQR$ (base$\angle$s, isos. Δ); $2\angle SQR + 46^o = 180^o$ ($\angle$sum of Δ),
$\angle SQR = 67^o$; $\angle QRP = \angle SQR = 67^o$ (alt.$\angle$s, SQ//RP);
$\because$ PR = PQ (given), $\therefore\ \angle QRP = \angle PQR = 67^o$;
$a + 67^o + 67^o = 180^o$ ($\angle$ sum of Δ), $\therefore\ a = 46^o$

3. $\angle C = \angle ABC$ (base$\angle$s, isos. Δ); $\therefore\ 2\angle C + 38^o = 180^o$ ($\angle$ sum of Δ),
$\angle C = 71^o$; $\angle BDC = \angle C = 71^o$ (base$\angle$s, isos. Δ);
$\therefore\ \angle ABD = 71^o - 38^o = 33^o$ (ext.$\angle$of Δ)

4. $\angle R = \angle PQR$ (base$\angle$s, isos. Δ); $\therefore\ 2\angle R + y = 180^o$ ($\angle$ sum of Δ),
$R = 90^o - \frac{y}{2}$; $x + \angle R = 90^o$ (ext.$\angle$of Δ), $x = 90^o - (90^o - \frac{y}{2}) = \frac{y}{2}$, $\therefore\ y = 2x$

5. $\angle CBD = a$ (base$\angle$s, isos. Δ); $\angle A = \angle ACB$ (base$\angle$s, isos. Δ);
$\angle A + \angle ACB = a$ (ext.$\angle$of Δ), $2\angle A = a$, $\angle A = \frac{a}{2}$;
$\therefore\ b = \angle D + \angle A = a + \frac{a}{2} = \frac{3a}{2}$ (ext.$\angle$of Δ)

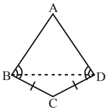
6. (a) In ΔACD, $\angle D = x$ (base$\angle$s, isos. Δ);
In ΔBCD, $\angle D = y$ (base$\angle$s, isos. Δ); $\therefore\ x = y$.
(b) $\angle ABC = \angle D + y = 2x$ (ext.$\angle$of Δ); $\angle ACB = \angle ABC = 2x$ (base$\angle$s, isos. Δ);
$x + 2x + 2x = 180^o$ ($\angle$ sum of Δ), $5x = 180^o$, $\therefore\ x = 36^o$

7. In ΔABC, $\angle BAC = \theta$ (base$\angle$s, isos. Δ), $\therefore\ \angle B = 180^o - 2\theta$ ($\angle$ sum of Δ).
In ΔADC, $\angle ADC = \theta$ (base$\angle$s, isos. Δ), $\therefore\ \angle DAB = \angle B = 180^o - 2\theta$.
$\angle B + \angle DAB = \angle ADC$ (ext.$\angle$of Δ), $2(180^o - 2\theta) = \theta$, $5\theta = 360^o$, $\therefore\ \theta = 72^o$

8. $\angle ABC = 18^\circ$ (base $\angle$s, isos. Δ); $\angle BCD = 18^\circ + 18^\circ = 36^\circ$ (ext. $\angle$ of Δ),
$\therefore$ $\angle BDC = 36^\circ$ (base $\angle$s, isos. Δ); $\angle EBD = 36^\circ + 18^\circ = 54^\circ$ (ext. $\angle$ of Δ).
$\because$ BD = DE (given), $\therefore$ $\angle BED = 54^\circ$ (base $\angle$s, isos. Δ);
$\angle EDF = 54^\circ + 18^\circ = 72^\circ$ (ext. $\angle$ of Δ); $\because$ DE = EF (given),
$\therefore$ $\angle EFD = 72^\circ$ (base $\angle$s, isos. Δ), $\therefore$ $\theta = 72^\circ + 18^\circ = 90^\circ$ (ext. $\angle$ of Δ)

9. $\angle DBC = 20^\circ + 28^\circ = 48^\circ$ (ext. $\angle$ of Δ); $\angle C = 180^\circ - 28^\circ - 20^\circ - 84^\circ = 48^\circ$ ($\angle$ sum of Δ).
$\because$ $\angle DBC = \angle C = 48^\circ$, $\therefore$ DB = DC (sides opp. eq. $\angle$s), $\therefore$ ΔBCD is isosceles.

10. Join BD. $\angle CBD = \angle CDB$ (base $\angle$s, isos. Δ), but $\angle ABC = \angle ADC$ (given),
$\therefore$ $\angle ABC - \angle CBD = \angle ADC - \angle CDB$, i.e. $\angle ABD = \angle ADB$,
$\therefore$ AB = AD (sides opp. eq. $\angle$s)



11. $\angle XYZ = \angle XZY$ (base $\angle$s, isos. Δ), but $\angle XYA = \angle AYZ$ and
$\angle XZA = \angle AZY$ (given), $\therefore$ $2\angle AYZ = 2\angle AZY$, $\angle AYZ = \angle AZY$,
$\therefore$ AY = AZ (sides opp. eq. $\angle$s), $\therefore$ ΔAYZ is also isosceles.

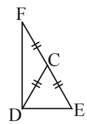
12. In ΔABD and ΔFEC, AB = FE (given), $\angle B = \angle E$ (given),
BD = BC + CD = DE + CD = EC ($\because$ BC = DE), $\therefore$ $\Delta ABD \cong \Delta FEC$ (S.A.S.),
$\therefore$ $\angle GCD = \angle GDC$ (corr. $\angle$s, $\cong$ Δs), $\therefore$ GC = GD (sides opp. eq. $\angle$s),
$\therefore$ ΔCDG is isosceles.

13. (a) In ΔABC and ΔCDA, AC = CA (common), $\angle ACB = a_2 = a_1 = \angle CAD$ (given),
$\angle CAB = a_1 + b_2 = a_2 + b_2 = \angle ACD$ (given), $\therefore$ $\Delta ABC \cong \Delta CDA$ (A.S.A.)
(b) $\because$ $\Delta ABC \cong \Delta CDA$ (proved), $\therefore$ BC = DA (corr. sides, $\cong$ Δs);
$\because$ $a_1 = a_2$ (given), $\therefore$ EA = EC (sides opp. eq. $\angle$s);
BE = BC – EC = DA – EA = DE, $\therefore \Delta BDE$ is isosceles.

14. $\angle BPQ = \angle BQP$ (base $\angle$s, isos. Δ); $\angle BPQ = x + z$ (ext. $\angle$ of Δ),
$\therefore$ $\angle BQP = \angle BQP = x + z$ (base $\angle$s, isos. Δ); $\angle CQR = y - z$ (ext. $\angle$ of Δ);
$\because$ $\angle BQP = \angle CQR$ (vert. opp. $\angle$s), $\therefore$ $x + z = y - z$, $2z = y - x$, $\therefore$ $z = \frac{1}{2}(y - x)$

15. $\because$ QR = RS (given), $\therefore$ $\angle RSQ = 40^\circ$ (base $\angle$s, isos. Δ);
$\angle SRQ = 180^\circ - 40^\circ - 40^\circ = 100^\circ$ ($\angle$ sum of Δ). $\angle PQR = 60^\circ$ (equi. Δ),
$\angle PRS = \angle PRQ + \angle SRQ = 60^\circ + 100^\circ = 160^\circ$. $\because$ PR = RS (given),
$\therefore$ $\angle PSR = \theta$ (base $\angle$s, isos. Δ), $2\theta + 160^\circ = 180^\circ$ ($\angle$ sum of Δ), $2\theta = 20^\circ$, $\therefore$ $\theta = 10^\circ$

16. $\because$ CD = CE (given), $\therefore$ $\angle CDE = \angle E$ (base $\angle$s, isos. Δ);
$\because$ CD = CF (given), $\therefore$ $\angle CDF = \angle F$ (base $\angle$s, isos. Δ);
$\angle CDE + \angle E + \angle CDF + \angle F = 180^\circ$ ($\angle$ sum of Δ), $2\angle CDE + 2\angle CDF = 180^\circ$,
$\angle CDE + \angle CDF = 90^\circ$, $\therefore$ $\angle FDE = 90^\circ$



17. $\angle A = \angle C$ (base $\angle$s, isos. Δ); $\angle BQC = \theta + \angle A$ (ext. $\angle$ of Δ);
$\angle BQC + \angle C = 3\theta$ (ext. $\angle$ of Δ); $\therefore$ $\theta + \angle A + \angle A = 3\theta$, $2\angle A = 2\theta$, $\angle A = \theta$,
$\therefore$ ΔAQB is isosceles (sides opp. eq. $\angle$s).

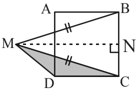
18. (a) In ΔABC, $\angle C = 2a$ (base $\angle$s, isos. Δ), $\therefore$ $2a + 2a + 100^\circ = 180^\circ$ ($\angle$ sum of Δ),
$4a = 80^\circ$, $a = 20^\circ$. $2a + a + 2b = 180^\circ$ (ext. $\angle$ of Δ), $100^\circ + a = 2b$, $2b = 120^\circ$,
$b = 60^\circ$. $\therefore$ $\angle DEA = 2a + b = 2(20^\circ) + 60^\circ = 100^\circ$ (ext. $\angle$ of Δ)
(b) In ΔABD and ΔAED, AD = AD (common), $\angle BAD = \angle DAE$ (given),

$\angle DEA = \angle B = 100^o$ (proved), $\therefore$ $\Delta ABD \cong \Delta AED$ (A.A.S.),
$\therefore$ DB = DE (corr.. sides, $\cong$ Δs)

19. (a) $\because$ QP = QR (given), $\therefore$ $\angle P = \angle QRP$ (base $\angle$s, isos. Δ);
$\because$ AP = AB (given), $\therefore$ $\angle P = \angle ABP$ (base $\angle$s, isos. Δ),
$\therefore$ AB//QD (corr. $\angle$s eq.)
(b) In ΔABC and ΔDRC, AB = DR (common), $\angle ABC = \angle DRC$ and
$\angle BAC = \angle RDC$ (alt. $\angle$s, AB//QD), $\therefore$ $\Delta ABC \cong \Delta DRC$ (A.S.A.),
$\therefore$ BC = RC (corr. sides, $\cong$ Δs), i.e. C is the mid-point of BR.

20. $\because$ $\Delta ABC \cong \Delta EDC$ (given), $\therefore$ $x = 68^o$ (corr. $\angle$s, $\cong$ Δs); $\because$ BC = DC (corr. sides, $\cong$ Δs),
$\therefore$ $\angle BDC = x = 68^o$ (base $\angle$s, isos. Δ), $y + 68^o + 68^o = 180^o$ ($\angle$ sum of Δ), $\therefore$ $y = 44^o$

21. Draw MN $\perp$ BC. In ΔBMN and ΔCMN, MN = MN (common),
MB = MC (given), $\angle MNB = \angle MNC = 90^o$ (construction),
$\therefore$ $\Delta BMN \cong \Delta CMN$ (R.H.S.), $\therefore$ $BN = CN = \frac{1}{2}BC$ (corr. sides, $\cong$ Δs),



$\therefore$ Area of $\Delta DMC = \frac{1}{2} \times DC \times CN = \frac{1}{2}(\sqrt{18})(\frac{1}{2}\sqrt{18}) = \frac{18}{4} = 4.5\ \text{cm}^2$

22. (a) $\because$ $\angle ACB = a$ (base $\angle$s, isos. Δ), $\therefore$ $\angle CBD = 2a$ (ext. $\angle$ of Δ),
$\therefore$ $\angle FDE = 2a$ (corr. $\angle$s, BC//DF). In ΔDEF, $\angle DFE = b$ (base $\angle$s, isos. Δ),
$\therefore$ $b + b + 2a = 180^o$ ($\angle$ sum of Δ), $2a + 2b = 180^o$, $\therefore$ $a + b = 90^o$
(b) $a + b + \angle AGE = 180^o$ ($\angle$sum of Δ), $90^o + \angle AGE = 180^o$, $\therefore$ $\angle AGE = 90^o$

23. Let $\angle PHQ = \angle PQH = a$ and $\angle RKQ = \angle RQK = b$ (base $\angle$s, isos. Δ).
In ΔPHQ, $\angle P = 180^o - 2a$ ($\angle$ sum of Δ);
In ΔRKQ, $\angle R = 180^o - 2b$ ($\angle$ sum of Δ);
In ΔHKQ, $a + b + \theta = 180^o$ ($\angle$ sum of Δ), $a + b = 180^o - \theta$;
$\angle PQR = \angle PQH + \angle RQK - \theta$, $100^o = a + b - \theta$,
$100^o = (180^o - \theta) - \theta$, $2\theta = 80^o$, $\therefore$ $\theta = 40^o$

24. (a) $\angle PLM = a$ (base $\angle$s, isos. Δ), $\therefore$ $\angle P = 180^o - 2a$ ($\angle$ sum of Δ);
$\angle RNM = b$ (base $\angle$s, isos. Δ), $\therefore$ $\angle R = 180^o - 2b$ ($\angle$ sum of Δ);
$\angle P + \angle R + \theta = 180^o$ ($\angle$ sum of Δ), $(180^o - 2a) + (180^o - 2b) + \theta = 180^o$,
$360^o - 2a - 2b + \theta = 180^o$, $\therefore$ $\theta = 2a + 2b - 180^o$
(b) $\theta = 2a + 2b - 180^o$, $2a + 2b = \theta + 180^o$, $\therefore$ $a + b = \frac{\theta}{2} + 90^o$;
$a + b + \angle LMN = 180^o$ (adj. $\angle$s on st. line), $\frac{\theta}{2} + 90^o + \angle LMN = 180^o$,
$\therefore$ $\angle LMN = 90^o - \frac{\theta}{2}$

25. (a) In ΔABC, $\angle A = \angle ACB$ (base $\angle$s, isos. Δ),
$\therefore$ $2\angle A + 20^o = 180^o$ ($\angle$ sum of Δ), $\angle A = 80^o$ and $\angle ACB = 80^o$.
In ΔACD, $\angle A = \angle ADC$ (base $\angle$s, isos. Δ),
$2\angle A + \angle ACD = 180^o$ ($\angle$ sum of Δ), $2(80^o) + \angle ACD = 180^o$,
$\angle ACD = 20^o$, $\therefore$ $\angle DCE = 80^o - 20^o = 60^o$.

In ΔDCE, $\angle$DEC = $\angle$DCE (base $\angle$s, isos. Δ),
$\angle$CDE +2(60°) = 180° ($\angle$ sum of Δ), $\therefore$ $\angle$CDE = 60°

(b) $\angle$EDB = 60° – 20° = 40° (ext. $\angle$ of Δ); $\therefore$ $\angle$EFD = 40° (base $\angle$s, isos. Δ);
$\angle$FEB = 40° – 20° = 20° (ext. $\angle$ of Δ); $\because$ $\angle$FEB = $\angle$B = 20°,
$\therefore$ BF = EF (sides opp. eq. $\angle$s), $\therefore$ AC = EF = BF = 6 cm

26. (a) In ΔABC and ΔCDE, BC = DE and AB = CD (given),
$\angle$ABC = $\angle$CDE = 180° – 94° = 86° (adj. $\angle$s on st. line),
$\therefore$ ΔABC $\cong$ vCDE (S.A.S.), $\therefore$ AC = CE (corr. sides, $\cong$ Δs),
$\therefore$ ΔACE is isosceles.

(b) $\because$ ΔABC $\cong$ ΔCDE (proved), $\therefore$ $\angle$BAC = y (corr. $\angle$s, $\cong$ Δs),
$x + y = 94°$ (ext. $\angle$ of Δ); But $35° + x + \theta + y = 180°$ (adj. $\angle$s on st. line),
$\therefore$ $\theta + 94° + 35° = 180°$, $\therefore$ $\theta = 51°$

(c) In ΔACE, AC = CE (proved), $\therefore$ $\angle$CAE = $\angle$CEA (base $\angle$s, isos. Δ),
$\therefore$ 2$\angle$CEA + 51° = 180° ($\angle$ sum of Δ), $\angle$CEA = 64.5°,
$\therefore$ $\angle$CED = 180° – 52° – 64.5° = 63.5° (adj. $\angle$s on st. line);
But x = $\angle$CED (corr. $\angle$s, $\cong$ Δs), $\therefore$ $x = 63.5°$ and $y = 94° - x = 94° - 63.5° = 30.5°$

27. (a) $AC = \sqrt{AB^2 + BC^2} = \sqrt{(4+3)^2 + 24^2} = \sqrt{625} = 25$ cm;

Area of ΔABC = $\dfrac{7 \times 24}{2} = \dfrac{BD \times 25}{2}$, $\therefore$ BD = 6.72 cm

(b) Let a = $\angle$BCE = $\angle$DCF. $\angle$BEC = 180° – 90° – a = 90° – a ($\angle$ sum of Δ);
$\angle$DFC = 180° – 90° – a = 90° – a ($\angle$ sum of Δ);
But $\angle$BFE = $\angle$DFC (vert. opp. $\angle$s), $\therefore$ $\angle$BFE = 90° – a = $\angle$BEC,
$\therefore$ ΔBEF is isosceles (sides opp., eq. $\angle$s)

(c) BF = BE = 4 cm (proved), $\therefore$ FD = BD – BF = 6.72 – 4 =3.72 cm

28. (a) Let $\angle$AQR = x and $\angle$APB = y. $\because$ BP = BA (given),
$\therefore$ $\angle$PAB = y (base $\angle$s, isos. Δ); $\because$ RQ = RA (given),
$\therefore$ $\angle$RAQ = x (base $\angle$s, isos. Δ);
$\angle$BAR = 180° – x – y (adj. $\angle$s on st. line),
$\angle$PQR = 180° – x – y ($\angle$ sum of Δ), $\therefore$ $\angle$BAR = $\angle$PQR

(b) In ΔABR and ΔRCQ, $\angle$BAR = $\angle$CRQ (proved),
$\angle$ABR = $\angle$RCQ (corr. $\angle$s, QC//AB), RA = RQ (given),
$\therefore$ ΔABR $\cong$ ΔRCQ (A.A.S.), $\therefore$ AB = CR (corr. sides, $\cong$ Δs)

Unit 11 More about triangles

1. (a) orthocentre (b) in-centre (c) circumcentre (d) centroid
2. According to triangle inequality:
$x < 3+8$, $x < 11$; $x+3 > 8$, $x > 5$; $x+8 > 3$, $x > -5$;
Combining the 3 cases, $5 < x < 11$, $\therefore$ *x can be 6, 7, 8 , 9 and 10.*
3. According to triangle inequality:

$XY + YZ > XZ$, $XY > XZ - YZ$ ….. (i)

$XW + WZ > XZ$, $XW > XZ - WZ$ …..(ii)

From (i) and (ii), $XY + XW > (XZ - YZ) + (XZ - WZ)$

$\therefore$ $XY + XW + (WZ + YZ) > (XZ - YZ) + (XZ - WZ) + (WZ + YZ)$

$XY + XW + WY > 2XZ$ $\therefore$ $XZ < \frac{1}{2}(XY + XW + WY)$

i.e. XZ is shorter than half of the perimeter of ΔWXY.

4. $\angle Q = 119^o - 57^o = 62^o$ (ext. $\angle$ of Δ)
   $\angle R = 180^o - 119^o = 61^o$ (adj. $\angle$s on st. line)
   $\because$ $\angle Q > \angle QRP > \angle P$, $\therefore$ *PR > PQ > QR (greater ∠, greater side)*
5. $\angle C = 180^o - 80^o - 65^o = 35^o$ ($\angle$ sum of Δ). $\angle A = 65^o - 25^o = 40^o$ (ext. $\angle$ of Δ)
   $\angle BDA = 180^o - 65^o = 115^o$ (adj. $\angle$s on st. line)
   In ΔBDC: $\because$ $\angle DBC > \angle BDC > \angle C$, $\therefore$ CD > BC > BD (greater ∠, greater side)
   In ΔBDA: $\because$ $\angle BDA > \angle A > \angle ABD$, $\therefore$ AB > BD > DA (greater ∠, greater side)
   In ΔABC: $\because$ $\angle A > \angle C$, $\therefore$ BC > AB (greater ∠, greater side)
   In sum, $\because$ CD > BC, BC > AB, AB > BD > DA, *i.e. DA < BD < AB < BC < CD*
6. AC, BC and CD
7. $\angle SPQ = 180^o - 46^o - 113^o = 21^o$ ($\angle$ sum of Δ), $\therefore$ $\angle SPQ \neq \angle SPR$,
   $\angle SQR = 180^o - 110^o - 24^o = 46^o$ ($\angle$ sum of Δ), $\therefore$ $\angle SQR = \angle SQP$,
   $\angle SRP = 180^o - 19^o - 21^o - 46^o - 46^o - 24^o = 24^o$ ($\angle$ sum of Δ),
   $\therefore \angle SRP = \angle SRQ$, $\therefore$ *QS and RS are angle bisectors.*
8. Let $\angle ABD = a = \angle CBD$. $\angle BCA = 180° - 90° - \angle CBD$ ($\angle$ sum of Δ), $= 90° - a$
   $\angle BAC = 180° - 90° - \angle ABD$ ($\angle$ sum of Δ), $= 90° - a$
   $\because$ $\angle DCA = \angle BAC$ (alt. ∠s, AB // DC), $\therefore$ $\angle DCA = 90° - a$
   $\because$ $\angle DCA = \angle BCA$, $\therefore$ *AC is an angle bisector of ΔBCD.*
9. (a) $AC = \sqrt{12^2 + 16^2} = \sqrt{400} = 20$ (Pyth. thm.), $\therefore$ EC = 20 – 10 = 10
   (b) $\because$ AE = EC = 10, $\therefore$ *BE is a median of ΔABC.*
   (c) If $BE \perp AC$, then in ΔABE, $BE = \sqrt{12^2 - 10^2} = \sqrt{44}$.
   If $BE \perp AC$, then in ΔBEC, $BE = \sqrt{16^2 - 10^2} = \sqrt{156}$.
   However, $\sqrt{44} \neq \sqrt{156}$, "$BE \perp AC$" must be false. $\therefore$ *BE is not an altitude of ΔABC.*
10. (a) $\because$ $EF \perp CD$ and EF bisects CD,
    $\therefore$ *EF is the perpendicular bisector of CD.*
    (b) In ΔCEF and ΔDEF, CF = CF (given), $\angle CFE = \angle DFE = 90°$ (given),
    EF = EF (common), $\therefore$ $\Delta CEF \cong \Delta DEF$ (S.A.S.), $\therefore$ CE = CE (corr. sides, ≅ Δs),
    $\therefore$ *ΔCDE is an isosceles triangle.*
11. (a) In ΔPTS and ΔRTS, PS = RS and $\angle PST = \angle RST$ (given), ST = ST (common),

$\therefore\ \Delta PTS \cong \Delta RTS$ (S.A.S.)

(b) $\because\ \Delta PTS \cong \Delta RTS$ (proved), $\therefore$ PT = RT (corr. sides, $\cong\Delta$s),

$\angle PTS = \angle RTS$ (corr. $\angle$s, $\cong\Delta$s), $\angle PTS + \angle RTS = 2\angle PTS = 180^o$ (adj. $\angle$s on st. line),

$\therefore \angle PTS = 90^o$, *i.e. QS is the perpendicular bisector of PR.*

12. In ΔACQ and ΔBCQ, AQ = BQ (radii), AC = BC (radii), CQ = CQ (common),

$\therefore\ \Delta ACQ \cong \Delta BCQ$ (S.S.S.), $\therefore\ \angle AQC = \angle BQC$ (corr. $\angle$s, $\cong\Delta$s),

$\therefore$ *QC is the angle bisector of* $\angle PQR$.

13. (a) $b_1 + a_1 + b_2 + a_2 = 180°$ ($\angle$ sum of Δ), $2a_1 + 2b_1 = 180°$ ($\because\ a_1 = a_2, b_1 = b_2$),

$a_1 + b_1 = 90°$, $\because\ \angle BDC = a_1 + b_1$ (ext. $\angle$ or Δ), $\therefore\ \angle BDC = 90°$

i.e. BD is an attitude of ΔABC.

(b) $\angle ABC = a_1 + b_2 = a_1 + b_1 = 90°$, $\therefore\ \Delta ABD$ is a right-angle triangle.

(c) BD, AB and BC.

14. Let $\angle ECD = a$. $\angle DBC = 90° - \angle ECD = 90° - a$ (ext. $\angle$ of Δ)

$\angle EDC = \angle ECD = a$ (base $\angle$s, isos. Δ)

$\because\ 90° + \angle BDE + \angle EDC = 180°$ (adj. $\angle$s on st. line)

$\therefore\ \angle DBE = 90° - \angle EDC = 90° - a$

$\because\ \angle DBC = 90° - a = \angle BDE$, $\therefore$ BE = ED (side opp. equal $\angle$s)

But ED = EC (give), $\therefore$ BE = EC, $\therefore$ DE is a median of ΔBDC

15. $\angle QPH = \angle RPH$ and $\angle QRH = \angle PRH$ (given),

$\angle QPR + \angle QRP + 50^o = 180^o$ ($\angle$sum ofΔ),

$\therefore\ 2\angle RPH + 2\angle PRH = 130^o$, $\angle RPH + \angle PRH = 65^o$,

$\angle PHR = 180^o - \angle RPH - \angle PRH = 180^o - 65^o = 115^o$ ($\angle$sum ofΔ)

16. (a) $\because$ AE = EB = 5cm, $\therefore$ *EC is a median of ΔABC.*

(b) $\because\ ED \perp AC$, $\therefore$ *ED is an altitude of ΔAEC, ΔAED and ΔCED.*

(c) $AB^2 = 5^2 = 25$, $EF^2 + BF^2 = 4^2 + 3^2 = 25$,

$\because\ AB^2 = EF^2 + BF^2$, $\therefore \angle BFE = 90^o$ (converse of Pyth. thm.)

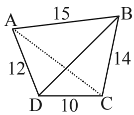
$\therefore$ *EF is an altitude of ΔEBC.*

(d) $ED = \sqrt{5^2 - 2^2} = 21$. In ΔCDE and ΔCFE, CE = CE (common) but ED $\neq$ EF,

$\therefore\ \Delta CDE$ is not congruent to ΔCFE, $\therefore\ \angle DCE \neq \angle FCE$.

Ans. EC is not an angle bisector of $\angle ACB$.

17. (a) $\because$ AB = 15, AD = 12, $\therefore$ $\angle ADB > \angle ABD$ *(greater side, greater $\angle$)*



(b) $\because$ BC = 14, CD, $\therefore$ $\angle BDC > \angle DBC$ (greater side, greater $\angle$)

$\therefore$ $\angle ADB + \angle BDC > \angle ABD + \angle DBC$, *i.e.* $\angle ADC > \angle ABC$

(c) Join AC. In ΔABC, $\therefore$ $\angle ACB > \angle CAB$ (greater side, greater $\angle$)

In ΔADC, $\angle ACD > \angle CAD$ (greater side, greater $\angle$)

$\therefore$ $\angle ACD + \angle ACB > \angle CAD + \angle CAB$, *i.e.* $\angle DCB > \angle DAB$

18. (a) $\angle R = x$ (base $\angle$s, isos. Δ), $\therefore$ $\angle KSQ = x + \theta$ (ext. $\angle$of Δ),

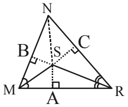
and $\angle TKP = x - \theta$ (ext. $\angle$of Δ)

(b) $\angle QKS = \angle TKP$ (vert. opp. $\angle$s), $\therefore$ $\angle QKS = x - \theta$, $\because$ $x + \theta > x - \theta$,

$\therefore$ $\angle KSQ > \angle QKS$, $\therefore$ QK > QS (greater $\angle$, greater sde)

$\therefore$ *It is impossible for QS to be longer than QK.*

19. (a) Draw SA $\perp$ MR, SB $\perp$ MN and SC $\perp$ NR.



In ΔMSA and ΔMSB, MS = MS (common),
$\angle$AMS = $\angle$BMS (given), $\angle$SAM = $\angle$SBM = 90°(construction),
$\therefore$ ΔMSA $\cong$ ΔMSB (A.A.S.), $\therefore$ SA = SB (corr. sides, $\cong$ Δs).
Similarly, ΔRSA $\cong$ ΔRSC (A.A.S.), $\therefore$ SA = SC (corr. sides, $\cong$ Δs).
$\therefore$ SA = SB = SC, *i.e. S is equidistant from MN, MR and NR.*

(b) In ΔSNB and ΔSNC, SN = SN (common), SB = SC (proved),
$\angle$SBN = $\angle$SCN (construction), $\therefore$ ΔSNB $\cong$ ΔSNC (R.H.S.),
$\therefore$ $\angle$SNB = $\angle$SNC (corr. $\angle$s, $\cong$ Δs), *i.e. NS is the angle bisector of $\angle$MNR.*

(c) *From (b), the three angle bisectors of a triangle intersect at a point.*

20. (a) In ΔADE and ΔBDE, AE = BE and $\angle$AED = $\angle$BED = 90^o (given),
DE = DE (common), $\therefore$ ΔADE $\cong$ ΔBDE (S.A.S.),
$\therefore$ AD = BD (corr. sides, $\cong$ Δs), $\therefore$ ΔBDA is an isosceles triangle.
Similarly, ΔADF $\cong$ ΔCDF (S.A.S.), $\therefore$ AD = CD (corr. sides, $\cong$ Δs),
$\therefore$ *ΔADC is an isosceles triangle.*

(b) BD = AD = CD (proved).
In ΔBDG and ΔCDG, BD = CD (proved), BG = CG (given),
DG = DG (common), $\therefore$ ΔBDG $\cong$ ΔCDG (S.S.S.),
$\therefore$ $\angle$BGD = $\angle$CGD (corr. $\angle$s, $\cong$ Δs),
$\angle$BGD + $\angle$CGD = 2$\angle$BGD = 180° (adj. $\angle$s on st. line), $\therefore$ $\angle$BGD = 90^o.
$\because$ BG = GC and BG $\perp$ DG, $\therefore$ *DG is the perpendicular bisector of BC.*

(c) From (b), the three perpendicular bisectors of a triangle intersect at a point.

(d) $\because$ DA = DB = DC, $\therefore$ *AD, BD and CD are radii of the circle passing through A, B and C with D as the centre.*

(e) AE = $\frac{1}{2}$BA $= 24 \times \frac{1}{2} = 12$ cm, and radius = AD,

$\therefore$ radius $= \sqrt{AE^2 + DE^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13\,\text{cm}$ (Pyth. thm.)

Unit 12 Quadrilaterals

In questions (1–4), the following reasons are simplified: (1) the properties of parallelogram as "//gram"; (2) $\angle$sum of Δ as "Δ"; (3) adj. $\angle$s on st. line as "st. line".

1. (a) $a + 4 = 20 - a$ (// gram), $2a = 16$, $\therefore$ $a = 8$
$x + 50^o = 3x - 70^o$ (// gram), $120^o = 2x$, $\therefore$ $x = 60^o$
$y + x + 50^o = 180^o$ (int. $\angle$s, AB//DC), $\therefore$ $y = 180^o - 60^o - 50^o = 70^o$

(b) $3x + 5 = 5x - 3$ (// gram), $8 = 2x$, $\therefore$ $x = 4$.
$7x - y = 7y + x$ (//gram), $7(4) - y = 7y + 4$, $\therefore$ $y = 3$

(c) $m + 155^o = 180^o$ (int. $\angle$s, QP//RS), $\therefore$ $m = 180^o - 155^o = 25^o$
$n + 2m = 180^o$ (int. $\angle$s, QP//PS), $\therefore$ $n = 180^o - 2(25^o) = 130^o$

(d) $\angle$PSR = c (// gram), $\angle$PSR + 70° = 180° (st. line), $\therefore$ $c = 180^o - 70^o = 110^o$.

$c+d+10^o+2d=180^o$ (Δ), $3d=180^o-110^o-10^o=60^o$, ∴ $d=20^o$

(e) $\angle$DFE $=180^o-102^o=78^o$ (st. line), $\angle$DEF $=\angle$DFE $=78°$ (base $\angle$s, isos.Δ),

$\angle$D $=180^o-2(78^o)=24^o$ (Δ), $2x=24^o$ (// gram), ∴ $x=12^o$

$y-x+24^o=180^o$ (int.$\angle$s, CD//BA), ∴ $y=180^o+12^o-24^o=168^o$

2. (a) $3x=90^o$ (rectangle), ∴ $x=30^o$. $2y-x+2x+y+90^o=180^o$ (Δ),

$3y+x=90^o$, $3y=90^o-30^o=60^o$, ∴ $y=20^o$

(b) $\angle$SRT $=90^o-65^o=25^o$, ∴ $c=\angle$SRT $=25^o$ (base$\angle$s, isos.Δ)

$e=180^o-25^o-25^o=130^o$ (Δ), $d=90^o-c=90^o-25^o=65^o$

(c) $\angle$PST $=y$ (rectangle), $y+\angle$PST $=100^o$ (ext.$\angle$ of Δ), $2y=100^o$,

∴ $y=50^o$. $x+\angle$PST $+90^o=180^o$ (Δ), ∴ $x=180^o-90^o-50^o=40^o$

(d) RS = 12 (rectangle), $12^2+(a-4)^2=(a+4)^2$ (Pyth. Thm.),

$144+a^2-8a+16=a^2+8a+16$, $144=16a$, ∴ $a=9$

(e) $a+90^o=4a+15^o$ (ext.$\angle$ of Δ), $75^o=3a$, ∴ $a=25^o$

$\angle$APQ $=a=25^o$ (base$\angle$s, isos.Δ), $\angle$QAP $=180^o-25^o-25^o=130^o$ (Δ),

∴ $b=\angle$QAP $=130^o$ (vert. opp.$\angle$s)

3. (a) $3m+8=13-2m$ (rhombus), $5m=5$, ∴ $m=1$. $6a=144^o$ (rhombus),

∴ a = 24^o. $4b+144^o=180^o$ (int.$\angle$s, BA//CD), 4b = 36^o, ∴ b = 9^o

(b) $4x=24^o$ (rhombus), ∴ $x=6^o$. $4x+y+90^o=180^o$ (Δ),

∴ $y=180^o-90^o-24^o=66^o$. $5z=90^o$ (rhombus), ∴ $z=18^o$

(c) $3x=51^o$ (alt.$\angle$s, SR//PQ), ∴ $x=17^o$. $y+11^o+2(3x)=180^o$ (Δ),

∴ $y=180^o-11^o-2(51^o)=67^o$

(d) $7m-6=5m-2$ (rhombus), $2m=4$, ∴ $m=2$. HG = 6 (rhombus),

HD = 7(2) – 6 = 8, ∴ $n=\sqrt{6^2+8^2}=\sqrt{100}=10$ (Pyth. Thm.)

4. (a) $3x=16-x$ (square), $4x=16$, ∴ $x=4$.

$y+2y=45^o$ (square), $3y=45^o$, ∴ $y=15^o$

(b) $2a+14^o=90^o$ (square), $2a=76^o$, ∴ $a=38^o$. $8b=4$ (square), ∴ $b=\frac{1}{2}$

(c) $\angle$ACB $=45^o$ (square), $m+45^o=110^o$ (ext.$\angle$ of Δ), ∴ $m=65^o$;

$m+n=180^o$ (int.$\angle$s, AD//BC), ∴ $n=180^o-65^o=115^o$

(d) $4y-3^o=45^o$ (rhombus), $4y=48^o$, ∴ $y=12^o$. QT = TR = 5 (square),

$x^2=5^2+5^2$ (Pyth. Thm.), $x^2=50$, ∴ $x=\sqrt{50}=5\sqrt{2}$

5. In ΔPSX and ΔRQY, PS = RQ (prop. of // gram), $\angle$P $=\angle$R (prop. of // gram),

PX = RY (given), ∴ ΔPSX ≅ ΔRQY (S.A.S.), ∴ QY = XS (corr. sides, ≅ Δs)

6. ∵ AB = DC and DC = EF (prop. of // gram), ∴ AB = EF;

∵ AB // DC and DC // EF (def. of // gram), ∴ AB // EF,

∴ ABFE is a // gram (2 sides eq. and //)

7. (a) In ΔABE and ΔCDF, AB = CD (prop. of // gram),

$\angle$BAE $=\angle$DCF (alt.$\angle$s, AB//DC), ∵ AF = CE (given),

$\therefore$ AE = AF + FE = CE + FE = CF, $\therefore$ ΔABE $\cong$ ΔCDF (S.A.S.)

(b) $\because$ ΔABE $\cong$ ΔCDF (proved), $\therefore$ BE = DF (corr. sides, $\cong$ Δs),
$\angle$BEA = $\angle$DFC (corr. $\angle$s, $\cong$ Δs), $\therefore$ BE // FD (alt. $\angle$s, eq.),
$\therefore$ BEDF is a // gram (2 sides eq. and //)

8. (a) In ΔPQN and ΔRSM, PQ = RS (prop. of // gram), $\angle$PNQ = $\angle$RMS = 90^o (given),
$\angle$QPN = $\angle$SRM (alt. $\angle$s, PQ//SR), $\therefore$ ΔPQN $\cong$ ΔRSM (A.A.S.)

(b) $\because$ ΔPQN $\cong$ ΔRSM (proved), $\therefore$ QN = SM (corr. sides, $\cong$ Δs),
$\because$ $\angle$PNQ = $\angle$RMS = 90^o (given), $\therefore$ QN//SM (alt. $\angle$s, eq.),
$\therefore$ QNSM is a // gram (2 sides eq. and //)

9. $\angle$FEG = $\angle$FGE = $(180^o - 30^o) \div 2 = 75^o$ (base $\angle$s, isos. Δ)
$\angle$CFG + 30^o = 90^o (square), $\angle$CFG = $90^o - 30^o = 60^o$.
$\because$ CF = FE = FG (square), $\therefore$ $\angle$FCG = $\angle$FGC = $(180^o - 60^o) \div 2$
= 60^o (base $\angle$s, isos. Δ), $\therefore$ ΔCFG is equilateral, and FG = CF = CG = CD,
$\angle$DCG = $90^o - 60^o = 30^o$. In ΔCDG and ΔFEG,
$\angle$DCG = $\angle$EFG = 30^o (proved), CD = FE (square), CG = FG (proved),
$\therefore$ ΔCDG $\cong$ ΔFEG (S.A.S.), $\therefore$ m = $\angle$CDG = 75^o (corr. $\angle$s, $\cong$ Δs).
n + $\angle$CDG = 90^o (square), $\therefore$ $n = 90^o - 75^o = 15^o$

10. $\angle$ADE = 60^o and AD = DE (equilateral Δ), $\angle$ADC = 90^o and AD = DC,
$\therefore$ $\angle$CDE = $90^o - 60^o = 30^o$ and DE = DC, $\therefore$ $\angle$DEC = x (base $\angle$s, isos. Δs),
$\therefore$ $2x + 30^o = 180^o$ ($\angle$ sum of Δ), $2x = 150^o$, $\therefore$ x = $\angle$DEC = 75^o
Similarly, $\angle$AEB = 75^o, but $\angle$AED = 60^o,
$\therefore$ $y + 75^o + 60^o + 75^o = 360^o$ ($\angle$s at a pt.), $\therefore$ $y = 150^o$

11. $\angle$ADC = $180^o - 115^o = 65^o$ (adj. $\angle$s on st. line),
$\therefore$ m = $\angle$ADC = 65^o (prop. of // gram), $\angle$DCG = m = 65^o (corr. $\angle$s, AB//DC),
$\therefore$ n = $\angle$DCG = 65^o (prop. of // gram). $\because$ EG = FG (given),
$\therefore$ $\angle$FEG = n = 65^o (base $\angle$s, isos. Δ), but $\angle$FED = 115^o (alt. $\angle s$, EF//AD),
$\therefore$ $p = 115^o - 65^o = 50^o$

12. (a) In ΔCMB and ΔCMN, CM = CM (common), $\angle$BCM = $\angle$NCM (given),
$\angle$B = $\angle$CNM = 90^o, $\therefore$ ΔCMB $\cong$ ΔCMN (A.A.S.)

(b) $\angle$NAM = 45^o (prop of square),
$\angle$NMA = $180^o - 90^o - 45^o = 45^o$ ($\angle$ sum of Δ),
$\therefore$ AN = MN (sides opp. eq. $\angle$s), but MN = MB (corr. sides $\cong$ Δs), $\therefore$ AN = MB

13. PS = QR = 10cm (prop. of // gram), PT = 10 – 3 = 7 = PQ
$\therefore$ $\angle$PQT = $\angle$PTQ (base $\angle$s, isos. Δ), but $\angle$PTQ = $\angle$RQT (alt. $\angle$s, QR//PS),
$\therefore$ $\angle$PQT = $\angle$RQT, i.e. QT bisects $\angle$PQR.

14. (a) $\angle$BCM = $180^o - 90^o - \angle$CBN = $90^o - \angle$CBN ($\angle$ sum of Δ),
$\angle$ABN + $\angle$CBN = 90^o (prop. of square), $\therefore$ $\angle$ABN = $90^o - \angle$CBN = $\angle$BCM

(b) In ΔCBM and ΔBAN, $\angle$BCM = $\angle$ABN (proved),
$\angle$MBC = $\angle$A = 90^o (prop. of square), BC = AB (prop. of square),
$\therefore$ ΔCBM $\cong$ ΔBAN (A.S.A.), $\therefore$ BN = CM (corr. sides, $\cong$ Δs)

15. In ΔBEP and ΔDEQ, BE = DE (prop. of // gram), $\angle$PBE = $\angle$QDE and
$\angle$BPE = $\angle$DQE (alt. $\angle$s, AB//DC), $\therefore$ ΔBEP $\cong$ ΔDEQ (A.A.S.),
$\therefore$ PE = QE (corr. sides, $\cong$ Δs)

16. In ΔAPD and ΔCPD, PD = PD (common), AD = CD (prop. of rhombus),
$\angle$ADP = $\angle$CDP (prop. of rhombus), $\therefore$ ΔAPD $\cong$ ΔCPD (S.A.S.),
$\therefore$ $\angle$APD = $\angle$CPD (corr. $\angle$s, $\cong$ Δs), i.e. PB bisects $\angle$APC.

17. AE = $y-1$ (rhombus), $(y+1)^2+(y-1)^2=10^2$ (Pyth. Thm.)
$y^2+2y+1+y^2-2y+1=100$, $2y^2=98$, $y^2=49$, $\therefore$ $y=\sqrt{49}=7$
$x=y+1=7+1=8$ (rhombus)

18. (a) $\angle$A = $\angle$BEA (base $\angle$s, isos. Δ), $\angle A=\dfrac{180^\circ-36^\circ}{2}=72^\circ$ ($\angle$ sum of Δ),
$\angle$C = $\angle$A = 72° (prop. of //gram); $\angle$BDC = $\angle$C = 72° (base $\angle$s, isos. Δ),
$\theta+36^o=\angle BDC=72^o$ (alt. $\angle$s, AB//DC), $\therefore$ $\theta=36^o$
(b) $\angle$EDB $=180^o-\angle A-\theta-36^o=180^o-72^o-36^o-36^o=36^o$ ($\angle$ sum of Δ),
$\because$ $\angle$EDB $=\theta=36^o$, $\therefore$ BE = DE (sides opp. eq. $\angle$s),
$\therefore$ ΔBED is isosceles.

19. (a) $\angle$SQN $=90^o+45^o=135^o$ (prop. of square),
$\angle$NQM = $\angle$SQM $=135^o\div 2=67.5^o$, $\angle$QPM = 45^o (prop. of square),
$\therefore$ $\angle$M $=67.5^o-45^o=22.5^o$ (ext. $\angle$ of Δ)
(b) $\angle$SQR = 45^o (prop. of square), $\angle$RQM $=67.5^o-45^o=22.5^o=\angle$M,
$\therefore$ RM = RQ (sides opp. eq. $\angle$s), $\therefore$ ΔQRM is isosceles.

20. AC = BC and $\angle$ACB = 60^o (equilateral Δ), $\therefore$ $\angle$ACD = $60^o+90^o=150^o$,
$\therefore$ $\angle$CDA = $\angle$CAD (base $\angle$s, isos. Δ),
$\angle CDA=\dfrac{180^\circ-150^\circ}{2}=15^\circ$ ($\angle$ sum of Δ), but $\angle$CDB = 45^o (prop. of square),
$\therefore$ $a=45^o-15^o=30^o$; $\angle$ADE $=90^o-15^o=75^o$, similarly, $\angle$AED = 75^o,
$\therefore$ $b+75^o+75^o=180^o$ ($\angle$ sum of Δ), $\therefore$ $b=30^o$

21. (a) In ΔPQT and ΔRQT, PQ = RQ (prop. of rhombus),
$\angle$PQT = $\angle$RQT (prop. of rhombus), QT = QT (common),
$\therefore$ ΔPQT $\cong$ ΔRQT (S.A.S.)
(b) $\because$ ΔPQT $\cong$ ΔRQT (proved), $\therefore$ $\angle$QTP = $\angle$QTR (corr. $\angle$s, $\cong$ Δs),
$\angle$QTP $=70^o\div 2=35^o$, $\angle$PQT $=180^o-35^o-90^o=55^o$ ($\angle$ sum of Δ),
but $\angle$RQS = $\angle$RSQ = $\angle$PQT = 55^o (prop. of rhombus),
$\therefore$ $\angle$QRS $=180^o-55^o-55^o=70^o$ ($\angle$ sum of Δ)

22. In ΔAED and ΔCGD, AD = CD and DE = DG (prop. of square),
$\angle$ADC = $\angle$EDG = 90° (prop. of square),
$\angle$ADE = $\angle$CDE + $\angle$ADC = $\angle$CDE + 90^o = $\angle$CDE + $\angle$EDG = $\angle$CDG,
$\therefore$ ΔAED $\cong$ ΔCGD (S.A.S.), $\therefore$ AE = CG (corr. sides, $\cong$ Δs)

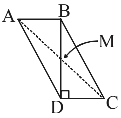
23. QN = NS (prop. of //gram), NS = LP (prop. of //gram), $\therefore$ QN = LP;
LP//NS (def. of //gram), i.e. LP//QS. $\therefore$ QNPL is a //gram (2 sides eq. and //),
$\therefore$ LN and QP bisect each other (prop. of //gram), $\therefore$ LM = MN

24. (a) In //gram PQRS, let QS and PR intersect at X. PX = XR and QX = XS (prop. of //gram). In //gram QTSU, let QS and TU intersect at Y, QY = YS and TY = YU (prop. of //gram). Since QX = XS and QY = YS, X and Y are the same point, i.e. QS, PR and TU are concurrent.
(b) In quadrilateral PTRU, X is the mid-point of TU and PR (proved),
$\therefore$ PTRU is a //gram (diags bisect each other)

25. $\angle CBF = 28^o$ (prop. of rectangle), $\angle BFC = 180^o - 28^o - 28^o = 124^o$ ($\angle$ sum of Δ),
$\angle AFE = 60^o$ (equilateral Δ), $\angle DFE + 60^o = 124^o$ (vert. opp. $\angle$s), $\angle DFE = 64^o$,
$\because$ DF = AF (prop. of rectangle) and AF = EF (equilateral Δ), $\therefore$ DF = EF,
$\therefore$ $\angle FED = \angle FDE$ (base $\angle$s, isos. Δ), $\therefore$ In ΔEFD, $\angle FED = \dfrac{180^\circ - 64^\circ}{2} = 58^o$,
$\therefore$ $\angle AED = \angle AEF + 58^o = 60^o + 58^o = 118^o$

26. (a) $\angle CDE = (5-2)\times 180^o \times \dfrac{1}{5} = 108^o$ ($\angle$ sum of polygon)
In ΔCDE, $\angle DCE = \angle DEC$ (base $\angle$s, isos. Δ),
$\therefore$ $\angle DCE = \dfrac{180^\circ - 108^\circ}{2} = 36^\circ$ ($\angle$ sum of isos. Δ)
(b) $\angle BCD = \angle CDE = 108^o$ (prop. of regular pentagon),
but $\angle BCF = \angle DCF$ (prop. of rhombus),
$\therefore$ $\angle DCF = 108^o \div 2 = 54^o$, $\therefore$ $\theta = 54^o - 36^o = 18^o$

27. DC = AB = 7 (prop. of // gram),



$\therefore$ $BD = \sqrt{25^2 - 7^2} = \sqrt{576} = 24$ (Pyth. Thm.),
$DM = BM = 24 \div 2 = 12$ (prop. of // gram),
$\therefore$ $CM = \sqrt{7^2 + 12^2} = \sqrt{193}$ (Pyth. Thm.),
$\because$ $AM = AC = \sqrt{193}$ (prop. of // gram), $\therefore$ $AC = 2\sqrt{193} = 27.8$
[*Method 2*: Produce CD to E such that AE $\perp$ CE. ED = AB = 7, AE = BD = 24,
$\therefore$ $AC = \sqrt{(7+7)^2 + 24^2} = \sqrt{772} = 27.8$]

28. (a) In ΔAXM and ΔCYM, $\angle MAX = \angle MCY$ (alt. $\angle$s, AB//DC),
$\angle AMX = \angle CMY = 90^o$ (vert. opp. $\angle$s), MX = MY (given),
$\therefore$ $\Delta AXM \cong \Delta CYM$ (A.A.S.)
(b) In ΔAXM and ΔCAD, $\angle MAX = \angle DCA$ (alt. $\angle$s, AB//DC),
$\angle AMX = \angle CDA = 90^o$, $\angle AXM = \angle CAD$ (3^{rd} $\angle$ of Δ),
$\therefore$ $\Delta AXM \sim \Delta CAD$ (A.A.A.)
(c) $AC = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ cm (Pyth. Thm.), $\because$ $\Delta AXM \cong \Delta CYM$ (proved),
$\therefore$ AM = CM (corr. sides, $\cong$ Δs), $\therefore$ $AM = 13 \div 2 = 6.5$ cm,

$\because\ \Delta AXM \sim \Delta CAD$ (proved), $\therefore\ \frac{XM}{AD} = \frac{AM}{CD}$, $XM = \frac{6.5}{12} \times 5 = \frac{65}{24}$,

$\therefore$ Area of $\Delta AXM = \frac{1}{2}(6.5)(\frac{65}{24}) = 8.8\ \ cm^2$

29. (a) $\because$ AB = BD (given), $\therefore\ \angle ADB = \angle DAB$ (base $\angle$ s, isos. Δ),
$\angle ABD = \angle BDC$ (alt. $\angle$ s, AB//DC),
$\therefore\ \angle CDA = \angle BDC + \angle ADB = \angle ABD + \angle DAB = \angle EDA$ (ext. $\angle$ of Δ),
(b) In ΔACD and ΔAED, AD = AD (common), DC = AB (prop. of // gram),
$\therefore$ DC = AB = BD = DE, $\angle CDA = \angle EDA$ (proved),
$\therefore\ \Delta ACD \cong \Delta AED$ (S.A.S.), $\therefore$ AC = AE (corr. sides $\cong \Delta$ s),
$\therefore\ \Delta ACE$ is isosceles.

30. (a) In ΔACE and ΔABD, AC = AB and AE = AD (equilateral Δ s),
$\angle EAC = 60^o + \angle DAC = \angle DAB$, $\therefore\ \angle EAC = \angle DAB$,
$\therefore\ \Delta ACE \cong \Delta ABD$ (S.A.S.), $\therefore$ EC = BD (corr. sides, $\cong \Delta$ s)
(b) In ΔACE and ΔDFE, AE = DE and CE = FE (equilateral Δ s),
$\angle AEC = 60^o - \angle CED = \angle DEF$, $\therefore\ \angle AEC = \angle DEF$,
$\therefore\ \Delta ACE \cong \Delta DEF$ (S.A.S.), $\therefore$ AC = DF (corr. sides, $\cong \Delta$ s)
(c) $\because$ BD = EC (proved) and EC = CF (equilateral Δ), $\therefore$ BD = CF,
$\because$ AC = DF (proved) and AC = BC (equilateral Δ), $\therefore$ DF = BC,
$\therefore$ BDFC is a //gram (opp. sides eq.)

31. (a) $\angle BPS = \angle BSP = \angle BRS = 45^o$ (prop. of square),
$\angle BPA = \angle SPA = 45^o \div 2 = 22.5^o$, $\angle ABP = \angle BRS = 45^o$ (corr. $\angle$ s, AB//SR),
$\therefore\ \angle BAC = \angle BPA + \angle ABP = 22.5^o + 45^o = 67.5^o$ (ext. $\angle$ of Δ) and,
$\angle BCA = \angle BSP + \angle SPA = 45^o + 22.5^o = 67.5^o$ (ext. $\angle$ of Δ),
$\therefore\ \angle BAC = \angle BCA$, $\therefore$ BA = BC (sides opp. eq. $\angle$ s)
(b) In ΔPBA and ΔPRT, $\angle BPA = \angle RPT$ (common),
$\angle PBA = \angle PRT$ and $\angle PAB = \angle PTR$ (corr. $\angle$ s, AB//SR),
$\therefore\ \Delta PBA \sim \Delta PRT$ (A.A.A.)
(c) $\because\ PB = \frac{1}{2}PR$ (prop. of square), $\therefore\ \frac{PB}{PR} = \frac{1}{2}$, $\because\ \Delta PBA \sim \Delta PRT$ (proved),
$\therefore\ \frac{BA}{RT} = \frac{PB}{PR} = \frac{1}{2}$ (corr. sides, $\sim \Delta$ s), but BA = BC (proved), $\therefore\ \frac{BC}{RT} = \frac{1}{2}$,
$\therefore$ 2BC = RT.

32. In ΔGBF and ΔGCD, $\angle G = \angle G$ (common), $\angle GBF = \angle GCD$ and
$\angle GFB = \angle GDC$ (corr. $\angle$ s, BF//CD), $\therefore\ \Delta GBF \sim \Delta GCD$ (A.A.A.),
$\therefore\ \frac{BF}{CD} = \frac{GF}{GD} = \frac{2}{2+1} = \frac{2}{3}$ (corr. sides, $\sim \Delta$ s), 3BF = 2CD, but BA = CD (prop. of //gram),
3(BA – FA) = 2CD, 3CD – 3FA = 2CD, $\therefore$ CD = 3FA, $\frac{CD}{FA} = 3$.

In ΔCDE and ΔAFE, $\angle$DEC = $\angle$FEA (vert. opp. $\angle$s), $\angle$CDE = $\angle$AFE and $\angle$DCE = $\angle$FAE (alt. $\angle$s, BA//CD), $\therefore$ ΔCDE ~ ΔAFE (A.A.A.),

$\therefore$ $\frac{DE}{EF}=\frac{CD}{FA}=3$ (corr. sides, ~ Δs), $\therefore$ DE : EF = 3 : 1

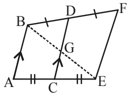
Unit 13 Mid-point theorem & intercept theorem

1. (a) $\because$ AB // CD // EF and BD = DF, $\therefore$ $x = 5$ (intercept thm), $y = 6$ (intercept thm)

   (b) $\because$ AC = CE and AD = DF (given), $\therefore$ $x=\frac{1}{2}\times 20=10$ (mid-pt thm)

   $\because$ AC = CE and BD = DE (given), $\therefore$ $y = 2(10) = 20$ (mid-pt thm)

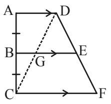
2. (a) $\because$ AC = CE and AB//CD (given),

   $\therefore$ BG = GE (intercept thm), $\therefore$ CG $=\frac{1}{2}\times 4=2$ (mid-pt thm),

   DG $= 7-2=5$, $a = 2(5) = 10$ (mid-pt thm)

   (b) $\because$ AB = BC and AD//BE (given),

   $\therefore$ DG = GC and DE = EF (intercept thm),

   $\therefore$ BG $=\frac{1}{2}\times 10=5$ (mid-pt thm), GE $=\frac{1}{2}\times 18=9$ (mid-pt thm),

   $\therefore$ $y=5+9=14$

3. (a) AC = AB = 16 cm (sides opp. eq. $\angle$s). $\because$ DG = GA and DF = FC (given),

   $\therefore$ EF//AC and GF $=\frac{1}{2}(16)=8$ (mid-pt thm).

   $\because$ BE = EA (given) and EF//AC (proved), $\therefore$ BG = GC (intercept thm)

   $\therefore$ $r=\frac{1}{2}\times 16=8$ (mid-pt thm)

   (b) $\because$ AE = EF and AB//EG (given), $\therefore$ BD = DF (intercept thm),

   $\therefore$ AB = 2(3) = 6 (mid-pt thm). $\angle$BAD = $\angle$GDC (corr. $\angle$s, AB//EG),

   $\angle$GDC = $\angle$GCD (given), $\therefore$ $\angle$BAD = $\angle$GCD,

   $\therefore$ y = AB = 6 (sides opp. eq. $\angle$s)

4. (a) $\because$ BC//EF (given), $\therefore$ $\frac{8}{12}=\frac{x}{4+5}$ (extension of int. thm.), $\frac{2}{3}=\frac{x}{9}$

   $\therefore$ $x=\frac{2}{3}\times 9=6$. $\because$ BD//EG (given), $\therefore$ $\frac{8}{12}=\frac{6+4}{5+y}$ (extension of int. thm.),

   $\frac{2}{3}=\frac{10}{5+y}$, $10+2y=30$, $\therefore$ $y=10$

   (b) $\because$ QR//ST (given), $\therefore$ $\frac{a}{b}=\frac{10}{5}=2$ (extension of int. thm.), $a=2b$

   $\because$ QT//SU (given), $\therefore$ $\frac{a+b}{6}=\frac{10}{5}=2$ (extension of int. thm.),

   $2b+b=12$, $3b=12$, $\therefore$ $b=4$ and $a=2(4)=8$

5. (a) $\because$ AD = DC and BE = EC (given), $\therefore$ AB//DE and $y = 2x$ (mid-p. thm),
$\therefore$ $a^o = \angle B = 90^o$ (corr. $\angle$s, AB//DE), $\therefore$ $a = 90$
$\angle CDE = 180^o - 90^o - 45^o = 45^o$ ($\angle$ sum of Δ), $\because$ $\angle CDE = \angle C = 45^o$,
$\therefore$ x = CE = 6 (sides opp. eq. $\angle$s). $y = 2(6) = 12$

(b) PQ = 2(3) = 6, QR $= \sqrt{10^2 - 6^2} = \sqrt{64} = 8$ (Pyth. thm.),

$\because$ QS = SP and ST//PR (given), $\therefore$ $y = \text{QT} = \frac{1}{2} \times 8 = 4$ (intercept thm)

$x = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ (Pyth. thm.)

6. (a) $\because$ DE//FG (given), $\therefore$ $\frac{p}{15} = \frac{15}{10} = \frac{3}{2}$ (extension of intercept thm.),

$\therefore$ $p = \frac{3}{2} \times 15 = 22.5$. $\because$ $\Delta ABC \sim \Delta EDC$ (A.A.A.),

$\therefore$ $\frac{q}{15} = \frac{20}{22.5}$ (corr. sides, $\sim\Delta$s), $\therefore$ $q = \frac{20}{22.5} \times 15 = 13\frac{1}{3}$

(b) $\because$ BC//DE (given), $\therefore$ $\frac{x-2}{4} = \frac{x}{6}$ (extension of intercept thm.),

$6x - 12 = 4x$, $2x = 12$, $\therefore$ $x = 6$.
$\because$ CF//AG (given) and AC = CE = 6 (proved), $\therefore$ $y = 8$ (intercept thm.)

7. $\because$ BE//KN (given), $\therefore$ $\frac{x}{3} = \frac{6}{5}$ (extension of intercept thm.),

$\therefore$ $x = \frac{6}{5} \times 3 = \frac{18}{5} = 3.6$. $\because$ $\Delta ADE \sim \Delta AMN$ (A.A.A.),

$\therefore$ $\frac{4}{y} = \frac{3.6}{3.6+3}$ (corr. sides, $\sim\Delta$s), $\frac{4}{y} = \frac{6}{11}$, $\therefore$ $y = 4 \times \frac{11}{6} = \frac{22}{3} = 7\frac{1}{3}$

8. EF // CD // AB and EC = CA, $\therefore$ AG = GF and BD = DE (intercept thm),

$\therefore$ CG $= \frac{1}{2} \times 9 = 4.5$ cm (mid-pt thm). CD $= \frac{1}{2} \times 15 = 7.5$ (mid-pt thm),

$\therefore$ GD $= 7.5 - 4.5 = 3$ cm

9. $\because$ AB = BD and AC = CE (given), $\therefore$ BC//DE (mid-pt thm),
$\therefore$ BCED is a trapezium.

10. $\because$ QH = HR and RN//HK (given), $\therefore$ NK = KQ (intercept thm.)
$\because$ PM = MH and RN//HK (given), $\therefore$ PN = NK (intercept thm.), $\therefore$ PN = KQ

11. $\because$ AB = BC and BF//CE (given), $\therefore$ AF = FE (intercept thm)
$\because$ AB = BC and BE//CD (given), $\therefore$ AE = ED (intercept thm)
Let AF = x, $\therefore$ FE = x, ED = $x + x = 2x$, AF : FE : ED = $x : x : 2x$ = 1 : 1 : 2

12. Join PR. $\because$ PA = AQ and RB = BQ (given),

$\therefore$ AB = $\frac{1}{2}$ PR and PR//AB (mid-pt thm).

$\because$ PD = DS and RC = CS (given), $\therefore$ DC = $\frac{1}{2}$PR and PR//DC (mid-pt thm),

AB = DC and AB//DC (proved), $\therefore$ ABCD is a // gram (2 sides eq. and //)

13. Join DE. $\because$ BE = EC and AD = DC (given),

$\therefore$ AB//DE and AB = 2ED (mid-pt thm), $\frac{AB}{ED} = \frac{2}{1}$.

$\because$ $\Delta ABG \sim \Delta EDG$ (A.A.A.), $\therefore \frac{BG}{GD} = \frac{AG}{GE} = \frac{AB}{ED} = \frac{2}{1}$ (corr. sides, $\sim \Delta$s),

$\therefore$BG : GD = 2 : 1, and AG : GE = 2 : 1

14. (a) $\because$ PA = AQ and PB = BR (given), $\therefore$ AD // QR (mid-pt thm)

(b) $\because$ PA = AQ (given) and AD//QC (proved), $\therefore$ PD = DC (intercept thm),

$\therefore$ AD = $\frac{1}{2}$QC (mid-pt thm). $\because$ PB = BR (given) and PD = DC (proved),

$\therefore$ DB = $\frac{1}{2}CR$, but QC = CR, $\therefore$ AD : DB = $\frac{1}{2}QC : \frac{1}{2}CR = 1:1$

15. HP // KQ and MP = PQ, $\therefore$ MH = HK (int. thm.). KR // HP and NR = RP,

$\therefore$ NK = HK (int. thm.). t cm = NK = HK = MH = 5 cm, $t = 5$.

KR = $\frac{1}{2}$HP = 3 cm (mid-pt. thm.). KQ = 2HP = 12cm (mid-pt. thm.)

$\therefore$ $3 + x = 12$, $x = 9$

16. $\because$ $\angle SQT = \angle QPR$ (given), $\because$ QT//PU (corr. $\angle$s eq.), QR//TU (given),

$\therefore$ RQTU is a //gram, $\therefore$ RU = 7 cm (prop. of // gram).

$\because$ PQ = QS and QR//SU (given), $\therefore$ $a = 7$ (intercept thm)

$\angle PQR = \angle S$ (corr. $\angle$s, QR//SU), but $\angle S = \angle P$, $\therefore$ $\angle PQR = \angle P$,

$\therefore$ QR = 7 (sides opp. eq. $\angle$s), $\therefore$ $b = 7$ (prop. of // gram)

17. (a) $\because$ QL = LP = 10 and QM = MR = 15, $\therefore$ LM//PR (mid-pt. thm.)

(b) $z = 2(14) = 28$ (mid-pt. thm.). $\because$ $\Delta LMN \sim \Delta RPN$ (A.A.A.),

$\therefore$ $\frac{x}{6} = \frac{y}{13} = \frac{28}{14} = 2$ (corr. sides, $\sim \Delta$s), $\therefore x = 2\times 6 = 12$ and $y = 2\times 13 = 26$

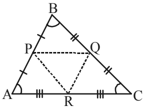
18. $\because$ BP = PA and BQ = QC (given), $\therefore$ PQ = $\frac{1}{2}\times$ AC and PQ//AC (mid-pt. thm.),

$\because$ DS = SA and DR = RC (given), $\therefore$ SR = $\frac{1}{2}\times$ AC and SR//AC (mid-pt. thm.),

$\therefore$ PQ = SR and PQ//SR, $\therefore$ PQRS is a // gram (2 sides eq. and //),

$\therefore$ PS = QR (prop. of // gram)

19. Let P, Q, R be the mid-points of AB, BC and AC respectively.
Join PQ, QR and PR. $\because$ BP = PA and BQ = QC
(construction), $\therefore$ PQ = $\frac{1}{2}$AC (mid-pt thm),



$\therefore$ PQ = AR = RC. Similarly, QR = BP = PA and PR = BQ = QC.

$\therefore$ $\Delta BPQ \cong \Delta PAR \cong \Delta QRC \cong \Delta RQP$ (S.S.S.)

20. (a) $\because$ RA = AQ and RB = BP (given), $\therefore$ AB $= \frac{1}{2}$ PQ (mid-pt. thm.),

$\because$ PC = CS and PB = BR (given), $\therefore$ BC $= \frac{1}{2}$ RS (mid-pt. thm.),

but PQ = RS (given), $\therefore$ AB = BC, $\therefore$ ΔABC is isosceles

(b) $\angle QPR = 180^o - 95^o - 15^o = 70^o$ ($\angle$ sum of Δ), $\because$ AB//QP (mid-pt. thm.),

$\therefore$ $\angle ABR = 70^o$ (corr. $\angle$s, AB//QP). BC//RS (mid-pt. thm.),

$\therefore$ $\angle RBC + 100^o = 180^o$ (int. $\angle$s, BC//RS), $\angle RBC = 80^o$,

$\therefore$ $\angle ABC = 70^o + 80^o = 150^o$

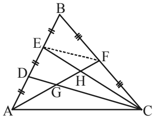
21. Join EF. BE = ED and BF = FC,

$\therefore$ DC = 2EF and DC//EF (mid-pt. thm.)

AD = DE and DC//EF (proved), $\therefore$ AG = GF (intercept thm.)

$\therefore$ EF = 2 DG (mid-pt. thm.). Let DG = x, $\therefore$ EF = $2x$,

DC = $4x$, GC = $4x - x = 3x$. $\therefore$ DG : GC = x : $3x$ = 1 : 3



22. (a) In ΔKHP and ΔKHQ, KH = KH (common), $\angle KHP = \angle KHQ = 90^o$ (given),

$\angle$HKP = $\angle$HKQ (given), $\therefore$ $\Delta KHP \cong \Delta KHQ$ (A.S.A.),

$\therefore$ HP = HQ (corr. sides, $\cong \Delta$s), SP = SR (given),

$\therefore$ HS//QR (mid-pt. thm.)

(b) $\because$ HP = HQ and HS//QR (proved), $\therefore$ PL = LK (intercept thm.),

$\therefore$ HL $= \frac{1}{2}$ QK (mid-pt. thm.), but QK = PK (corr. sides, $\cong \Delta$s), $\therefore$ HL $= \frac{1}{2}$ PK

23. (a) $\because$ $\Delta MUW \sim \Delta SPW$ (A.A.A.),

$\therefore$ $\frac{PW}{UW} = \frac{PS}{UM} = \frac{18}{6} = 3$ (corr. sides, $\sim \Delta$s), PW = 3 UW,

PU = 3UW + UW = 4UW. $\because$ QR//TU (given),

$\therefore$ $\frac{PU}{7} = \frac{12}{9} = \frac{4}{3}$ (extension of int. thm.), PU $= \frac{28}{3}$, $\therefore$ 4UW $= \frac{28}{3}$, UW $= \frac{7}{3}$

(b) $\because$ $\Delta WQR \sim \Delta WMU$ (A.A.A.), $\therefore$ $\frac{QR}{MU} = \frac{WR}{WU}$ (corr. sides, $\sim \Delta$s),

$$\frac{QR}{6} = \frac{7 + \frac{7}{3}}{\frac{7}{3}} = \frac{21+7}{7} = 4, \quad \therefore \text{ QR} = 4 \times 6 = 24$$

24. (a) $\because$ QH = HR and PK = KR (given), $\therefore$ HK//QP (mid-pt. thm.),

$\therefore$ $\angle KHR = 90^o$ (corr. $\angle$s, HK//QP),

$\therefore$ $\angle KHQ = 180^o - 90^o = 90^o$ (adj. $\angle$s on st. line).

In ΔQHK and RHK, QH = RH (given), KH = KH (common),

$\therefore$ $\angle KHQ = \angle KHR = 90°$, $\therefore$ $\Delta QHK \cong \Delta RHK$ (S.A.S.),

$\therefore$ QK = RK (corr. sides, $\cong \Delta$s), $\therefore$ ΔRKQ is isosceles

(b) $\because$ QK = RK (proved) and PK = PK (given), $\therefore$ PK = QK,

$\therefore$ ΔPKQ is isosceles

25. In ΔDEB and ΔDEF, BD = DF = 7cm (given), DE = DE (common),
$\angle$DEB = $\angle$DEF = 90^o (given), ∴ ΔDEB ≅ ΔDEF (R.H.S.)
∴ EF = BE = 5cm (corr. sides, ≅ Δs). ∵ $\angle$DEF = $\angle$ AFC = 90^o (given),
∴ DE//AF (corr. $\angle$ s eq.), BE = EF = 5cm (proved),
∴ DA = BD = 7cm (intercept thm.).
∵ BD = DA (proved) and DF//AC (given),
∴ FC = BF = 5 + 5 = 10cm (intercept thm.)

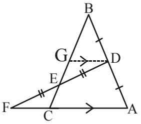
26. ∵ CD//BH (given), ∴ $\frac{DH}{HA} = \frac{CB}{BA} = \frac{7}{3}$ (extension of intercept thm.),

AH = $\frac{3}{7}$ DH. ∵ DF//HG (given),

∴ $\frac{DE}{DH} = \frac{EF}{FG} = \frac{15}{12} = \frac{5}{4}$ (extension of int. thm.), DE = $\frac{5}{4}$ DH,

∴ $\frac{AD}{DE} = \frac{AH + DH}{DE} = \frac{\frac{3}{7}DH + DH}{\frac{5}{4}DH} = \frac{10}{7} \div \frac{5}{4} = \frac{8}{7}$, ∴ AD : DE = 8 : 7

27. (a) Draw GD//FA,



∵ BD = DA (given) and GD//FA (construction),
∴ BG = GC (intercept thm.), ∴ AC = 2DG (mid-pt. thm.),
In ΔDEG and ΔFEC, DE = FE (given),
$\angle$ DEG = $\angle$ FEC (vert. opp. $\angle$ s),
$\angle$ GDE = $\angle$ CFE (alt. $\angle$ s, GD//FC), ∴ ΔDEG ≅ ΔFEC (A.S.A.),
∴ DG = FC (corr. sides, ≅ Δ s), $\frac{AC}{FC} = \frac{2DG}{DG} = 2$, ∴ AC : FC = 2 : 1

(b) ∵ ΔDEG ≅ ΔFCE (proved), ∴ GE = CE (corr. sides, ≅ Δ s),
BE = BG + GE = GC + GE = EC + GE + GE = 3EC, ∴ BE : EC = 3: 1

28. (a) In ΔBAD and ΔBCD, BD = BD (common), $\angle$ BDA = $\angle$ BDC = 90^o (given),
$\angle$ ABD = $\angle$ CBD (given), ∴ ΔBAD ≅ ΔBCD (A.S.A.),
∴ AD = CD (corr. sides, ≅ Δ s), i.e. D is the mid-pt. of AC

(b) ∵ AE = EF (given) and AD = DC (proved), ∴ DE//CF (mid-pt thm)

(c) DE = $\frac{1}{2}$ CF (mid-pt thm), i.e. DE = $\frac{1}{2}$(BC – BF),

but AB = BC (corr. sides, ≅ Δ s), ∴ DE = $\frac{1}{2}$(AB – BF)

29. (a) ∵ AM = MB and CN = NB (given), ∴ MN//AC (mid-pt. thm.)
In ΔABC and ΔMBN, $\angle$ B = $\angle$ B (common),
$\angle$ BAC = $\angle$ BMN (corr. $\angle$ s, MN//AC),
$\angle$ BCA = $\angle$ BNM (corr. $\angle$ s, MN//AC), ∴ ΔABC ~ ΔMBN (A.A.A.)
In ΔABC and ΔCBM, $\angle$ B = $\angle$ B (common), $\angle$ BAC = $\angle$ BCM (given),
$\angle$ BCA and $\angle$ BMC (3^{rd} $\angle$ of Δ), ∴ ΔABC ~ ΔCBM (A.A.A.)

Ans. ΔMBN *and* ΔCBM *are similar to* ΔABC.

(b) $\because$ $\Delta ABC \sim \Delta CBM$ (proved), $\therefore$ $\frac{BA}{BC} = \frac{BC}{BM}$ (corr. sides, $\sim \Delta$s),

$\therefore$ $BC^2 = BM \times BA$

(c) BA = 2BM and BC = 2BN (given), $\because$ $BC^2 = BM \times BA$,

$\therefore$ $(2BN)^2 = BM \times 2BM$, $4BN^2 = 2BM^2$, $2 = \frac{BM^2}{BN^2}$, $\frac{BM}{BN} = \sqrt{2}$,

$\therefore$ $BM : BN = \sqrt{2} : 1$

Unit 14 Measures of central tendency

1. (a) $\text{Mean} = \frac{49}{8} = 6.125$, Mode = 8, $\text{Median} = \frac{6+8}{2} = 7$

(b) $-14, -11, -11, -10, -7$; $\text{Mean} = \frac{-53}{5} = -10.6$, $\text{Mode} = -11$, $\text{Median} = -11$

(c) $-18, -13, -10, 13, 15, 18$; $\text{Mean} = \frac{5}{6}$, No mode, $\text{Median} = \frac{-10+13}{2} = 1.5$

(d) $-8^o C, -5^o C, -4^o C, -1^o C, 0^o C, 0^o C, 2^o C$;

$\text{Mean} = \frac{-16}{7} = -2\frac{2}{7}{}^o C$, Mode = 0°C, Median = −1°C

(e) $2.4m^2, 6.2m^2, 7m^2, 9.4m^2, 10m^2, 11.5m^2, 12m^2$;

$\text{Mean} = \frac{58.5}{7} = 8.36m^2$, No mode, $\text{Median} = 9.4m^2$

(f) $20.4, 23.2, 24.1, 24.3, 24.7, 24.7, 28.5$;

$\text{Mean} = \frac{169.9}{7} = 24.3^o C$, Mode = 24.7° C, Median = 24.3°C

(g) $x-2, x-1, x, x, 3x, 5x$; $\text{Mean} = \frac{12x-3}{6}$, Mode = x, $\text{Median} = \frac{x+x}{2} = x$

$60cm, 0.6m, 75cm, 1m, 1.3m$;

$\text{Mean} = \frac{425}{5} = 85cm$, Mode = $60cm$, Median = $75cm$

2. (a) $\text{Mean} = \frac{12\times12+13\times10+14\times15+15\times13}{12+10+15+13} = \frac{679}{50} = 13.58$, Median = 14, Mode = 14

$\text{Mean} = \frac{1\times6+2\times15+3\times16+4\times1+5\times2}{6+15+16+1+2} = \frac{98}{40} = 2.45$, Median = 2, Mode = 3

3. (a) The class marks are 145, 245, 345, 445 and 545.

Total number of shoes = 7 + 16 + 33 + 24 + 20 = 100;

$\therefore$ Mean = $(145\times7+245\times16+345\times33+445\times24+545\times20)\div100$

$= 37900 \div 100 = \$379$

(b) The modal class is $\$300 - \$390$.

4\. (a)

Weight (kg)	30 – 34	35 – 39	40 – 44	45 – 49	50 – 54
Frequency	3	14	9	4	6

(b) The modal class is 35*kg* – 39*kg*.

5\. (a) Weight mean score of Apple $=\dfrac{72\times3+85\times3+74\times2}{3+3+2}=\dfrac{619}{8}=77.4$

Weight mean score of Banana $=\dfrac{64\times3+87\times3+76\times2}{8}=\dfrac{605}{8}=75.6$

Weight mean score of Cherry $=\dfrac{74\times3+67\times3+84\times2}{8}=\dfrac{591}{8}=73.9$

(b) Apple achieved the best result.

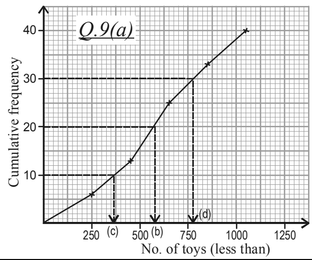
6\. (a) The modal class is 161*cm*–165*cm*.

(b) Mean $=\dfrac{153\times3+158\times9+163\times14+168\times12+173\times2}{40}=\dfrac{6525}{40}\approx163.1cm$

7\. (a) $A = 25$, $B = 11500$ (b) Median salary = \$11500

8\. The class marks are
14.5, 24.5, 34.5, 44.5, 54.5, 64.5 and 74.5.
Mean = (14.5×18+24.5×11+34.5×15+ 44.5×26 +54.5×22+64.5×30+74.5×28)÷150 = 7425 ÷ 150 = 49.5

9\. (b) From the graph, the median daily production is 575.
(c) From the graph, the lower quartile is 375.
(d) From the graph, the upper quartile is 775.



10\. (a)

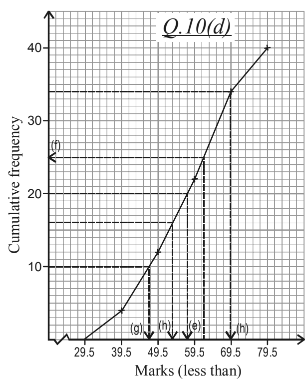
Marks	Class boundaries	Class mark(x)	Frequency (f)	fx
30 – 39	29.5 – 39.5	34.5	4	138
40 – 49	39.5 – 49.5	44.5	8	356
50 – 59	49.5 – 59.5	54.5	10	545
60 – 69	59.5 – 69.5	64.5	12	774
70 – 79	69.5 – 79.5	74.5	6	447
		Total:	40	2260

(b) Mean mark $=\dfrac{2260}{40}=56.5$.

The modal class is 60 – 69.

(c)

Marks less than	Cumulative frequency
39.5	4
49.5	12
59.5	22
69.5	34
79.5	40



(e) From the graph, the median mark is 57.5.

(f) Percentage $=\dfrac{40-25}{40}\times100\%=37.5\%$

(g) 75% of the students pass means 25% of the students fail, from the graph, the passing mark is 47.

(h) $40\times40\%=16$, from the graph, the 40^{th} percentile is 43.5.

$40 \times 85\% = 34$, from the graph, the 85th percentile is 69.5.

11. Mean fare $= \dfrac{11.9 \times 12 + 6.9 \times 8}{20} = \dfrac{198}{20} = \$9.9$

12 $\dfrac{3 + x + 2x + (x+6) + 11}{5} = 8$, $4x + 20 = 40$, $\therefore\ x = 5$

13. Salary in December $= 9000 \times 12 - 8500 \times 11 = \$14500$

14. Mean $= \dfrac{a+b+c+d+17+19}{6} = \dfrac{15 \times 4 + 17 + 19}{6} = \dfrac{96}{6} = 16$

15. Mean weight $= \dfrac{48 \times 25 + 35 \times 15}{40} = \dfrac{1725}{40} = 43.125kg$

16. The last number $= 14 \times 10 - 15 \times 9 = 5$

17. Correct mean mark $= \dfrac{65 \times 40 - 17 + 71}{40} = \dfrac{2654}{40} = 66.35$

18. (a) Mean age = 22 + 9 = 31

 (b) Mean age of the remaining 10 members $= \dfrac{31 \times 11 - 36}{10} = \dfrac{305}{10} = 30.5$

19. $\because$ x is the median, $\therefore\ 6 \le x \le 8$, $\therefore$ Possible values of x are 6, 7 and 8.

20. $\dfrac{17 + 20 + y + 27 + 31 + 36}{6} = \dfrac{y+27}{2}$, $\dfrac{131+y}{6} = \dfrac{y+27}{2}$,

 $131 + y = 3y + 81$, $2y = 50$, $\therefore\ y = 25$

21. $\dfrac{3 + x + 8 + 9}{4} = 6.5$, $20 + x = 26$, $\therefore\ x = 6$. $\dfrac{3 \times 5 + 6 \times 6 + 8 \times 9 + 9 \times y}{5 + 6 + 9 + y} = 7.1$,

 $\dfrac{123 + 9y}{20 + y} = 7.1$, $123 + 9y = 142 + 7.1y$, $1.9y = 19$, $\therefore\ y = 10$

22. (a) 2,4,5,6,6,10,12,15; Mean $= \dfrac{60}{8} = 7.5$, Mode = 6, Median $= \dfrac{6+6}{2} = 6$

 (b) (i) Mean $= \dfrac{7.5 \times 8 + 12}{9} = 8$, Mode = 6 and 12, Median = 6

 (ii) Mean $= \dfrac{7.5 \times 8 - 4}{7} = 8$, Mode = 6, Median = 6

 (iii) Mean $= \dfrac{7.5 \times 8 + 0 + 0}{10} = 6$, Mode = 0 and 6, Median $= \dfrac{5+6}{2} = 5.5$

 (iv) Mean $= 7.5 + 4 = 11.5$, Mode = 6 + 4 = 10, Median = 6 + 4 = 10

 (v) Mean $= 7.5 \times 2 = 15$, Mode $= 6 \times 2 = 12$, Median $= 6 \times 2 = 12$

23. $\dfrac{x}{6}, \dfrac{x}{2}, \dfrac{2x}{3}, \dfrac{3x}{4}, \dfrac{6x}{5}$; $\dfrac{2x}{3} = 10$, $\therefore\ x = 15$

24. $\dfrac{4 + x + y + 7}{4} = 6$, $x + y + 11 = 24$, $y = 13 - x$; $\dfrac{3 + y + 2x}{3} = 7$, $3 + (13 - x) + 2x = 21$,

 $16 + x = 21$, $\therefore\ x = 5$, $\therefore\ y = 13 - 5 = 8$

25. $\dfrac{k\times n-3-7-12}{n-3}=k$, $nk-22=nk-3k$, $3k=22$, $\therefore\ k=\dfrac{22}{3}$

26. Median $=20\times 9-25\times 4-16\times 4=16$

27. $a+b+c=12\times 5-9\times 2=42$, but $b=a+2$ and $c=a+4$,
$\therefore\ a+(a+2)+(a+4)=42$, $3a+6=42$, $3a=36$, $\therefore\ a=12$,
$b=12+2=14$ and $c=12+4=16$

28. (a) $x-9, x-5, x+3, x+7, x+9$; Median $=x+3$

(b) $\dfrac{(x-9)+(x-5)+(x+3)+(x+7)+(x+9)}{5}=\dfrac{x+3}{2}$, $\dfrac{5x+5}{5}=\dfrac{x+3}{2}$,
$2x+2=x+3$, $\therefore\ x=1$

29. $\because$ Mode = 33 and $p<q$, $\therefore\ p=33$.
$\because$ Median = 37, $\therefore\ \dfrac{q+39}{2}=37$, $q+39=74$, $\therefore\ q=35$

30. (a) Mean mark $=\dfrac{76\times 40+58\times 32}{72}=\dfrac{4896}{72}=68$

(b) Correct mean mark $=\dfrac{76\times 39+58\times 32-48+84}{72}=\dfrac{4856}{72}=67.4$

31. (a) $\dfrac{63\times 1+84\times 2+77\times 1+n\times 3}{1+2+1+3}=83$, $\dfrac{308+3n}{7}=83$, $308+3n=581$,
$3n=273$, $\therefore\ n=91$

(b) The new result = $\dfrac{63\times 1+84\times 2+77\times 1+91\times 4}{1+2+1+4}=84$,
$\therefore$ Percentage change = $\dfrac{84-83}{83}\times 100\%=1.20\%$ (increase)

32. (a) $15+32+x+y+6+4=100$, $y=43-x$;
$\dfrac{0\times 15+1\times 32+2x+3y+4\times 6+5\times 4}{100}=1.75$, $2x+3y+76=175$,
$2x+3(43-x)=99$, $-x=-30$, $\therefore\ x=30$, $\therefore\ y=43-30=13$

(b) Modal number = 1 (c) Median number = 2

33. (a) 32, 36, 40, 48, 52, 60; Mean $=\dfrac{268}{6}=44\dfrac{2}{3}$, Median $=\dfrac{40+48}{2}=44$

(b) Let x be the number removed, $\dfrac{268-x}{5}=44\dfrac{2}{3}-\dfrac{22}{15}=\dfrac{216}{5}$, $268-x=216$,
$x=52$. *Ans. The removed number is 52.* $\therefore$ The new median is 40.

34. $\because$ Mode = 16, $\therefore\ r+1=16$, $\therefore\ r=15$. $\dfrac{(p+1)+q}{2}=11$, $p+1+q=22$,
$p=21-q$; $\dfrac{4+6+6+p+(p+1)+(p+1)+q+q+15+16+16+16}{12}=11$,
$3p+2q+81=132$, $3(21-q)+2q=51$, $-q=-12$, $\therefore\ q=12$, $\therefore\ p=21-12=9$

35. (a) Median = y = 36; $\frac{x}{36} = \frac{3}{4}$, $\therefore$ $x = 27$; $\frac{z}{36} = \frac{2}{4}$, $\therefore$ $z = 18$

(b) Let $x = 3k$, $y = 4k$, $z = 2k$, $\frac{3k + 3k + 4k + 2k + 2k}{5} = 8.4$

$14k = 42$, $k = 3$, $\therefore$ $x = 9$, $y = 12$, $z = 6$

36. (a) Every datum is multiplied by 3, $\therefore$ Mean = $3x$, Mode = $3y$, Median = $3z$

(b) Each number of data is trebled, $\therefore$Mean = x, Mode = y, Median = z

(c) Every datum is multiplied by –1 and then plus 4,

$\therefore$ Mean = $4 - x$, Mode = $4 - y$, Median = $4 - z$

37. $x + 10$ and $x + 16$ are the greatest, and $4 - x < 8 - x$. $\because$ $8 - x$ is the median,

$\therefore$ $8 - x \geq x$, $2x \leq 8$, $x \leq 4$, since x is positive, $\therefore$ x = 1, 2, 3 or 4

38. (a) $18 - n > 15, n < 3 \ldots$ (i); $18 - n > n + 1, 17 > 2n$. $n < 8.5 \ldots$ (ii)

Combining the two cases, $n < 3$. *Ans. Possible values of n are 0, 1 and 2.*

(b) $n + 1 > 15$, $n > 14 \ldots$ (i); $n + 1 > 18 - n$, $2n > 17$, $n > 8.5 \ldots$ (ii);

$18 - n \geq 0$, $n \leq 18 \ldots$ (iii) Combining the three cases, $14 < n \leq 18$.

Ans. Possible values of n are 15, 16, 17 and 18.

(c) If median is 14, $10 + 18 - n > n + 1 + 15$, $12 > 2n$, $n < 6$; on the other hand,

mode = 14 when n = 0, 1 or 2.

$\therefore$ It is possible for both the median and modal age to be 14.

39. (a) Mean = $\frac{8400 + 10000 + 14000 + 20000 + 25000 + 28000 + 120000}{8} = \$28175$,

Median = $\frac{14000 + 20000}{2} = \$17000$

(b) Median is a better measure of central tendency since it is not affected by the extreme data ($0 and $120,000).

40. (a) Extreme data exist and these data are much higher than the average.

(b) Extreme data exist and these data are much lower than the average.

(c) If Class A is chosen, the reason should be: (1) there are some very able students in Class A; or (2) there are some very weak students in Class B. If Class B is chosen, the reason should be: the difference between the mean and the median is smaller in Class B, and therefore the difference among students are not so great in Class B as in Class A.

Unit 15 Introduction to probability

1. P(not W) = $\frac{8 + 6}{10 + 8 + 6} = \frac{14}{24} = \frac{7}{12}$

2. Favourable outcomes: 3, 6, 9, … 27, 30; $\therefore$ P(correct date) = $\frac{10}{31}$

3. P(good apple) = $\frac{100 - 16}{100 - 1} = \frac{84}{99} = \frac{28}{33}$

4. P(red or Queen) = $\frac{26+2}{52}=\frac{28}{52}=\frac{7}{13}$

5. (a) Favourable outcomes: 6, 12, 18, 24, 30; $\therefore$ P(even and multiple of 3) $=\frac{5}{30}=\frac{1}{6}$

(b) Favourable outcomes: 1 – 9, 11, 13, 17, 19, 23, 29;

$\therefore$ P(prime number or smaller then 10) $=\frac{15}{30}=\frac{1}{2}$

(c) P(an integer) $=\frac{30}{30}=1$ (d) P(divisible by 40) $=\frac{0}{30}=0$

(e) Favourable outcomes: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30. $\therefore$ P(factor of 60) $=\frac{11}{30}$

Let n be the number of \$2 coins. $\frac{18}{18+n}=\frac{2}{3}, 54=36+2n, 18=2n, n=9$

Ans. The number of \$2 coins is 9.

7. P(2nd one is \$2 coin) $=\frac{3}{7+3-1}=\frac{3}{9}=\frac{1}{3}$

8. Number of possible outcomes $=2\times2\times2=8$; Favourable outcomes: BBB, BBG, BGB, GBB;

$\therefore$ P(not more than 1 girl) $=\frac{4}{8}=\frac{1}{2}$

9. (a) P(6) $=\frac{20}{500}=\frac{1}{25}$ (b) P(smaller than 4) $=\frac{100+160+140}{500}=\frac{400}{500}=\frac{4}{5}$

10. Number of green balls $=12\times\frac{51}{200}=3.06\approx3$

Number of blue balls $=12\times\frac{69}{200}=4.14\approx4$

Number of white balls $=12\times\frac{80}{200}=4.8\approx5$

11. Number of possible outcomes $=2\times2\times2=8$.

Favourable outcomes: HTT, THT, TTH; $\therefore$ P(1H and 2T) $=\frac{3}{8}$.

Favourable outcome: TTT; $\therefore$ P(no H) = $\frac{1}{8}$

12. (a) Number of possible outcomes $=6\times6=36$

Favourable outcomes: (5,5), (5,6), (6,5), (6,6); $\therefore$ P(both not less than 5) $=\frac{4}{36}=\frac{1}{9}$

(b) Favourable outcomes: (1,1),(2,2), (2,1), (3,2), (3,3), (3,1), ….., (6,6), (6,5), (6,4), (6,3), (6,2), (6,1); $\therefore$ P(1st no. not smaller than 2nd no.) $=\frac{21}{36}=\frac{7}{12}$

(c) Unfavourable outcomes: (1,3), (2,2), (2,6), (3,1), (3,5), (4,4), (5,3), (6,2), (6,6);

$\therefore$ P(sum is not multiple of 4) $=\dfrac{36-9}{36}=\dfrac{27}{36}=\dfrac{3}{4}$

(d) Favourable outcomes: (1,4), (2,4), (3,4), (5,4), (6,4), (4,1), (4,2), (4,3), (4,5), (4,6);

$\therefore$ P('4' occurs exactly once) $=\dfrac{10}{36}=\dfrac{5}{18}$

(e) Favourable outcomes: (1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5)

$\therefore$P(product is odd) $=\dfrac{9}{36}=\dfrac{1}{4}$

(f) Favourable outcomes: (2,3), (3,2); $\therefore$ P('2' and '3') $=\dfrac{2}{36}=\dfrac{1}{18}$

(g) Favourable outcomes: (1,2), (2,1), (2,3), (3,2), ….. , (5,4), (5,6), (6,5);

$\therefore$ P(difference =1) $=\dfrac{10}{36}=\dfrac{5}{18}$

13. His expected age = $13\times\dfrac{5}{36}+14\times\dfrac{20}{36}+15\times\dfrac{11}{36}= 14\dfrac{1}{6}$=14.2 years old.

14. (a) P (6-mark region) = $\dfrac{5^2}{40^2}=\dfrac{1}{64}$, P (3-mark region) = $\dfrac{20^2-5^2}{40^2}=\dfrac{15}{64}$

P (1-mark region) = $\dfrac{40^2-20^2}{40^2}=\dfrac{3}{4}$

(b) The expected score = $6\times\dfrac{1}{64}+3\times\dfrac{15}{64}+1\times\dfrac{3}{4}= 1\dfrac{35}{64}=1.55$

15. Favourable outcomes: (O,F), (F,O), (R,O), (O,R), (T,O), (O,T)

No. of possible outcome $=4\times3=12$, $\therefore$ P(meaningful English word) $=\dfrac{6}{12}=\dfrac{1}{2}$

16. Favourable outcomes: (49, 52), (49, 60), (52, 49), (60, 52), (52, 60), (60, 49)

No. of possible outcomes $=4\times3=12$, $\therefore$ P(exceed 100 kg) $=\dfrac{6}{12}=\dfrac{1}{2}$

17. (a) There are 20 possible outcomes: Favourable outcomes: (2,3), (2,9), (2,11), (3,2), (9,2), (11,2), (11,12), (12,11); $\therefore$ P(prime number) $=\dfrac{8}{20}=\dfrac{2}{5}$

(b) Favourable outcomes: (2,3), (3,2), (3,12), (9,11), (11,9), (12,3);

No. of possible outcomes = $5\times4=20$, $\therefore$ P(divisible by 5) $=\dfrac{6}{20}=\dfrac{3}{10}$

18. Number of possible outcomes $=10\times10\times10=1000$,

$\therefore$ P(open the safe in 1^{st} trial) $=\dfrac{1}{1000}$

19. Let n be the number of girls,

$\therefore$ P(getting a girl) $=\dfrac{n}{n+55\%n}=\dfrac{n}{1.55n}=\dfrac{100}{155}=\dfrac{20}{31}$

20. Let n be the total number of students,

$\therefore$ P(boy not wearing glasses) $= \dfrac{n \times 40\% \times (1-30\%)}{n} = \dfrac{40}{100} \times \dfrac{70}{100} = \dfrac{7}{25}$

21. Number of students passed both subjects $= 200 - (200 \times 27\% + 200 \times 32\% - 16) = 98$,

 $\therefore$ P(the student passed both subjects) $= \dfrac{98}{200} = \dfrac{49}{100}$

22. The possible position of the yellow ball: YOOO, <u>OYOO</u>, <u>OOYO</u>, OOOY

 $\therefore$ P(at least one orange ball separated by yellow ball) $= \dfrac{2}{4} = \dfrac{1}{2}$

23. P(get a marked fish) = $\dfrac{50}{x} = \dfrac{4}{50}, 4x = 2500, x = 625.$

 Ans. The approximate number of fish is 625.

24 (a) Favourable outcomes: 3004, 3008, 3012, …., 3096, 3100;

 $\therefore$ P(multiple of 4) $= \dfrac{25}{100} = \dfrac{1}{4}$

 (b) Favourable outcomes: 3003, 3006, 3009, …., 3096, 3099;

 $\therefore$ P(multiple of 3) $= \dfrac{33}{100}$

 (c) There are 8 multiples of 12: 3012, 3024, 3036, …., 3096;

 $\therefore$ P(either multiple of 3 or multiple of 4) $= \dfrac{25+33-8}{100} = \dfrac{50}{100} = \dfrac{1}{2}$

25. Let n be the number of white balls. $\because$ P(W) $= \dfrac{4}{9} < \dfrac{1}{2}$,

 $\therefore$ the no. of black balls $= n + 4$. $\dfrac{n}{n+n+4} = \dfrac{4}{9}$, $9n = 8n + 16$, $n = 16$,

 $\therefore$ Total number of balls $= 16 + 16 + 4 = 36$

26. The last 2 digits must be a number divisible by 4. Favourable outcomes: 0, 4, 8;

 $\therefore$ P(divisible by 4) $= \dfrac{3}{10}$

27. Assume the seat of B is fixed. Number of possible seats of A = 4,

 number of seats not next to B = 2, $\therefore$ P(A doesn't sit next to B) $= \dfrac{2}{4} = \dfrac{1}{2}$

28. Number of possible outcomes $= 4 \times 4 = 16$. Favourable outcomes: ES, SE (E: east, S: south).

 $\therefore$ P(reach $(-1,2)$ after 2 moves) $= \dfrac{2}{16} = \dfrac{1}{8}$

29. Number of favourable outcomes = $3 \times 4 = 12$.

 There are 2 unfavourable outcomes: (R, R), (R,R)

 $\therefore$ P(different colours) $= \dfrac{12-2}{12} = \dfrac{10}{12} = \dfrac{5}{6}$

30. Favourable outcomes: 101, 102, …., 109; 110, 120, …, 190,

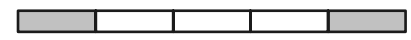
 201, 202, …., 209; 210, 220, …., 290,

……

901, 902, …., 909; 910, 920, …., 990;

$\therefore$ P(exactly one digit is 0) $= \dfrac{9\times9+9\times9}{900} = \dfrac{162}{900} = \dfrac{9}{50}$

31. Divide the rod into 5 equal parts. If the rod is cut at a point in the shaded regions, the longer part will be at least 4 times as long as the shorter part.



$\therefore$ *The required probability* $= \dfrac{2}{5}$

32. (a) No. of possible outcomes $= 2\times2\times2 = 8$. Unfavourable outcomes: all go to restaurant B, $\therefore$ P(at least one go to A) $= \dfrac{8-1}{8} = \dfrac{7}{8}$

(b) P(same restaurant) $= \dfrac{2}{8} = \dfrac{1}{4}$

33. Let the envelopes and the letters for Adam, Benjamin and Cart be E_A, E_B, E_C and L_A, L_B, L_C respectively.
Possible outcomes: (E_AL_A, E_BL_B, E_CL_C), (E_AL_A, E_BL_C, E_CL_B), (E_AL_B, E_BL_A, E_CL_C), (E_AL_B, E_BL_C, E_CL_A), (E_AL_C, E_BL_A, E_CL_B), (E_AL_C, E_BL_B, E_CL_A).

(i) P(only Adam right) $= \dfrac{1}{6}$

(ii) P(none of them right) $= \dfrac{2}{6} = \dfrac{1}{3}$

(iii) P(all of them right) $= \dfrac{1}{6}$

34. $\begin{cases} \dfrac{y}{24} = k, y = 24k \text{ ......(i)} \\ \dfrac{y+12}{24+12} = 2k, y = 72k - 12 \text{ ....(ii)} \end{cases}$

Sub. (i) into (ii), $24k = 72k - 12$, $48k = 12$, $k = \dfrac{1}{4}$. Put $k = \dfrac{1}{4}$ into (i), $y = 24(\dfrac{1}{4}) = 6$.

Ans. The solutions are $y = 6$ *and* $k = \dfrac{1}{4}$.

35. (a) $\begin{cases} m + n = 1 \text{ ......(i)} \\ m = 4n \text{ ......(ii)} \end{cases}$

Sub. (ii) into (i), $4n + n = 1$, $5n = 1$, $\therefore n = \dfrac{1}{5}$

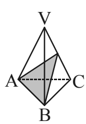
(b) Let x be the number of red marbles, $\dfrac{x}{y} = m = 4(\dfrac{1}{5}), x = \dfrac{4y}{5} = 0.8y$

Ans. The number of red marbles is 0.8y.

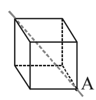
Unit 16 3-D figures

1. A and C.
2. B.
3. (b) There are 6 planes of reflection ($\because$ there are 6 edges).

3. (a)

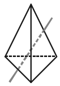


4. (a)



4. (b) Order 2. It is drawn by joining the mid-points of two opposite sides of the cube. Since there are 6 pairs of opposite sides in a cube, 6 axes of rotational symmetry of order 2 can be found.

(c) No. of axes of rotational symmetry of order 2 = 6
No. of axes of rotational symmetry of order 3 = 4
No. of axes of rotational symmetry of order 4 = 3

5. (a) 

(b) Order 3. It is drawn by joining the vertex and the centre of the opposite base of the tetrahedron. Since there are 4 vertices in a tetrahedron, 4 axes of rotational symmetry of order 3 can be found.

(c) No. of axes of rotational symmetry of order 2 = 3

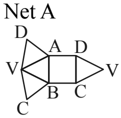
No. of axes of rotational symmetry of order 3 = 4

6. C, E, F, G, H, J and K 7. 3 8. (a) S (b) Q 9. D

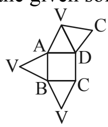
10. (a) Rectangular (right) pyramid. (b) G (c) B

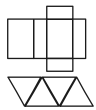
11. (A) and (C) are the nets of the given solid.

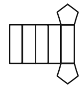
Net A



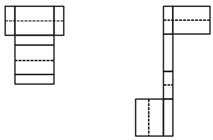
Net C

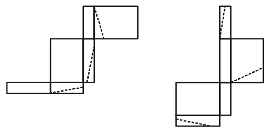


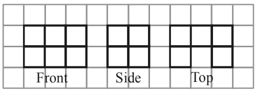
12. (a) 

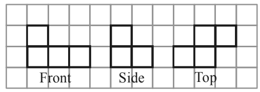
(b) 

(c)

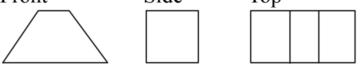
13. (a) 

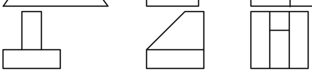
(b) 

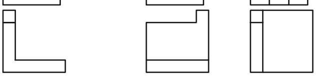
14. (a) 

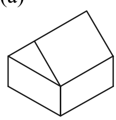
(b) 

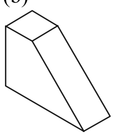
15. (a) Front Side Top

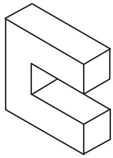


(b) 

(c) 

16. (a) 

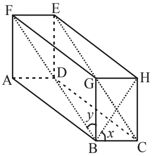
(b) 

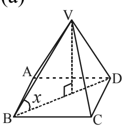
(c) 

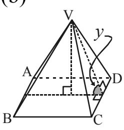
17. (a) When F = 8, F = 18, V – 18 + 8 = 2, ∴ V = 12

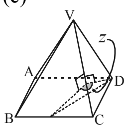
(b) When F = 9, V = 14, 14 – E + 9 = 2, ∴ E = 21

(c) When V = 20, E = 30, 20 – 30 + F = 2, ∴ F = 12

18. 

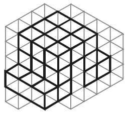
19. (a) 

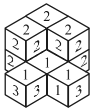
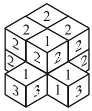
(b) 

(c) 

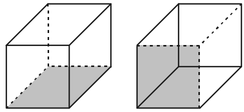
20. (a) 4, 4 (b) 6,6 (c) 6, 1 (d) 9, 9

(e) infinitely many, infinitely many (f) infinitely many, 1

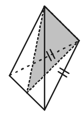
21. (a)


(b)

or


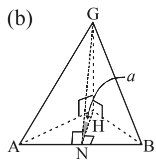
22. $y - z + x = 2$ (Euler's formula), $z - x = y - 2$;
when $F = x + 1$, $E = z + 5$, $V - (z + 5) + (x + 1) = 2$,
$\therefore\ V = 6 + z - x = 6 + (y - 2) = y + 4$.
Ans. The number of vertices is y + 4.

23.


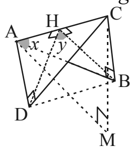
24. (a) $\because$ The internal triangle has 3 right angles, making a total sum of interior angles of 270°.
(b) $\because$ The lowest step is also the highest step, i.e. starting from the lowest step at the left, the steps keep rising and return to the starting position.

25. (a) A pyramid whose base is an isosceles triangle has one plane of reflectional symmetry but no axes of rotational symmetry. Therefore the statement is not true.

(b) A pyramid whose base is a parallelogram has one axis of rotational symmetry (from the vertex to the centre of the base) but no plane of reflectional symmetry. Therefore the statement is not true.

26. (a) $GH = FC = 6\text{cm}$. $AH^2 = DH^2 + DA^2 = 8^2 + 8^2 = 128$,
$\therefore GA = \sqrt{AH^2 + GH^2} = \sqrt{128 + 6^2} = \sqrt{164} = 2\sqrt{41}$ cm
$BH^2 = CH^2 + CB^2 = 4^2 + 8^2 = 80$,
$\therefore\ GB = \sqrt{BH^2 + GH^2} = \sqrt{80 + 6^2} = \sqrt{116} = 2\sqrt{29}$ cm
$\sin\angle GAH = \dfrac{GH}{GA} = \dfrac{6}{2\sqrt{41}}$, $\therefore\ \angle GAH = 27.94°$,
which is the angle between AG and ABCD.
$\sin\angle GBH = \dfrac{GH}{GB} = \dfrac{6}{2\sqrt{29}}$, $\therefore\ \angle GBH = 33.85°$,
Ans. The angle between BG and ABCD is 33.85°.

(b)


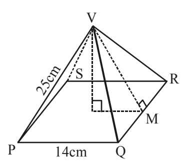
(c)
$HN = BC = 8\text{cm}$;
$\tan a = \dfrac{GH}{HN} = \dfrac{6}{8}$, $\therefore\ a = 36.87°$

27. (a)


(b) ΔDHB is an isosceles triangle.
(c) $AC = \sqrt{AD^2 + DC^2} = \sqrt{12^2 + 12^2} = \sqrt{288} = 12\sqrt{2}$ cm
BH and DH are half of the diagonal in Fig 27a,
$\therefore\ BH = DH = \frac{1}{2}BD = \frac{1}{2}AC = 6\sqrt{2}$ cm
(d) $\sin x = \dfrac{CM}{AC} = \dfrac{8}{\sqrt{288}}$, $\therefore\ x = 28.1°$. $\because$ ΔDHB is isosceles,
$$\therefore\ \sin(\frac{y}{2}) = \frac{\frac{1}{2}DB}{DH} = \frac{14.3}{2(6\sqrt{2})} = 0.8426,\quad \frac{y}{2} = 57.4°,\quad \therefore\ y = 114.8°.$$

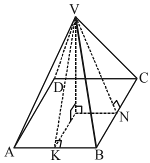
Unit 17 Pyramids, cones & spheres

1. (a) $VM = \sqrt{25^2 - (\frac{14}{2})^2} = \sqrt{576} = 24$ cm
Total surface area $= 14^2 + \dfrac{14 \times 24}{2} \times 4 = 196 + 672 = 868\,\text{cm}^2$



(b) Height $=\sqrt{24^2-(\frac{14}{2})^2}$ $=\sqrt{527}$ $=23.0$ cm

Volume $=\frac{1}{3}\times14^2\times\sqrt{527}$ $=1499.8\text{ cm}^3$



2. (a) Volume $=\frac{1}{3}\times20\times10\times12$ $=800\text{ cm}^3$

(b) VN $=\sqrt{12^2+(\frac{10}{2})^2}$ $=\sqrt{169}$ $=13$ cm

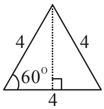
VK $=\sqrt{12^2+(\frac{20}{2})^2}$ $=\sqrt{244}$ $=2\sqrt{61}$ $=15.6$ cm

(c) Total surface area $=(\frac{13\times20}{2}+\frac{2\sqrt{61}\times10}{2})\times2+10\times20$ $=616.2\text{ cm}^2$

3. Diagonals of a rhombus are perpendicular to and bisect each other,

$\therefore$ base area $=(\frac{1}{2}\times24\times\frac{10}{2})\times2$ $=120\text{ cm}^2$ $\therefore$ Volume $=\frac{1}{3}\times120\times16$ $=640\text{ cm}^3$

4. Let h cm be the height of the base. $\frac{h}{4}=\sin60^\circ=\frac{\sqrt3}{2}$, $h=2\sqrt3$



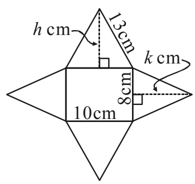
$\therefore$ Total surface area $=\frac{4\times2\sqrt3}{2}\times4$ $=16\sqrt3$ $=27.7\text{ cm}^2$

5. (a) Let h cm and k cm be the two slant heights.

$h=\sqrt{13^2-(\frac{10}{2})^2}=\sqrt{144}=12$, $k=\sqrt{13^2-(\frac{8}{2})^2}=\sqrt{153}$

$\therefore$ Total surface area $=(\frac{12\times10}{2}+\frac{\sqrt{153}\times8}{2})\times2+10\times8$

$=299.0\text{ cm}^2$



(b) Height of the pyramid $=\sqrt{12^2-(\frac{8}{2})^2}$ $=\sqrt{128}$ cm

$\therefore$ Volume $=\frac{1}{3}\times10\times8\times\sqrt{128}=301.7\text{ cm}^3$

6. (a) Let y cm be the side of the base. $\frac{1}{3}\times y^2\times15=2000$, $y^2=400$,

$y=\sqrt{400}=20$. $\therefore$ Area of the base $=20^2$ $=400\text{ cm}^2$

(b) Slant height $=\sqrt{15^2+(\frac{20}{2})^2}$ $=\sqrt{325}$ cm

$\therefore$ Length of slant edge $=\sqrt{(\sqrt{325})^2+(\frac{20}{2})^2}$ $=\sqrt{425}$ $=20.6$ cm

(c) Total surface area $=\frac{\sqrt{325}\times20}{2}\times4+20^2$ $=1121.1\text{ cm}^2$

7. Volume $=\frac{4}{3}\pi(\frac{7}{2})^3$ $=\frac{343\pi}{6}$ $=179.6\text{ cm}^3$

8. Volume = $\frac{4}{3}\pi(11)^3 \times \frac{1}{2} = \frac{2662\pi}{3}$ cm^3

Total surface area = $4\pi(11)^2 \times \frac{1}{2} + \pi(11)^2 = 242\pi + 121\pi = 363\pi$ cm^2

9. Let r cm be the radius. $4\pi r^2 = 100$, $r^2 = \frac{25}{\pi}$, $r = \sqrt{\frac{25}{\pi}}$

∴ Diameter = $2 \times \sqrt{\frac{25}{\pi}} = 5.64$ cm

10. Let r cm be the radius. $4\pi r^2 = 320$, $r = \sqrt{\frac{80}{\pi}}$

∴ Volume = $\frac{4}{3}\pi(\sqrt{\frac{80}{\pi}})^3 = 538.3$ cm^3

11. (a) Let r cm be the new radius. $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3^3 + 5^3 + 7^3)$,

$r^3 = 3^3 + 5^3 + 7^3 = 495$, ∴ $r = \sqrt[3]{495} = 7.91$. *Ans. The radius is 7.91 cm.*

(b) Original surface area = $4\pi(3^2 + 5^2 + 7^2) = 332\pi$ cm^2,

new surface area = $4\pi(\sqrt[3]{495})^2 = 250.3\pi$ cm^2.

∴ % change in surface area = $\frac{250.3\pi - 332\pi}{332\pi} \times 100\% = -24.6\%$

Ans. The surface area decreases by 24.6%.

12. Let R and r be the radii of original and new spheres respectively. $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \times 8$,

$R^3 = 8r^3$, $R = \sqrt[3]{8r^3} = 2r$, $r = \frac{R}{2}$.

∴ % change in total surface area = $\frac{4\pi r^2 \times 8 - 4\pi R^2}{4\pi R^2} \times 100\%$

$= \frac{32\pi(\frac{R}{2})^2 - 4\pi R^2}{4\pi R^2} \times 100\% = \frac{4\pi R^2}{4\pi R^2} \times 100\% = 100\%$ (increase)

13. External and internal radii are $\frac{15}{2}$ cm and $\frac{15 - 2\times 2}{2} = \frac{11}{2}$ cm respectively.

Volume of hollow sphere = $\frac{4}{3}\pi[(\frac{15}{2})^3 - (\frac{11}{2})^3] = 1070.235$ cm^3

∴ Weight = $1070.235 \times 150 = 160535$ g

14. Let h cm be the rise in water level. $\pi(6)^2 h = \frac{4}{3}\pi(\frac{3}{2})^3 \times 10$, $36h = 45$, $h = 1.25$

Ans. The rise in water level is 1.25 cm.

15. (a) Volume = $\frac{1}{3}\pi(6)^2(7) = 84\pi = 263.9$ cm^3. Slant edge = $\sqrt{6^2 + 7^2} = \sqrt{85}$ cm

$\therefore$ Total surface area $= \pi(6)(\sqrt{85}) + \pi(6)^2 = 286.9$ cm^2

(b) Height $= \sqrt{13^2 - 5^2} = \sqrt{144} = 12$ cm

$\therefore$ Volume $= \frac{1}{3}\pi(5)^2(12) = 100\pi = 314.2$ cm^3

Total surface area $= \pi(5)(13) + \pi(5)^2 = 90\pi = 282.7$ cm^2

16. Let r cm be the base radius, $2\pi r = 30$, $r = \frac{15}{\pi}$, $\therefore$ Slant edge $= \sqrt{(\frac{15}{\pi})^2 + 20^2} = 20.562$,

$\therefore$ Curved surface area $= \pi(\frac{15}{\pi})(20.562) = 308.4$ cm^2

17. Let ℓ cm be the length of slant edge, $\pi(5)(\ell) = 40\pi$, $\ell = 8$,

$\therefore$ Height $= \sqrt{8^2 - 5^2} = \sqrt{39}$. $\therefore$ Volume $= \frac{1}{3}\pi(5)^2(\sqrt{39}) = 163.5$ cm^3

18. Let r cm be the base radius, $\frac{1}{3}\pi(r)^2(12) = \pi(6)^2(10)$, $4r^2 = 360$, $r^2 = 90$,

$\therefore$ $r = \sqrt{90} = 9.5$. *Ans. The base radius is 9.5 cm.*

19. Let r cm be the base radius, then the height is $2r$ cm.

$\frac{1}{3}\pi r^2(2r) = 1152\pi$, $r = \sqrt[3]{1728} = 12$, $\therefore$ Slant edge $= \sqrt{12^2 + (2\times 12)^2} = \sqrt{720} = 12\sqrt{5}$

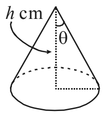
$\therefore$ Curved surface area $= \pi(12)(12\sqrt{5}) = 144\sqrt{5}\pi$ cm^2

20. $\theta = 60° \div 2 = 30°$. Let h cm be the height. $\tan 30° = \frac{15}{h}$, $h = \frac{15}{\tan 30°} = 15\sqrt{3}$

$\therefore$ Slant edge $= \sqrt{15^2 + (15\sqrt{3})^2} = 30$ cm,

$\therefore$ Total surface area $= \pi(15)(30) + \pi(15)^2 = 675\pi = 2120.6$ cm^2

Volume $= \frac{1}{3}\pi(15)^2(15\sqrt{3}) = 1125\sqrt{3}\pi = 6121.6$ cm^3



21. Let r cm be the base radius of the cone, $2\pi r = 2\pi(\frac{14}{2}) \times \frac{1}{2}$, $r = \frac{7}{2}$

$\therefore$ Height $= \sqrt{(\frac{14}{2})^2 - (\frac{7}{2})^2} = \frac{7\sqrt{3}}{2}$, $\therefore$ Volume $= \frac{1}{3}\pi(\frac{7}{2})^2(\frac{7\sqrt{3}}{2}) = \frac{343\sqrt{3}\pi}{24} = 77.8$ cm^3

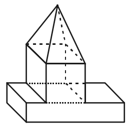
22. (a) Curved surface area $= \pi(9)^2 \times \frac{280°}{360°} = 63\pi = 197.9$ cm^2

(b) Let r cm be the base radius of the cone. $\pi r(9) = 63\pi$, $r = 7$,

$\therefore$ height $= \sqrt{9^2 - 7^2} = 4\sqrt{2}$, $\therefore$ Volume $= \frac{1}{3}\pi(7)^2(4\sqrt{2}) = 290.3$ cm^3

23. Volume $= 5(1)(1) + (1)(5-2-1)(3-1) + \frac{1}{3}(1)(5-2-1)(2) = 5 + 4 + \frac{4}{3} = 10\frac{1}{3}$ cm^3

24. (a) Volume $= \frac{1}{3}\pi(5)^2(12) + \pi(5)^2(25-12-5) + \frac{4}{3}\pi(5)^3 \times \frac{1}{2}$

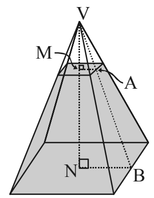


$= 100\pi + 200\pi + \frac{250\pi}{3} = \frac{1150\pi}{3} = 1204.3 \text{ cm}^3$

(b) Slant edge of the cone $= \sqrt{5^2+12^2} = \sqrt{169} = 13$ cm,
height of cylinder = 25 – 12 – 5 = 8 cm

$\therefore$ Total sruface area $= \pi(5)(13) + 2\pi(5)(8) + 4\pi(5)^2 \times \frac{1}{2} = 195\pi = 612.6 \text{ cm}^2$

25. (a) $\because \Delta VMA \sim \Delta VNB$, $\therefore \frac{MA}{NB} = \frac{VM}{VN}$, $\frac{MA}{5} = \frac{4}{4+8}$, $MA = \frac{5}{3}$



$\therefore$ Length of base of upper pyramid $= \frac{5}{3} \times 2 = \frac{10}{3}$ cm

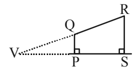
$\therefore$ Volume of frustum $= \frac{1}{3}\pi[(10)^2(12) - (\frac{10}{3})^2(4)] = \frac{10400\pi}{27}$

$= 1210.1 \text{ cm}^3$

(b) $VB = \sqrt{12^2+5^2} = 13$, $VA = \sqrt{4^2 + (\frac{5}{3})^2} = = \sqrt{\frac{169}{9}} = \frac{13}{3}$

$\therefore$ Total surface area $= 10^2 + (\frac{10}{3})^2 + 4 \times \frac{1}{2} \times (10 \times 13 - \frac{10}{3} \times \frac{13}{3}) = 342.2 \text{ cm}^2$

26. (a) $\because \Delta VQP \sim \Delta VRS$, $\therefore \frac{VP}{VS} = \frac{QP}{RS}$, $\frac{VP}{VP+10} = \frac{3}{6} = \frac{1}{2}$, $VP = 10$



$\therefore VQ = \sqrt{3^2+10^2} = \sqrt{109}$, $VR = \sqrt{6^2 + (10+10)^2} = \sqrt{436}$

$\therefore$ Lateral surface area $= \pi(6)(\sqrt{436}) - \pi(3)(\sqrt{109}) = 295.2 \text{ cm}^2$

(b) Volume $= \frac{1}{3}\pi[(6)^2(10+10) - (3)^2(10)] = 210\pi = 659.7 \text{ cm}^3$

27. The space is one-eighths of a sphere with radius 1 m.

$\therefore$ Volume of space $= \frac{4}{3}\pi(1)^3 \times \frac{1}{8} = \frac{\pi}{6} = 0.524 \text{ m}^3$

28. (a) Slant height $= \sqrt{(2.6x)^2 - (\frac{2x}{2})^2} = \sqrt{5.76x^2} = 2.4x$ cm

(b) $\frac{(2x)(2.4x)}{2} \times 4 + (2x)^2 = 1360$, $13.6x^2 = 1360$, $x^2 = 100$, $\therefore x = \sqrt{100} = 10$

(c) Height of pyramid $= \sqrt{(2.4x)^2 - x^2} = \sqrt{24^2 - 10^2} = \sqrt{476}$,

$\therefore$ Volume $= \frac{1}{3}(20)^2\sqrt{476} = 2909.0 \text{ cm}^3$

29. $AD = AH = AB = \sqrt[3]{a}$ cm.

$\therefore$ Volume of tetrahedron BADH $= \frac{1}{3} \times$ area of $\Delta ABH \times AD = \frac{1}{3}(\frac{\sqrt[3]{a} \times \sqrt[3]{a}}{2})(\sqrt[3]{a})$

$= \frac{1}{6}(\sqrt[3]{a})^3 = \frac{a}{6} \text{ cm}^3$. $\therefore$ Volume of remaining solid $= a - \frac{a}{6} = \frac{5a}{6}$ cm^3

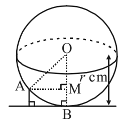
30. (a) Capacity $= \frac{4}{3}\pi(1)^3 \times \frac{1}{2} + \pi(1)^2(10-1) = \frac{29}{3}\pi = 30.4 \text{ cm}^3$

(b) Let h cm be the height of water level of the cylindrical part.

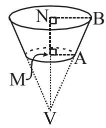
$\pi(1)^2(h) + \frac{4}{3}\pi(1)^3 \times \frac{1}{2} = \frac{29}{3}\pi \times \frac{2}{3}$, $h + \frac{2}{3} = \frac{58}{9}$, $h = \frac{52}{9}$

$\therefore$ Total area of wet surface $= 2\pi(1)(\frac{52}{9}) + 4\pi(1)^2 \times \frac{1}{2} = \frac{104\pi}{9} + 2\pi = \frac{122\pi}{9} = 42.6 \text{ cm}^2$

31. Let r cm be the radius. $AM^2 + OM^2 = OA^2$, $3^2 + (r-1)^2 = r^2$,
$9 + r^2 - 2r + 1 = r^2$, $10 = 2r$, $r = 5$.
$\therefore$ Surface area $= 4\pi(5)^2 = 100\pi = 314.2 \text{ cm}^2$
Volume $= \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3} = 523.6 \text{ cm}^3$



32. (a) Capacity $= \pi(4)^2(5-4) + \frac{4}{3}\pi(4)^3 \times \frac{1}{2} = \frac{176\pi}{3} \text{ cm}^3$



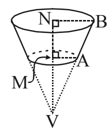
(b) $\because \Delta VAM \sim \Delta VBN$, $\therefore \frac{VM}{VM+6} = \frac{4}{6} = \frac{2}{3}$, VM = 12,

$\therefore$ VN = 12 + 6 = 18.

$\therefore$ Volume of mould $= \frac{1}{3}\pi[(6)^2(18) - (4)^2(12)] - \frac{176\pi}{3} = \frac{280\pi}{3} \text{ cm}^3$,

$\therefore$ Weight $= \frac{280\pi}{3} \times 0.8 = 234.6$ g

33. (a) $\because \Delta VAM \sim \Delta VBN$, $\therefore \frac{VM}{VN} = \frac{AM}{BN}$, $\frac{VM}{VM+h} = \frac{6}{8} = \frac{3}{4}$,



VM = $3h$, $\therefore$ VN = $3h + h = 4h$

$\therefore \frac{1}{3}\pi[(8)^2(4h) - (6)^2(3h)] = 148\pi$, $\frac{148\pi h}{3} = 148\pi$, $h = 3$

Ans. Height of the frustum is 3 cm.

(b) $VA = \sqrt{6^2 + (3\times3)^2} = \sqrt{117}$, $VB = \sqrt{8^2 + (4\times3)^2} = \sqrt{208}$,

$\therefore$ Total surface area $= \pi(8)(\sqrt{208}) - \pi(6)(\sqrt{117}) + \pi(6)^2 + \pi(8)^2 = 472.7 \text{ cm}^2$

34. (a) Let r cm be the radius. $\text{Volume}_{\text{hemisphere}} : \text{Volume}_{\text{cone}} = \frac{4}{3}\pi r^3 \times \frac{1}{2} : \frac{1}{3}\pi r^2(r) = \frac{2}{3} : \frac{1}{3} = 2:1$

(b) Let base area of the tank = A cm^2, rise in water level = h cm.

$A(6) : A(h) = 2 : 1$, $\therefore$ rise in water level $= 6 \times \frac{1}{2} = 3$ cm

35. (a) $\frac{1}{3}\pi(\frac{5}{2})^2(x) + \pi(\frac{5}{2})^2(8 - \frac{5}{2}) + \frac{4}{3}\pi(\frac{5}{2})^3 \times \frac{1}{2} = \pi(\frac{5}{2})^2(x+8)(1-20\%)$,

$\frac{25x}{12} + \frac{275}{8} + \frac{125}{12} = \frac{25}{4} \times \frac{4}{5}(x+8)$, $\frac{5x}{12} + \frac{55}{8} + \frac{25}{12} = x + 8$,

$10x + 165 + 50 = 24x + 192$, $23 = 14x$, $\therefore x = \frac{23}{14}$

(b) Slant edge fo circular cone = $\sqrt{(\frac{5}{2})^2+(\frac{23}{14})^2}=2.9915$ cm,

$\therefore$ Total surface area = $\pi(\frac{5}{2})(2.9915)+\pi(5)(8-\frac{5}{2})+4\pi(\frac{5}{2})^2\times\frac{1}{2}=149.2\ \text{cm}^2$

$\therefore$ Cost = 149.2 × 0.4 = \$59.7

36. (a) Let h_1 cm and h_2 cm be the heights of cones A and (A+B) respectively.

$\frac{h_1}{30}=\frac{5}{15}$, $h_1=10$; $\frac{h_2}{30}=\frac{12}{15}$, $h_2=24$.

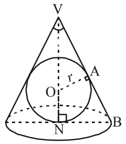
$\therefore$ Volume of frustum B = $\frac{1}{3}\pi[(12)^2(24)-(5)^2(10)]=3357.3\ \text{cm}^3$.

(b) Lateral surface area of frustum B = $\pi(12)(\sqrt{12^2+24^2})-\pi(5)(\sqrt{5^2+10^2})$

$=\pi(12\times12\sqrt{5}-5\times5\sqrt{5})=119\sqrt{5}\pi\ \text{cm}^2$

(c) Total surface area = $\pi(15)(\sqrt{30^2+15^2})-119\sqrt{5}\pi+\pi(15^2+12^2-5^2)=1825.3\ \text{cm}^2$

37. (a) $\angle OVA=60^\circ\div2=30^\circ$. In ΔOVA, $\sin30^\circ=\frac{OA}{OV}$, $\frac{1}{2}=\frac{r}{OV}$, OV = $2r$.

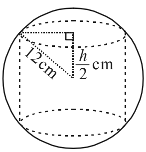


Height of cone = OV + ON = $2r+r=3r$

(b) In ΔVNB, $\tan30^\circ=\frac{BN}{VN}$, $\frac{1}{\sqrt{3}}=\frac{BN}{3r}$, BN = $\sqrt{3}r$

$\therefore$ $\text{Volume}_{\text{cone}}:\text{Volume}_{\text{sphere}}=\frac{1}{3}\pi(\sqrt{3}r)^2(3r):\frac{4}{3}\pi r^3=3\pi r^3:\frac{4}{3}\pi r^3=9:4$

38. (a) Base radius of cylinder = $\sqrt{12^2-(\frac{h}{2})^2}=\sqrt{144-\frac{h^2}{4}}$ cm,



$\therefore V=\pi\left(\sqrt{144-\frac{h^2}{4}}\right)^2(h)=\pi h(144-\frac{h^2}{4})=\frac{\pi h}{4}(576-h^2)$

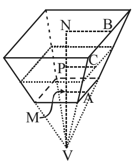
(b) $\frac{2h}{3}=12$, $h=18$. $\therefore$ $V=\frac{18\pi}{4}(576-18^2)=1134\pi$

$\therefore$ $\text{Volume}_{\text{cylinder}}:\text{Volume}_{\text{wood remained}}=1134\pi:[\frac{4}{3}\pi(12)^3-1134\pi]$

$=1134\pi:1170\pi=63:65$

39. (a) $\because$ $\Delta VMA\sim\Delta VNB$, $\therefore$ $\frac{VM}{VN}=\frac{AM}{BN}$, $\frac{VM}{VM+12}=\frac{4}{8}$, 2VM = VM + 12,

VM = 12, $\therefore$ VN = 12 + 12 = 24.



$\therefore$Volume of frustum = $\frac{1}{3}(16)^2(24)-\frac{1}{3}(8)^2(12)=1792\ \text{cm}^3$

(b) $\because$ $\Delta VMA\sim\Delta VPC$, $\therefore$ $\frac{VM}{VP}=\frac{AM}{CP}$, $\frac{12}{12+9}=\frac{4}{CP}$,

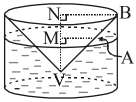
CP = 7. $\therefore$ Length of side of water surface = 7 × 2 = 14 cm

$\therefore$ Volume of water $= \frac{1}{3}(14)^2(21) - \frac{1}{3}(8)^2(12) = 1116 \text{ cm}^3$

(c) Water flows from the pipe to the frustum in 1 second $= \pi(0.8)^2(6) \text{ cm}^3$

Time taken $= 1116 \div \pi(0.8)^2(6) = 92.5$ seconds.

40. MN = 12 – 9 = 3 cm, VN = 9 cm, $\therefore$ VM = 9 – 3 = 6 cm



$\because$ ΔVMA ~ ΔVNB, $\therefore \frac{AM}{8} = \frac{VM}{VN} = \frac{6}{9}$, $AM = \frac{16}{3}$

Let h cm be the new water level. $\pi(8)^2 h = \pi(8)^2(9) - \frac{1}{3}\pi(AM)^2(VM)$,

$(8)^2 h = (8)^2(9) - \frac{1}{3}(\frac{16}{3})^2(6)$, $64h = 576 - \frac{512}{9}$, $\therefore h = \frac{73}{9} = 8.11$

Ans. The new water level is 8.11 cm.

Unit 18 Area and volume of similar solids

1. Ratio of heights = 1.2 : 0.9 = 4 : 3, $\therefore$ Ratio of volumes $= 4^3 : 3^3 = 64 : 27$
2. Ratio of capacities = 162 : 750 = 27: 125

   $\therefore$ Ratio of surface areas $= (\sqrt[3]{27})^2 : (\sqrt[3]{125})^2 = 9 : 25$

3. Ratio of diameters = (10 + 2 × 2) : 10 = 7 : 5

   $\therefore$ External surface area : internal surface area $= 7^2 : 5^2 = 49 : 25$

4. Ratio of heights = 9 : 12 = 3: 4, ratio of volumes $= 3^3 : 4^3 = 27 : 64$

   $\therefore$ Weight of bigger pyramid $= 270 \times \frac{64}{27} = 640$g

5. Ratio of volumes = 1 kg : 125 g = 1000 : 125 = 8 : 1, ratio of surface areas

   $= (\sqrt[3]{8})^2 : (\sqrt[3]{1})^2 = 4 : 1$, $\therefore$ Cost of painting the larger solid = 18 × 4 = \$72

6. (a) Let the original radius and area be r_1, A_1; those of the new ones be r_2, A_2.

   $$\left(\frac{r_2}{r_1}\right)^2 = \frac{A_2}{A_1} = \frac{A_1(1+69\%)}{A_1} = \frac{1.69}{1}, \quad \frac{r_2}{r_1} = \sqrt{\frac{1.69}{1}} = \frac{1.3}{1},$$

   $\therefore$ Percentage increase in radius $= \frac{1.3r_1 - r_1}{r_1} \times 100\% = 30\%$

   (b) $$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{1.3}{1}\right)^3 = \frac{2.197}{1},$$

   $\therefore$ Percentage increase in volume $= \frac{2.197V_1 - V_1}{V_1} \times 100\% = 119.7\%$

7. Let the original length, area and volume be r_1, A_1, V_1; those of the new ones be r_2, A_2, V_2.

   $$\left(\frac{r_2}{r_1}\right)^3 = \frac{V_2}{V_1} = \frac{V_1(1-27.1\%)}{V_1} = \frac{0.729}{1}, \quad \frac{r_2}{r_1} = \sqrt[3]{\frac{0.729}{1}} = \frac{0.9}{1}, \quad \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{0.9}{1}\right)^2 = \frac{0.81}{1},$$

$\therefore$ Percentage change in area $= \dfrac{0.81A_1 - A_1}{A_1} \times 100\% = -19\%$ (decrease)

8. $\dfrac{\text{Vol. of small pendulum}}{\text{Vol. of big pendulum}} = \dfrac{1}{4}$. Let h cm be the height of a small pendulum, $\left(\dfrac{h}{16}\right)^3 = \dfrac{1}{4}$,

$\dfrac{h}{16} = \sqrt[3]{\dfrac{1}{4}}$, $h = 10.1$ *Ans. The height of a small pendulum is 10.1 cm.*

9. $V_A : (V_A + V_B) : (V_A + V_B + V_C) = y^3 : (2y)^3 : (3y)^3 = 1 : 8 : 27$

$\therefore$ $V_A : V_B : V_C = 1 : (8 - 1) : (27 - 8) = 1 : 7 : 19$

10. PQ : PR : PS = 2 : (2+1) : (2+1+3) = 2 : 3 : 6,

$\therefore$ Area of circle I : area of circle II : area of circle III = $2^2 : 3^2 : 6^2 = 4 : 9 : 36$

11. $\dfrac{\text{Vol. of small pyramid}}{\text{Vol. of big pyramid}} = \left(\sqrt{\dfrac{49}{64}}\right)^3 = \left(\dfrac{7}{8}\right)^3 = \dfrac{343}{512}$, $\dfrac{\text{weight of frustum}}{\text{weight of big pyramid}} = \dfrac{512 - 343}{512} = \dfrac{169}{512}$

$\therefore$ Weight of frustum $= 2 \times \dfrac{169}{256} = 1.32$ kg

12. $\left(\dfrac{AQ}{AB}\right)^3 = \dfrac{216}{216 + 513} = \dfrac{216}{729} = \dfrac{8}{27}$, $\dfrac{AQ}{AB} = \sqrt[3]{\dfrac{8}{27}} = \dfrac{2}{3}$, $\therefore$ $AQ : QB = 2 : (3 - 2) = 2 : 1$

13. $\dfrac{\text{Old volume}}{\text{New volume}} = \dfrac{60}{60 + 420} = \dfrac{1}{8}$, $\dfrac{\text{Old wet area}}{\text{New wet area}} = \left(\sqrt[3]{\dfrac{1}{8}}\right)^2 = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$,

$\therefore$ Increase in wet surface area = $25 \times 4 - 25 = 75\text{cm}^2$

14. (a) $\dfrac{\text{Old volume}}{\text{New volume}} = \left(\dfrac{16}{24}\right)^3 = \left(\dfrac{2}{3}\right)^3 = \dfrac{8}{27}$.

$\therefore$ Volume of water added $= 400 \times \dfrac{27}{8} - 400 = 950\text{cm}^3$

(b) $\dfrac{\text{Old wet surface area}}{\text{New wet surface area}} = \left(\dfrac{16}{24}\right)^2 = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$,

$\therefore$ Percentage change in wet surface area $= \dfrac{9 - 4}{4} \times 100\% = 125\%$ (increase)

15. PT = 3TR, $\therefore$ PR = 3TR + TR = 4TR; QR = 3TR, $\therefore$ QR = 3SR + SR = 4SR

In ΔPQR and ΔTSR, $\angle$R = $\angle$R (common), $\dfrac{PR}{TR} = \dfrac{4TR}{TR} = 4$, $\dfrac{QR}{SR} = \dfrac{4SR}{SR} = 4$,

$\therefore$ $\Delta PQR \sim \Delta TSR$ (ratio of 2 sides, inc. $\angle$)

$\therefore$ $\dfrac{\text{Area of } \Delta STR}{\text{Area of } \Delta PQR} = \left(\dfrac{TR}{PR}\right)^2 = \left(\dfrac{1}{4}\right)^2 = \dfrac{1}{16}$, $\dfrac{\text{Area of } \Delta STR}{\text{Area of } PQST} = \dfrac{1}{16 - 1} = \dfrac{1}{15}$

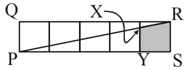
16. $\because$ $\Delta AEF \sim \Delta ADB$ (A.A.A.), $\therefore$ $\dfrac{\text{Area of } \Delta AEF}{\text{Area of } \Delta ADB} = \left(\dfrac{AE}{AD}\right)^2 = \left(\dfrac{2ED}{AE+ED}\right)^2 = (\dfrac{2ED}{3ED})^2 = \dfrac{4}{9}$

$\because$ $\Delta CGH \sim \Delta CBD$ (A.A.A.), $\therefore$ $\dfrac{\text{Area of } \Delta CGH}{\text{Area of } \Delta CBD} = \left(\dfrac{GC}{BC}\right)^2 = \left(\dfrac{GC}{BG+GC}\right)^2 = \left(\dfrac{GC}{2GC}\right)^2 = \dfrac{1}{4}$

Since area of ΔADB = area of ΔCBD,

$\therefore$ Percentage of ABCD shaded = $\dfrac{4}{9} \times 50\% + \dfrac{1}{4} \times 50\% = 34.7\%$

17. $\because$ $\Delta PXY \sim \Delta PRS$ (A.A.A.),



$\therefore$ $\dfrac{\text{Area of } \Delta PXY}{\text{Area of } \Delta PRS} = \left(\dfrac{PY}{PS}\right)^2 = \left(\dfrac{4}{5}\right)^2 = \dfrac{16}{25}$,

Area of $\Delta PRS = \dfrac{6 \times 5}{2} = 15\text{cm}^2$, $\therefore$ Shaded area = $15 - 15 \times \dfrac{16}{25} = 5.4 \text{ cm}^2$

18. (a) $\because$ $\Delta ABC \sim \Delta ADE$ (A.A.A.), $\therefore$ $\left(\dfrac{BC}{DE}\right)^2 = \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \dfrac{50}{50+112} = \dfrac{25}{81}$,

$\therefore$ BC : DE $= \sqrt{25} : \sqrt{81} = 5 : 9$

(b) $\because$ AC : AE = BC : DE = 5 : 9, $\therefore$ AC : CE = 5 : (9 − 5) = 5 : 4,
AE : AG = (5+4) : (5+4+4) = 9 : 13. $\because$ $\Delta ADE \sim \Delta AFG$ (A.A.A.),

$\therefore$ DE : FG = AE : AG = 9 : 13. $\dfrac{\text{Area of } \Delta ADE}{\text{Area of } \Delta AFG} = \left(\dfrac{DE}{FG}\right)^2 = \left(\dfrac{9}{13}\right)^2 = \dfrac{81}{169}$

$\therefore$ Area of $\Delta AFG = (50 + 112) \times \dfrac{169}{81} = 338 \text{ cm}^2$

19. Let A_1, A_2, A_3 be the curved surface areas of portions I, II, III respectively.

$\dfrac{A_1}{A_1 + A_2} = \left(\dfrac{2h}{3h}\right)^2 = \dfrac{4}{9}$, $9A_1 = 4(A_1 + 36)$, $5A_1 = 144$, $A_1 = 28.8$

$\dfrac{A_1}{A_1 + A_2 + A_3} = \left(\dfrac{2h}{4h}\right)^2 = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$, $\dfrac{28.8}{28.8 + 36 + A_3} = \dfrac{1}{4}$, $115.2 = 28.8 + 36 + A_3$, $A_3 = 50.4$.

Ans. The curved surface areas of portions I and III are 28.8 cm² and 50.4 cm² respectively.

20. Vol. of B : Vol. of C = 3 : 2, Vol. of A : Vol. of B = $(\sqrt{4})^3 : (\sqrt{1})^3 = 8 : 1 = 24 : 3$

$\therefore$ Vol. of A : Vol. of B : Vol. of C = 24 : 3 :2,
$\therefore$ Vol. of A : Vol. of C = 24 : 2 = 12 : 1

21. (a) Volume of M : volume of N = $1^3 : 2^3$ = 1: 8
(b) Let the radius of A be $2x$ cm, radius of B be $3x$ cm, water risen in cylinder B be h cm.

$$\frac{\text{Vol. of N}}{\text{Vol. of M}} = \frac{\text{Vol. of water risen in B}}{\text{Vol. of water risen in A}}, \quad \therefore \quad \frac{\pi(3x)^2 h}{\pi(2x)^2(6)} = \frac{8}{1}, \quad \frac{9h}{24} = 8, \quad h = \frac{64}{3}$$

Ans. The rise in water level in B is $21\frac{1}{3}$ *cm.*

22. (a) Vol. of space : Vol. of vessel = $(12 - 4)^3 : 12^3 = 8 : 27$

$\therefore$ Vol. of water : Vol. of vessel = $(27 - 8) : 27 = 19 : 27$

Let h cm be the depth of water now. $\frac{h}{12} = \sqrt[3]{\frac{19}{27}}$, $h = 12 \times \sqrt[3]{\frac{19}{27}} = 10.7$

Ans. The depth of water now is 10.7 cm.

(b) Let the curved surface area of the vessel, the curved area of the original space, the original wet lateral surface and the new wet surface be x, A_{space}, $A_{original}$ and A_{new}.

$$\frac{A_{space}}{x} = \left(\sqrt[3]{\frac{8}{27}}\right)^2 = \frac{4}{9}, \quad \therefore \quad \frac{A_{original}}{x} = \frac{9-4}{9} = \frac{5}{9}, \quad A_{original} = \frac{5}{9}x$$

$$\frac{A_{new}}{x} = \left(\sqrt[3]{\frac{19}{27}}\right)^2 = \frac{(\sqrt[3]{19})^2}{9}, \quad \therefore \; A_{new} = \frac{(\sqrt[3]{19})^2}{9}x \qquad \frac{A_{new}}{A_{originald}} = \frac{(\sqrt[3]{19})^2}{9} \div \frac{5}{9} = \frac{(\sqrt[3]{19})^2}{5}.$$

$\therefore$ Percentage change in curved wet surface areas $= \frac{(\sqrt[3]{19})^2 - 5}{5} \times 100\% = 42.4\%$ (increase)

23. (a) Volume of water = $\frac{4}{3}\pi(3)^3 \times \frac{1}{2} + \pi(3)^2(19 - 3) = 18\pi + 144\pi = 162\pi \text{ cm}^3$

(b) Let $V\text{cm}^3$ be the volume of oil. $\frac{162\pi}{V + 162\pi} = \left(\frac{3}{3+1}\right)^3 = \frac{27}{64}$,

$10368\pi = 27V + 4374\pi, \quad V = 222\pi$

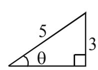
Let h cm be the depth of oil in the glass-tube. $\frac{4}{3}\pi(3)^3 \times \frac{1}{2} + \pi(3)^2(h - 3) = 222\pi$,

$18\pi + 9\pi(h - 3) = 222\pi, \quad 9(h - 3) = 204, \quad h = 25.7$

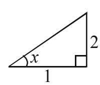
Ans. The depth of oil in the glass-tube is 25.7 cm.

Unit 19 Trigonometric relations

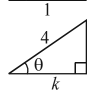
1. (a) $\sin\theta = 0.6 = \frac{3}{5}, \quad \cos\theta = \frac{\sqrt{5^2 - 3^2}}{5} = \frac{4}{5} = 0.8$



(b) $\tan x = 2 = \frac{2}{1}, \quad \sin x = \frac{2}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}}$



(c) $\cos\theta = \frac{k}{4}, \quad \tan\theta = \frac{\sqrt{4^2 - k^2}}{k} = \frac{\sqrt{16 - k^2}}{k}$



2. $$\frac{4\sin\theta - 3\cos\theta}{10\sin\theta + \cos\theta} = \frac{4\tan\theta - 3}{10\tan\theta + 1} = \frac{4(\frac{5}{13}) - 3}{10(\frac{5}{13}) + 1} = \frac{20 - 39}{50 + 13} = \frac{-19}{63}$$

3. (a) $=(\frac{1}{2})(\frac{1}{2})+(\frac{\sqrt{3}}{2})(\frac{\sqrt{3}}{2})=\frac{1}{4}+\frac{3}{4}=1$

(b) $\frac{\sin^2 45^\circ}{\cos 60^\circ}-\frac{\tan^2 30^\circ}{\cos 30^\circ}=\frac{(\frac{1}{\sqrt{2}})^2}{\frac{1}{2}}-\frac{(\frac{1}{\sqrt{3}})^2}{\frac{\sqrt{3}}{2}}=1-\frac{2}{\sqrt{3}}=1-\frac{2\sqrt{3}}{9}=\frac{9-2\sqrt{3}}{9}$

(c) $=(\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}})^2=(\frac{2}{\sqrt{3}})^2=\frac{4}{3}$ (d) $=1\div\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=\sqrt{2}-\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}$

4. (a) $\tan\theta=\frac{\cos 47^\circ}{\sin 47^\circ}=\frac{1}{\tan 47^\circ}=\tan(90^\circ-47^\circ)=\tan 43^\circ$, $\therefore\ \theta=43^\circ$

(b) $\tan^2\theta=\frac{1}{3}, \tan\theta=\frac{1}{\sqrt{3}}=\tan 30^\circ$, $\therefore\ \theta=30^\circ$

(c) $\sin\theta=\cos(40^\circ+\theta)=\sin[90^\circ-(40^\circ+\theta)]=\sin(50^\circ-\theta)$,

$\therefore\ \theta=50^\circ-\theta,\ \theta=25^\circ$

5. (a) $=\sin\theta\cos\theta(\sin^2\theta+\cos^2\theta)=\sin\theta\cos\theta(1)=\sin\theta\cos\theta$

(b) $=\sin^2\theta(\frac{\sin^2\theta}{\cos^2\theta}+1)=\sin^2\theta(\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta})=\sin^2\theta(\frac{1}{\cos^2\theta})=\tan^2\theta$

(c) $=\frac{(\sin\theta-1)-(1+\sin\theta)}{(1+\sin\theta)(\sin\theta-1)}=\frac{-2}{-(1-\sin^2\theta)}=\frac{2}{\cos^2\theta}$

(d) $=\frac{\sin\theta+\cos\theta}{1-\frac{\sin^2\theta}{\cos^2\theta}}=\frac{\cos^2\theta(\sin\theta+\cos\theta)}{\cos^2\theta-\sin^2\theta}=\frac{\cos^2\theta(\sin\theta+\cos\theta)}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$

$=\frac{\cos^2\theta}{\cos\theta-\sin\theta}=\frac{\cos\theta}{1-\tan\theta}$

6. (a) L.H.S. $=\cos^4\theta-\sin^4\theta=(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)$

$=(1-\sin^2\theta-\sin^2\theta)(1)=1-2\sin^2\theta=$ R.H.S.

(b) L.H.S. $=(1-\cos y)(1+\cos y)=1-\cos^2 y=\sin^2 y$

R.H.S. $=\cos^2 y\tan^2 y=\cos^2 y\frac{\sin^2 y}{\cos^2 y}=\sin^2 y=$ L.H.S.

(c) R.H.S. $=\frac{\tan\theta}{1+\tan^2\theta}=\frac{\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}=\frac{\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}=\sin\theta\cos\theta=$L.H.S.

7. (a) $\cos^2 35^\circ+\cos^2 55^\circ=\sin^2(90^\circ-35^\circ)+\cos^2 55^\circ=\sin^2 55^\circ+\cos^2 55^\circ=1$

(b) $\tan 59^\circ\cdot\tan 31^\circ=\frac{1}{\tan(90^\circ-59^\circ)}\cdot\tan 31^\circ=\frac{1}{\tan 31^\circ}\cdot\tan 31^\circ=1$

(c) $\sin^2 22^\circ-\cos^2 68^\circ=\cos^2(90^\circ-22^\circ)-\cos^2 68^\circ=\cos^2 68^\circ-\cos^2 68^\circ=0$

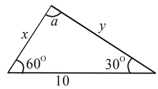
8. (a) $=\frac{\tan\theta-\frac{1}{\tan\theta}}{\tan\theta+\frac{1}{\tan\theta}}=\frac{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}=\frac{\frac{\sin^2\theta-\cos^2\theta}{\sin\theta\cos\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}=\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta}=\sin^2\theta-\cos^2\theta$

(b) $=\frac{2\cos\theta\sin^2\theta}{\cos^2\theta\sin\theta}=\frac{2\sin\theta}{\cos\theta}=2\tan\theta$

(c) $\tan(30^\circ - x)\times\tan(60^\circ + x) = \dfrac{1}{\tan[90^\circ - (30^\circ - x)]}\times\tan(60^\circ + x)$

$= \dfrac{1}{\tan(60^\circ + x)}\times\tan(60^\circ + x) = 1$

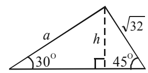
9. (a) $\because\ a = 180^\circ - 60^\circ - 30^\circ = 90^\circ$



$\therefore\ \sin 30^\circ = \dfrac{x}{10},\quad x = 10\sin 30^\circ = 10(\dfrac{1}{2}) = \underline{\underline{5}}$

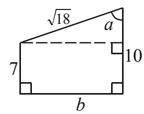
$\therefore\ \sin 60^\circ = \dfrac{y}{10},\quad y = 10\sin 60^\circ = 10(\dfrac{\sqrt{3}}{2}) = \underline{\underline{5\sqrt{3}}}$

(b) $\sin 45^\circ = \dfrac{h}{\sqrt{32}},\quad h = \sqrt{32}\sin 45^\circ$,



$\sin 30^\circ = \dfrac{h}{a},\quad a = \dfrac{h}{\sin 30^\circ},\quad \therefore\ a = \dfrac{\sqrt{32}\sin 45^\circ}{\sin 30^\circ} = \dfrac{(4\sqrt{2})(\frac{1}{\sqrt{2}})}{\frac{1}{2}} = 2\times 4 = \underline{\underline{8}}$

(c) $\because\ \cos a = \dfrac{10-7}{\sqrt{18}} = \dfrac{3}{3\sqrt{2}} = \dfrac{1}{\sqrt{2}},\quad \therefore\ a = \underline{\underline{45^\circ}}$



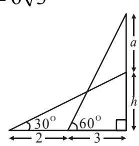
$\because\ \sin a = \dfrac{b}{\sqrt{18}},\quad b = \sqrt{18}\sin 45^\circ = 3\sqrt{2}(\dfrac{1}{\sqrt{2}}) = 3$

(d) $\dfrac{x+2}{12} = \cos 30^o,\quad x+2 = \dfrac{\sqrt{3}}{2}\cdot 12,\quad x = 6\sqrt{3} - 2$

10. In ΔCDB, $\mathrm{DC} = 9\tan 30^\circ = \dfrac{9\sqrt{3}}{3} = 3\sqrt{3}$

In ΔCAB, $x + \mathrm{DC} = 9\tan 60^\circ = 9\sqrt{3},\quad x = 9\sqrt{3} - \mathrm{DC} = 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3}$

11. $\tan 30^\circ = \dfrac{h}{2+3} = \dfrac{h}{5},\quad h = 5\tan 30^\circ = \dfrac{5\sqrt{3}}{3}$



$\tan 60^\circ = \dfrac{a+h}{3},\quad a + h = 3\sqrt{3},\quad a = 3\sqrt{3} - h = 3\sqrt{3} - \dfrac{5\sqrt{3}}{3} = \dfrac{4\sqrt{3}}{3}$

12. $\mathrm{AD} = 80 - \dfrac{18}{\tan 30^o} - \dfrac{18}{\tan 45^o} = 80 - 18\sqrt{3} - 18 = (62 - 18\sqrt{3})$ cm

13. $\because\ \tan\theta = \dfrac{1}{\tan(90^\circ - \theta)},\quad \therefore\ \tan\theta\cdot\tan(90^o - \theta) = 1.$ The given expression

$= (\tan 1^o \times \tan 89^o)\times(\tan 2^o\times\tan 88^o)\times.....(\tan 44^o\times\tan 46^o)\times\tan 45^o = (1)(1).....(1)\tan 45^o = 1$

14. (a) L.H.S. $= \cos^4\theta - \sin^4\theta + 2\sin^2\theta = (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) + 2\sin^2\theta$

$= \cos^2\theta - \sin^2\theta + 2\sin^2\theta = \cos^2\theta + \sin^2\theta = 1 =$ R.H.S.

(b) L.H.S. $= \dfrac{\cos\theta}{1+\sin\theta}\cdot\dfrac{1-\sin\theta}{1-\sin\theta} = \dfrac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta} = \dfrac{\cos\theta(1-\sin\theta)}{\cos^2\theta} = \dfrac{1-\sin\theta}{\cos\theta} =$ R.H.S.

(c) L.H.S. $= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta)$

$= (1)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$= (\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) - 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta = (1)^2 - 3\sin^2\theta\cos^2\theta$ = R.H.S.

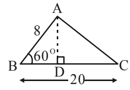
15. (a) $=\dfrac{\cos^2\theta+(\sin\theta+1)^2}{\cos\theta(1+\sin\theta)}=\dfrac{\cos^2\theta+\sin^2\theta+2\sin\theta+1}{\cos\theta(1+\sin\theta)}$

$=\dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}=\dfrac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=\dfrac{2}{\cos\theta}$

(b) $=\dfrac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta-1}{\cos\theta}}=\dfrac{\sin^2\theta}{\cos\theta(\cos^2\theta-1)}=\dfrac{\sin^2\theta}{\cos\theta(-\sin^2\theta)}=\dfrac{-1}{\cos\theta}$

(c) $=\dfrac{1+\cos\theta-(1-\cos^2\theta)}{1+\cos\theta}=\dfrac{\cos\theta(1+\cos\theta)}{1+\cos\theta}=\cos\theta$

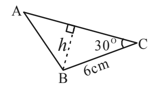
16. In ΔABD, $AD=AB\sin 60^\circ=(8)(\frac{\sqrt{3}}{2})=4\sqrt{3}$,



$BD=AB\cos 60^\circ=(8)(\frac{1}{2})=4$, $\therefore$ DC = BC – BD = 20 – 4 = 16,

$AC=\sqrt{AD^2+DC^2}=\sqrt{(4\sqrt{3})^2+(16)^2}=\sqrt{16(3+16)}=4\sqrt{19}$

17. $h=6\sin 30^\circ=3\,cm$. Area of $\Delta ABC=\frac{1}{2}\times h\times AC=27$,

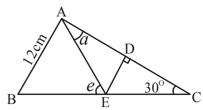


$AC=\dfrac{2\times 27}{h}=\dfrac{54}{3}=18\,cm$

18. ΔABE is equilateral, $\therefore$ $e=60^o$, AE = 12 cm

$a=e-30^o$ (ext. $\angle$ of Δ), $a=60^o-30^o=30^o$

$\therefore$ ΔAEC is isosceles (base $\angle$ s equal), $\therefore$AD = DC



In ΔAED, ED = AE $\sin a$ = $12\sin 60^o=12(\frac{\sqrt{3}}{2})=$ $6\sqrt{3}$ cm

AD = AE $\cos a$ $=12\cos 60^o=12(\frac{1}{2})=$ 6 cm

Area of ΔAEC $=2\cdot(\frac{1}{2}\cdot AD\cdot ED)=6\cdot 6\sqrt{3}=36\sqrt{3}$ cm^2

19. $\because$ $\sin(m+n)=\dfrac{\sqrt{3}}{2}$, $\therefore$ $m+n=60^\circ\ldots(1)$, $\because$ $\cos(m-n)=\dfrac{\sqrt{3}}{2}$, $\therefore$ $m-n=30^\circ$ …(2),

(2) + (1), $2m=90^\circ$, $\therefore$ $m=45^\circ$, From (1), $45^\circ+n=60^\circ$, $\therefore$ $n=\underline{\underline{15^\circ}}$

20. $18\cos^2 x+5(1-\cos^2 x)=9$, $13\cos^2 x=4$, $\cos x=\sqrt{\dfrac{4}{13}}=\dfrac{2}{\sqrt{13}}$, $\therefore$ $\tan x=\dfrac{\sqrt{13-2^2}}{2}=\dfrac{3}{2}$

21. $a^2\sin^2\theta=2^2=4$, and $a^2\cos^2\theta=3^2=9$, $\therefore$ $a^2\sin^2\theta+a^2\cos^2\theta=4+9=13$,

$\therefore$ $a^2(\sin^2\theta+\cos^2\theta)=13$, $a^2(1)=13$, $a=\sqrt{13}$

22. $\sin\theta-\sqrt{2}\cos\theta=0$, $\sin\theta=\sqrt{2}\cos\theta$, $\tan\theta=\sqrt{2}=\dfrac{\sqrt{2}}{1}$, $\sin\theta\cos\theta=(\dfrac{\sqrt{2}}{\sqrt{3}})(\dfrac{1}{\sqrt{3}})=\dfrac{\sqrt{2}}{3}$

23. $(\sin\theta-\cos\theta)^2=(\frac{1}{3})^2$, $(\sin^2\theta+\cos^2\theta)-2\sin\theta\cos\theta=\dfrac{1}{9}$,

$1-2\sin\theta\cos\theta=\frac{1}{9}$, $2\sin\theta\cos\theta=\frac{8}{9}$, $\therefore \sin\theta\cos\theta=\frac{4}{9}$

24. $(\sin\theta+\cos\theta)^2=(\sin^2\theta+\cos^2\theta)+2\sin\theta\cos\theta=1+2(\frac{13}{10})=\frac{18}{5}$

$\therefore \quad \sin\theta+\cos\theta=\sqrt{\frac{18}{5}}=\frac{3\sqrt{2}}{\sqrt{5}}=\frac{3\sqrt{10}}{5}$

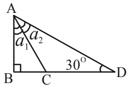
25. $\frac{1}{\frac{1}{\cos\theta}-1}-\frac{1}{\frac{1}{\cos\theta}+1}=\frac{1}{\frac{1-\cos\theta}{\cos\theta}}-\frac{1}{\frac{1+\cos\theta}{\cos\theta}}=\cos\theta(\frac{1}{1-\cos\theta}-\frac{1}{1+\cos\theta})$

$=\cos\theta[\frac{(1+\cos\theta)-(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}]=\cos\theta(\frac{2\cos\theta}{1-\cos^2\theta})=\frac{2\cos^2\theta}{\sin^2\theta}=\frac{2}{\tan^2\theta}=\frac{2}{a^2}$

26. $\tan 30^o=\frac{\text{BC}}{\text{AC}}=\frac{\text{BC}}{\text{2MC}}$, $\therefore \quad \frac{\sqrt{3}}{3}=\frac{1}{2}(\frac{\text{BC}}{\text{MC}})$, $\frac{2\sqrt{3}}{3}=\frac{\text{BC}}{\text{MC}}$

But $\tan\theta=\frac{\text{BC}}{\text{MC}}$, $\therefore \quad \tan\theta=\frac{2\sqrt{3}}{3}$

27. $a_1=a_2=\frac{180^o-90^o-30^o}{2}=30^o$, $\therefore$ AC = CD (sides opp. eq. $\angle$s)



But $\frac{\text{BC}}{\text{AC}}=\sin a_1=\sin 30^o=\frac{1}{2}$, $\therefore$ 2BC = AC = CD, $\frac{\text{BC}}{\text{CD}}=\frac{1}{2}$,
BC : CD = 1 : 2

28. Let BD = a, $\therefore$ DC = $3a$, BC = $a + 3a = 4a$. $\tan 60^\circ=\frac{\text{DC}}{\text{AC}}=\frac{3a}{\text{AC}}$,

$\text{AC}=\frac{3a}{\sqrt{3}}=\sqrt{3}a$. $\tan\theta=\frac{\text{AC}}{\text{BC}}=\frac{\sqrt{3}a}{4a}=\frac{\sqrt{3}}{4}$

29. AM $\perp$ BM and $\angle$BAM = 45° (prop. of square), $\angle$BPM = 60° (equil. Δ).
In ΔBMP, BM = PM tan 60° = $\sqrt{3}$PM ; in ΔBMA, BM = AM tan 45° = AM.
$\therefore$ AM = $\sqrt{3}$PM. $\because$ ΔABM ~ ΔPQM, $\therefore$ $\frac{\text{AB}}{\text{PQ}}=\frac{\text{AM}}{\text{PM}}$, $\frac{\text{AB}}{y}=\frac{\sqrt{3}\text{PM}}{\text{PM}}$, AB = $\sqrt{3}y$.

30. (a) $\angle$ANM =$\angle$ACB = 90°, $\therefore$ MN//BC (corr. $\angle$s equal),
$\therefore$ $\therefore \theta_1=\theta_2$ (alt. $\angle$s, MN//BC)

(b) tan $\angle$A = $\frac{MN}{AN}=1$, $\therefore$ $\angle$A = 45°. In Δ ABC, $\frac{BC}{AC}$ = tan 45°, $\therefore$ BC = AC

MN//BC (proved) and MB = AM (given)

$\therefore$ NC =AN (intercept theorem) $=\frac{1}{2}$AC. $\tan\theta_1=\tan\theta_2=\frac{NC}{BC}=\frac{\frac{1}{2}AC}{AC}=\frac{1}{2}$

Unit 20 Applications of trigonometry

1. (a) area of ABCD = 14 × 9 sin 60° = 109.1 cm^2

(b) area of ΔPQR = $\frac{20\times 15\sin 72^\circ}{2}$ = 142.7 cm^2

(c) the height $= \sqrt{26^2 - \left(\frac{18}{2}\right)^2} = \sqrt{595}$, $\therefore$ area of ΔMNR = $\frac{18 \times \sqrt{595}}{2}$ = 219.5 cm^2

(d) area of ΔABC = $\frac{5 \times 5 \sin 60^\circ}{2}$ = 10.8 cm^2

2. The hexagon can be cut into 6 congruent isosceles triangles whose vertical angle $= \frac{360^\circ}{6} = 60^\circ$. Let h cm be the height of the isosceles triangle. $\tan\frac{60^\circ}{2} = \frac{8 \div 2}{h}$,

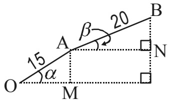
$h = \frac{4}{\tan 30^\circ}$. $\therefore$ Area of the hexagon = 6 × $(\frac{1}{2} \times 8 \times \frac{4}{\tan 30^\circ}) = 96\sqrt{3}$ cm^2.

3. Let θ be the angle of inclination, $\tan\theta = \frac{1}{15}$, θ = 3.81°.

Ans. The angle of inclination is 3.81°.

4. Horizontal distance = $\sqrt{116^2 - 30^2} = \sqrt{12556}$, $\therefore$ Gradient = $\frac{30}{\sqrt{12556}}$ = 0.268.

5. $\tan\alpha = \frac{1}{10}$, $\alpha = 5.71°$; $\therefore$ AM = 15 sin 5.71°;



$\tan\beta = \frac{1}{12}$, $\beta = 4.76°$; $\therefore$ BN = 20 sin 4.76°;

$\therefore$ Vertical distance = 15 sin 5.71° + 20 sin 4.76° = 3.15 km

6. Vertical distance = 400 – 350 = 50,
horizontal distance = 25000 × 4 ÷ 100 = 1000.
Let θ be the angle of inclination, $\tan\theta = \frac{50}{1000}$, θ = 2.86°.

Ans. The angle of inclination is 2.86°.

7. Fiona's height = 2.14 tan 35°. Let the new shadow be x m long.

$\frac{\text{Fiona's height}}{\text{the new shadow}} = \tan 60°$, the new shadow = $\frac{2.14 \tan 35°}{\tan 60°}$ = 0.865

Ans. The length of the new shadow is 0.865m.

8. The angles of elevation are also 47° and 63°.

$\therefore$ Distance between A and B = $\frac{22}{\tan 47°} + \frac{22}{\tan 63°}$ = 31.7 m

9. The angles of elevation are also 24° and 36°.

$\therefore$ Distance between the cars = $\frac{120}{\tan 24°} - \frac{120}{\tan 36°}$ = 104.4 m

10. (a) $\frac{h}{AD} = \tan 40°$, $\therefore$ AD = $\frac{h}{\tan 40°}$. $\frac{h}{AC} = \tan 25°$, $\therefore$ AC = $\frac{h}{\tan 25°}$

(b) $\frac{h}{\tan 25°} - \frac{h}{\tan 40°} = 75$, $h\left(\frac{1}{\tan 25°} - \frac{1}{\tan 40°}\right) = 75$,

h = 75 ÷ $\left(\frac{1}{\tan 25°} - \frac{1}{\tan 40°}\right)$ = 79. *Ans. The height of the cliff is 79 m.*

11. $\dfrac{PQ}{AQ} = \tan 38°$, $AQ = \dfrac{PQ}{\tan 38°}$; $\dfrac{PQ}{QB} = \tan 22°$, $QB = \dfrac{PQ}{\tan 22°}$;

$\dfrac{PQ}{\tan 38°} + \dfrac{PQ}{\tan 22°} = 120$, $PQ\,(\dfrac{1}{\tan 38°} + \dfrac{1}{\tan 22°}) = 120$,

$PQ = 120 \div (\dfrac{1}{\tan 38°} + \dfrac{1}{\tan 22°}) = 32$. *Ans. The height of the lighthouse is 32 m.*

12. (a) $a = 90° - 40° - 15° = 35°$, $b = 40°$ (alt. ∠s, // lines),

$y = 45°$ (alt. ∠s, // lines), $x = 40° + a$ (alt. ∠s, // lines) $= 40° + 35° = 75°$

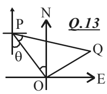
(b) $40° + a = 40° + 35° = 75°$, ∴ the compass bearing of B from A is N75°E.

(c) $180° - y = 180° - 45° = 135°$, ∴ the true bearing of B from C is 135°.

(d) The true bearing of C from A is 040°.

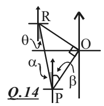
13. $\theta = 42°$, $\theta + 39° = 42° + 39° = 81°$.

Ans. Bearing of Q from P is S81°E.



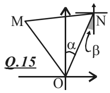
14. $\beta = 35°$; $\alpha = 45° - \beta = 45° - 35° = 10°$; $\theta = \alpha = 10°$.

Ans. Compass bearing of P from R is S10°E.



15. $\alpha = 60° - (360° - 318°) = 18°$; $\beta = \alpha = 18°$

∴ True bearing of M from N = $180° + \beta + 60° = 180° + 18° + 60° = 258°$.



16. ∵ $\angle POQ = 65° + 25° = 90°$, ∴ $\tan\angle OQP = \dfrac{24}{30}$, $\angle OQP = 38.7°$;

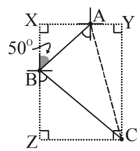
$38.7° + 25° = 63.7°$. *Ans. Compass bearing of P from Q is N63.7°W.*

17. $OA = 80 \times 2 = 160$; $OB = 60 \times 2 = 120$;

$\angle AOB = 360° - 325° + 55° = 90°$, ∴ $AB = \sqrt{160^2 + 120^2} = 200$ km

Ans. Distance between A and B after 2 hours is 200 km.

18. $AX = 1.5 \sin 50°$; $BX = 1.5 \cos 50°$; $BZ = 3 \cos 50°$; $CZ = 3 \sin 50°$;
$AY = CZ - AX = 3 \sin 50° - 1.5 \sin 50° = 1.149$,
$CY = BX + BZ = 1.5 \cos 50° + 3 \cos 50° = 2.893$,
$AC = \sqrt{AY^2 + CY^2} = \sqrt{1.149^2 + 2.893^2} = 3.1124$ km $= 3112.4$ m
∴ Shortest time = $3112.4 \div 50 = 62.2$ min.



19. (a) $BZ = 240 \cos 60°$; $BY = 150 \sin 74°$;

∴ $AX = BZ + BY = 240 \cos 60° + 150 \sin 74° = 264.19 = 264.2$

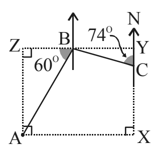
Ans. He is 264.2 m west of the starting point.

(b) $AZ = 240 \sin 60°$; $CY = 150 \cos 74°$;

∴ $CX = AZ - CY = 240 \sin 60° - 150 \cos 74° = 166.5$

Ans. He is 166.5 m south of the starting point.

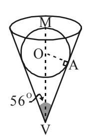
(c) Distance from the starting point $= \sqrt{AX^2 + CX^2} = \sqrt{264.19^2 + 166.5^2}$
$= 312.3$ m

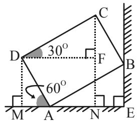


20. $OV = MV - MO = 8 - r$. In ΔOVA, $\dfrac{r}{8-r} = \sin(\dfrac{56°}{2})$,

$r = 8 \sin 28° - r \sin 28°$, $r = \dfrac{8 \sin 28°}{1 + \sin 28°} = 2.56$.

Ans. The radius of the sphere is 2.56 cm.





21. (a) DM = 20 sin 60°=17.3; CF = 30 sin 30°;
CN = DM + CF = 20 sin 60° + 30 sin 30°= 32.3.
Ans. Distances from C and D to AE are 32.3 cm and 17.3 cm respectively.
(b) AM = 20 cos 60°; AE = 30 cos 30°;
ME = AM + AE = 20 cos 60°+ 30 cos 30° = 36.0
Ans. Distance from D to BE is 36.0 cm.

22. (a) Distance between H and K = $180 \times \dfrac{15}{60} = 45$ km

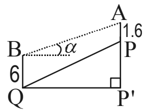
(b) Let x km be the perpendicular distance from A to HK.

$$\therefore \ \frac{x}{\tan 44°} + \frac{x}{\tan 79°} = 45,\quad x = 45 \div \left(\frac{1}{\tan 44°} + \frac{1}{\tan 79°}\right) = 36.6$$

Ans. The altitude of the helicopter is 36.6 km.

23. (a) Let P'Q be the horizontal distance between P and Q.
P'Q = 4 × 400 = 1600 m, P′P = 300 – 100 = 200 m,

$$\therefore \text{ Actual length of road PQ} = \sqrt{200^2 + 1600^2} = 200\sqrt{65} = 1612.5 \text{ m}$$



(b) Gradient of road PQ = $\dfrac{200}{1600} = \dfrac{1}{8}$

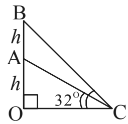
Let θ be the angle of inclination, $\tan\theta = \dfrac{1}{8}$, θ = 7.13°

Ans. The angle of inclination of road PQ is 7.13°.

(c) Vertical distance between A and B = 200 + 1.6 – 6 = 195.6 m,
horizontal distance = P'Q = 1600 m.

Let α be the angle of depression, $\tan\alpha = \dfrac{195.6}{1600}$, α = 6.97°.

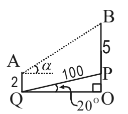
Ans. The angle of depression from the man to the tree is 6.97°.



24. Let 2h be the height of the building. $\dfrac{h}{OC} = \tan 32°$;

$\tan\angle BCO = \dfrac{2h}{OC} = 2\tan 32°$, ∴ ∠BCO = 51.3°

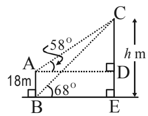
Ans. The angle of depression from the top of building is 51.3°.



25. Their horizontal distance = OQ = 100 cos 20°; $OP = 100\sin 20°$;
their vertical distance = OP + PB – AQ $= 100\sin 20° + 5 - 2$

$= 100 \sin 20° + 3$; $\tan\alpha = \dfrac{3 + 100\sin 20°}{100\cos 20°}$, α = 21.6°

Ans. The angle of depression required is 21.6°.



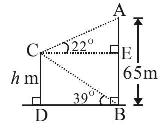
26. Let h m be the height of the building, ∴ CD = (h – 18) m. In △BCE,

BE = $\dfrac{h}{\tan 68°}$; in △ACD, AD = $\dfrac{h-18}{\tan 58°}$. ∵ BE = AD,

$\therefore \ \dfrac{h}{\tan 68°} = \dfrac{h-18}{\tan 58°}$, h tan 58° = h tan 68° – 18 tan 68°,

$h = \dfrac{18\tan 68°}{\tan 68° - \tan 58°} = 50.9$. *Ans. The height of the building is 50.9 m.*

27. (a) In $\triangle ACE$, $CE = \dfrac{65-h}{\tan 22^\circ}$; in $\triangle CDE$, $BD = \dfrac{h}{\tan 39^\circ}$.



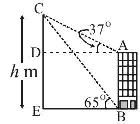
$\because$ CE = BD, $\therefore \dfrac{65-h}{\tan 22^\circ} = \dfrac{h}{\tan 39^\circ}$,

65 tan 39° – h tan 39° = h tan 22°,

$\therefore h = \dfrac{65\tan 39^\circ}{\tan 22^\circ + \tan 39^\circ} = 43.364 \approx 43.4.$

(b) Distance between two buildings = $\dfrac{43.364}{\tan 39^\circ}$ = 53.6 m

28. Let h m be the vertical height of the balloon,



$\therefore$ CD = (h – 80) m. In $\triangle ACD$, $AD = \dfrac{h-80}{\tan 37^\circ}$; in $\triangle BCE$,

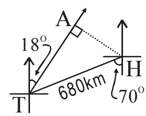
$BE = \dfrac{h}{\tan 65^\circ}$. $\because$ AD = BE, $\therefore \dfrac{h-80}{\tan 37^\circ} = \dfrac{h}{\tan 65^\circ}$,

h tan 65° – 80 tan 65° = h tan 37°, $h = \dfrac{80\tan 65^\circ}{\tan 65^\circ - \tan 37^\circ} = 123.3.$

Ans. The vertical height of the balloon is 123.3 m.

29. $\dfrac{y}{BE} = \tan 45^\circ = 1$, BE = y; AD = BE = y; CD = AD tan θ, $\therefore$ $y - x = y$ tan θ,

$y = \dfrac{x}{1-\tan\theta}$



30. (a) Typhoon is nearest Hong Kong when TA ⊥ AH.

∠ATH = 70° – 18° = 52°, $\dfrac{AH}{TH}$ = sin 52°,

AH = 680 sin 52° = 535.8 km. *Ans. The shortest distance from Hong Kong is 535.8 km.*

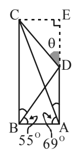
(b) AT = 680 cos 52° = 418.65, $\therefore$ Time taken = 418.65 ÷ 160 = 2 h 37 min.

Ans. It will be nearest Hong Kong at 2 h 37 min after 11:00 a.m., that is 1:37 p.m.

31. CE = AB; BC= AB tan 69°; AE = BC = AB tan 69°; AD = AB tan 55°;

$\tan\theta = \dfrac{CE}{AE-AD} = \dfrac{AB}{AB\tan 69^\circ - AB\tan 55^\circ} = \dfrac{1}{\tan 69^\circ - \tan 55^\circ}$, θ = 40.4°.

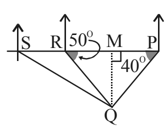
Ans. Bearing of C from D is N40.4°W.



32. (a) $PR = 60 \times \dfrac{20}{60} = 20$km; $\because$∠PQR = 180° – 50° – 40° = 90°,

$\therefore$QR = PR cos 50° = 20 cos 50° = 12.9 km

(b) QM = QR sin 50° = 20cos50°sin50° = 9.848;



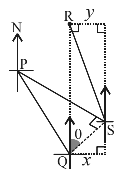
MS = RM + RS = 20cos50°cos50° + 10 = 18.2635;

$\tan\angle MSQ = \dfrac{QM}{MS} = \dfrac{9.848}{18.2635}$, ∠MSQ = 28.3°.

$\therefore$ True bearing of Q from S = 90° + 28.3° = 118.3°

(c) $SQ = \sqrt{MS^2 + QM^2} = \sqrt{18.2635^2 + 9.848^2}$ = 20.7 km

$\therefore$ Time taken = (20.7 ÷ 60) × 60 = 20.7 min



33. (a) The ship is nearest to Q when PS ⊥ SQ. In $\triangle PQS$,

∠QPS = 60° – 32° = 28°, $\therefore$ SQ = 40 sin 28° = 18.8.

θ = 180° – 90° – 28° – 32° = 30°.

Ans. At that instant the ship was 18.8 km in the direction of N30°E from Q.

(b) $\dfrac{x}{\text{SQ}} = \cos(90^\circ - 30^\circ)$, $x = (40\sin 28°)\cos 60°$; $y = x$,

$\therefore$ $y = (40\sin 28°)\cos 60°$. $\dfrac{y}{\text{RS}} = \sin 20^\circ$, $\therefore$ RS $= \dfrac{(40\sin 28°)\cos 60°}{\sin 20°}$ = 27.453 km

In ΔPQS, $\dfrac{PS}{PQ} = \cos 28°$, PS = 40 cos 28° = 35.318 km

$\therefore$ Time taken = (27.453 + 35.318) ÷ 28 = 2 h 15 min.

Ans. The ship reached R at 3:45 p.m.

34. Let AB = a; BC $= \sqrt{a^2 + a^2} = \sqrt{2}a$; AC $= \sqrt{a^2 + (\sqrt{2}a)^2} = \sqrt{3}a$

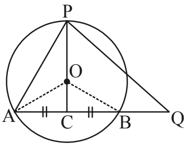
$\therefore$ $\cos\angle ACB = \dfrac{BC}{AC} = \dfrac{\sqrt{2}a}{\sqrt{3}a} = \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{\sqrt{6}}{3}$

35. $\angle$R = $\angle$P = 52° (base $\angle$s, isos. Δ); $\angle$AQD = 180° – 2(52°) = 76°

In ΔAQD, $\dfrac{AD}{AQ} = \sin 76°$, AQ = $\dfrac{6}{\sin 76^\circ}$, $\angle$PBA = $\angle$R = 52° (corr. $\angle$s, AB // QR),

$\therefore$ PA = AB = 6 cm (sides opp. eq. $\angle$s). $\therefore$ PQ = PA + AQ = 6 + $\dfrac{6}{\sin 76^\circ}$ = 12.2 cm

36. (a) In ΔOAC and ΔOBC, OA = OB (radii), OC = OC (common),
AC = BC (given), $\therefore$ ΔOAC $\cong$ ΔOBC (S.S.S.), $\therefore$ $\angle$OCA = $\angle$OCB (corr. $\angle$s, $\cong$ Δs).
$\angle$OCA + $\angle$OCB = 180° (adj. $\angle$s on st. line),
$\angle$PCA = $\angle$OCA = 180° ÷ 2 = 90°



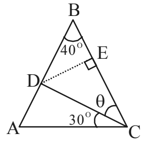
(b) AC = 8 ÷ 2 = 4, OC $= \sqrt{5^2 - 4^2} = 3$, PC = 5 + 3 = 8,

$\therefore$ AP $= \sqrt{AC^2 + PC^2} = \sqrt{4^2 + 8^2}$ = 8.94 cm

(c) $\dfrac{\text{PC}}{\text{CQ}} = \tan 56°$, CQ $= \dfrac{8}{\tan 56°}$ = 5.396,

$\therefore$ Area of ΔAPQ $= \dfrac{1}{2} \times$ PC $\times$ AQ $= \dfrac{1}{2}$(8) (4 + 5.396) = 37.6 cm^2

37. $\angle$BAC = $\angle$BCA (base $\angle$s, isos. Δ),



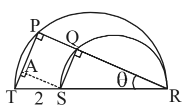
$\therefore$ θ + 30° = $\dfrac{180° - 40°}{2}$ ($\angle$ sum of Δ), θ = 40°,

$\therefore$ CD = BD (sides opp. eq. $\angle$s),

$\therefore$ ΔDCE $\cong$ ΔDBE (A.A.S.). CE = BE (corr. sides, $\cong$ Δs),

CE $= \dfrac{1}{2}$ BC = 6. In ΔDCE, $\dfrac{CE}{CD} = \cos 40°$, $\therefore$ CD = $\dfrac{6}{\cos 40°}$ = 7.83cm

38. Draw SA $\perp$ PT, $\therefore$ APQS is a rectangle.



$\therefore$ $\angle$AST = θ (corr. $\angle$s, AS // PR).

In ΔSTA, $\dfrac{AS}{TS} = \cos\theta$, AS = 2 cos θ.

$\because$ PQ = AS (rectangle property), $\therefore$ PQ = 2 cos θ

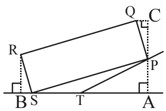
39. (a) In ΔEAB and ΔFBE, $\angle$A = $\angle$B (prop. of square);
$\angle$ADE + 90° = $\angle$DEB (ext. $\angle$ of Δ), $\angle$ADE + 90° = 90° + $\angle$BEF,
$\therefore$ $\angle$ADE = $\angle$BEF; $\angle$AED = $\angle$BFE (3^{rd} $\angle$ of Δ); $\therefore$ ΔEAD ~ ΔFBE (A.A.A.)

(b) Let the side of the square be x.

$\because$ ΔFBE ~ ΔEAD (proved), $\therefore$ $\frac{BF}{AE}=\frac{BE}{AD}$, $\frac{BF}{\frac{1}{2}x}=\frac{\frac{1}{2}x}{x}$, BF = $\frac{1}{4}x$,

$\therefore$ CF = $x-\frac{1}{4}x=\frac{3}{4}x$, $\therefore$ $\tan\theta=\frac{CF}{CD}=\frac{\frac{3}{4}x}{x}=\frac{3}{4}$

40. In ΔAPT, $\frac{PA}{PT}=\sin 28°$, PA = 12 sin 28°.



In ΔPSA, $\sin\angle PSA=\frac{PA}{PS}=\frac{12\sin 28^\circ}{18}$, $\angle$PSA = 18.24°;

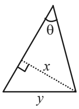
$\angle$BRS = $\angle$PSA = 18.24°. In ΔRBS, $\frac{RB}{6}=\cos 18.24^\circ$;

$\therefore$ Height of R = RB = 6 cos 18.24° = 5.70 cm

$\therefore$ Height of Q = AC = CP + PA = RB + PA = 5.70 + 12 sin 28° = 11.3 cm

41. (a) θ = 180° – 60° – 75° = 45° ($\angle$ sum of Δ);

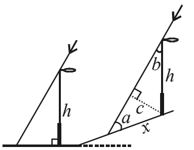
$\frac{x}{8}=\sin 45°$, x = 8 sin 45°; $\frac{x}{y}=\sin 60°$, $\therefore$ $y=\frac{x}{\sin 60°}=\frac{8\sin 45°}{\sin 60°}$ = 6.53



(b) a = 60° – 20° = 40°, b = 180° – 90° – 60° = 30° ($\angle$ sum of Δ)

(c) $\frac{h}{2.3}=\tan 60°$, h = 2.3 tan 60° = 3.98

Ans. *The height of the lamppost is 3.98 m.*



(d) $\frac{c}{h}=\sin b$, c = 2.3 tan 60° sin 30°; $\frac{c}{x}=\sin a$,

$\therefore$ $x=\frac{2.3\tan 60^\circ\sin 30^\circ}{\sin 40^\circ}=3.10$

Ans. *The length of shadow on the slope is 3.10 m.*

Unit 21 Coordinates – Distance & slope

1. (a) Slope $=\frac{0-(-30)}{24-0}=\frac{5}{4}$

(b) Slope $=\frac{25-(-75)}{-18-(-20)}=\frac{100}{2}=50$

(c) Slope $=\frac{8-8}{2-(-3)}=\frac{0}{5}=0$

(d) Slope $=\frac{10-4}{6-6}=\frac{6}{0}=$ undefined

(e) Slope $=\frac{-18.5-(-9.5)}{-8.5-0.5}=\frac{-9}{-9}=1$

(f) Slope $=[3\frac{1}{3}-(-1\frac{1}{2})]\div[-1\frac{3}{4}-(-2\frac{1}{3})]=\frac{29}{6}\times\frac{12}{7}=\frac{58}{7}$

2. (a) Distance $=\sqrt{(5-2)^2+(12-16)^2}=\sqrt{3^2+(-4)^2}=\sqrt{25}=5$

(b) Distance $=\sqrt{[-10-(-4)]^2+[5-(-1)]^2}=\sqrt{(-6)^2+6^2}=\sqrt{72}=6\sqrt{2}$

(c) Distance $=\sqrt{(-4\frac{2}{3}-6\frac{4}{5})^2+(\frac{3}{5}-\frac{1}{4})^2+}=\sqrt{(-\frac{172}{15})^2+(\frac{7}{20})^2}=11.5$

(d) Distance $=\sqrt{(-4.2-1.7)^2+[-10-(-6.8)]^2}=\sqrt{45.05}=6.71$

3. (a) Slope of the line segment $=\dfrac{4-8}{9-3}=\dfrac{-4}{6}=\dfrac{-2}{3}$,

$\therefore$ slope of the lines parallel to it is $\dfrac{-2}{3}$.

(b) Slope of the line segment $=\dfrac{-2-5}{3-(-2)}=\dfrac{-7}{5}$,

$\therefore$ slope of the lines parallel to it is $-\dfrac{7}{5}$.

4. (a) Slope of the line segment $=\dfrac{1-(-1)}{7-6}=\dfrac{2}{1}=2$,

$\therefore$ slope of the lines perpendicular to it $=-1\div 2=-\dfrac{1}{2}$

(b) the given line is vertical (x-coordinates equal),
$\therefore$ slope of the lines perpendicular to it = 0.

(c) the given line is horizontal (y-coordinates equal),
$\therefore$ slope of the lines perpendicular to it is undefined.

5. (a) $\dfrac{9-3}{-2-a}=\dfrac{3}{5},\ 30=-6-3a,\ 3a=-36,\ \therefore\ a=-12$

(b) $(\dfrac{9-3}{-2-a})(-\dfrac{3}{2})=-1,\ 18=-4-2a,\ 2a=-22,\ \therefore\ a=-11$

6. (a) Slope $=\dfrac{-3-9}{10-(-10)}=\dfrac{-12}{20}=-\dfrac{3}{5}$

(b) Let PQ cuts the x-axis and y-axis at $(x, 0)$ and $(0, y)$ respectively.

$\dfrac{9-0}{-10-x}=-\dfrac{3}{5},\ 45=30+3x,\ 3x=15,\ \therefore\ x=5$

$\dfrac{9-y}{-10-0}=-\dfrac{3}{5},\ 45-5y=30,\ 5y=15,\ \therefore\ y=3$

Ans. PQ cuts the x-axis at (5, 0) and cuts the y-axis (0, 3).

7. $\sqrt{(8-k)^2+[-12-(-2)]^2}=k,\ \sqrt{64-16k+k^2+(-10)^2}=k$,

$k^2-16k+164=k^2,\ 16k=164,\ \therefore\ k=10.25$

8. (a) the slope $=\dfrac{7-(-2)}{4-3}=9$, $\therefore$ the angle of inclination $=\tan^{-1}(9)=83.7°$

(b) the slope $=\dfrac{1-(-8)}{-5-6}=\dfrac{9}{-11}$, $\tan^{-1}(\dfrac{9}{-11})=-39.3°$,

$\therefore$ the angle of inclination = 180° – 39.3° = 140.7°

9. $\dfrac{7-1}{-7-1}=\dfrac{t-1}{3-1},\ \dfrac{6}{-8}=\dfrac{t-1}{2},\ \dfrac{3}{-2}=t-1,\ t=-\dfrac{1}{2}$

10. $(\dfrac{9-18}{15-12})(\dfrac{16-12}{9-k})=-1,\ (\dfrac{-9}{3})(\dfrac{4}{9-k})=-1,\ 12=9-k,\ \therefore k=-3$

11. (a) $m_{PQ}=\frac{1-(-2)}{5-1}=\frac{3}{4}$, $m_{QR}=\frac{0-(-2)}{4-1}=\frac{2}{3}$, $\because$ $m_{PQ}\neq m_{QR}$, $\therefore$ P, Q, R are not collinear.

(b) $m_{AB}=\frac{3-(-3)}{5-2}=\frac{6}{3}=2$, $m_{BC}=\frac{3-1}{5-4}=2$, $\because$ $m_{AB}=m_{BC}$, $\therefore$ A, B, C are collinear.

12. (a) $BC=\sqrt{(-2-8)^2+(3-1)^2}=\sqrt{104}$, $AC=\sqrt{[-2-(-3)]^2+[(3-(-2)]^2}=\sqrt{26}$,

$AB=\sqrt{[8-(-3)]^2+[1-(-2)]^2}=\sqrt{130}$, $\because$ $BC^2+AC^2=104+26=130=AB^2$,

$\therefore$ ΔABC is right-angled.

(b) Area of $\Delta ABC=\frac{1}{2}(AC)(BC)=\frac{1}{2}(\sqrt{26})(\sqrt{104})=\frac{1}{2}(\sqrt{26})(2\sqrt{26})=26$ sq. units

13. $m_{PQ}=\frac{-1-4}{3-1}=-\frac{5}{2}$, $m_{RS}=\frac{2-(-3)}{-4-(-2)}=-\frac{5}{2}$, $m_{QR}=\frac{-3-(-1)}{-2-3}=\frac{-2}{-5}=\frac{2}{5}$, $m_{SP}=\frac{4-2}{1-(-4)}=\frac{2}{5}$.

$\because$ $m_{PQ}\times m_{QR}=m_{RS}\times m_{QR}=m_{PQ}\times m_{SP}=m_{RS}\times m_{SP}=(\frac{-5}{2})(\frac{2}{5})=-1$,

$\therefore$ PQ$\perp$QR, RS$\perp$QR, PQ$\perp$SP, RS$\perp$SP. $PQ=\sqrt{(3-1)^2+(-1-4)^2}=\sqrt{29}$,

$QR=\sqrt{[-3-(-1)]^2+(-2-3)^2}=\sqrt{29}$, $RS=\sqrt{[2-(-3)]^2+[-4-(-2)]^2}=\sqrt{29}$,

$SP=\sqrt{(4-2)^2+[1-(-4)]^2}=\sqrt{29}$, $\therefore$ PQ = QR = RS = SP.

Ans. P, Q, R, S are the vertices of a square.

14. $\sqrt{(k-17)^2+[3-(-29)]^2}=40$, $(k-17)^2+32^2=40^2$, $(k-17)^2=576$,

$k-17=\pm\sqrt{576}=\pm 24$, $k=24+17$ or $-24+17$, $\therefore$ $k=41$ or -7.

15. $PQ=25=\sqrt{(x-6)^2+(-12-8)^2}$, $25^2=(x-6)^2+400$,

$(6-x)^2=225$, $\therefore$ $6-x=15$ or -15, $\therefore$ $x=21$ or -9

16. Let coordinates of N be $(-6, n)$. $n=\sqrt{[-6-(-2)]^2+(n-8)^2}$,

$n^2=16+n^2-16n+64$, $16n=80$, $\therefore$ $n=5$. *Ans. The coordinates of N are (–6, 5).*

17. $\sqrt{[(n+1)-n]^2+[2-(1-n)]^2}=\sqrt{26}$, $1+(n+1)^2=26$, $(n+1)^2=25$,

$n+1=\pm 5$, $n+1=5$ or $n+1=-5$, $\therefore$ $n=4$ or $n=-6$.

18. Let the coordinates of P be $(0, p)$. $\sqrt{(0-9)^2+(p-3)^2}=\sqrt{[0-(-7)]^2+[p-(-5)]^2}$,

$81+p^2-6p+9=49+p^2+10p+25$, $16p=16$, $\therefore$ $p=1$

Ans. The coordinates of P are (0, 1).

19. Distance $=\sqrt{(1-m^2)^2+(2m-0)^2}=\sqrt{1-2m^2+m^4+4m^2}=\sqrt{m^4+2m^2+1}=\sqrt{(m^2+1)^2}$

$= (m^2 + 1)$ units.

20. Let the coordinates of P be $(p, 0)$. $p=\sqrt{(20-p)^2+(-12-0)^2}$, $p^2 = 400 - 40p + p^2 + 144$,

$40p = 544$, $p = 13.6$. $\therefore$ Area of $\Delta OPQ=\frac{1}{2}(13.6)(12)=81.6$ sq. units

21. $AB=\sqrt{[0-(-6)]^2+(-2-0)^2}=\sqrt{40}=2\sqrt{10}$, $BC=AB=2\sqrt{10}$,

$\therefore$ area of $\Delta ABC = \frac{1}{2}(2\sqrt{10})(6)=6\sqrt{10}$ sq. units

22. $AB=\sqrt{[19-(-21)]^2+(12-3)^2}=\sqrt{1681}=41$, $\because$ AP : BP = 5 : 2, $\therefore$ AB : BP = 3 : 2,

$\therefore$ $BP=\frac{2}{3}AB=\frac{2\times 41}{3}=27\frac{1}{3}$

23. Angle of inclination of L = 180° – 45° = 135°. $\therefore$ slope of L = tan 135° = –1

$\therefore$ $\frac{y-0}{x-(-3)}=-1$, $y=-x-3$, $x=-3-y$

24. (a) $m_{AD}=\frac{2-(-6)}{3-(-5)}=\frac{8}{8}=1$, $\therefore$ angle of inclination of AD = $\tan^{-1}(1)$ = 45°,

$\theta+45°=90°$ (ext. ∠ of Δ), $\theta = 45°$

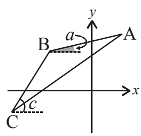
(b) $m_{L_2}=\tan\beta=\frac{7-0}{4-2}=\frac{7}{2}=3.5$, $\therefore$ $\beta\approx 74°$. $m_{L_1}=\tan\alpha=\frac{4-0}{5-(-4)}=\frac{4}{9}$,

$\therefore$ $\alpha\approx 24°$. Let γ be the vert . opp. ∠ of θ.

$\gamma=\beta-\alpha$ = 74°–24° = 50° (ext. ∠ of Δ); $\therefore$ $\theta=\gamma$ = 50°

25. Slope of AB = tan a = $\frac{11-15}{-4-2}=\frac{-4}{-6}=\frac{2}{3}$, $\therefore$ a = 33.69°

slope of BC = tan c = $\frac{-1-11}{-12-(-4)}=\frac{-12}{-8}=\frac{3}{2}$, $\therefore$ c = 56.31°



∠ABC = (180° – c) + a = 180° – 56.31° + 33.69° ≈ 157°

26. (a) When $x = 0$, $2(0) + 4y - 12 = 0$, $4y = 12$, $y = 3$. When $y = 0$, $2x + 4(0) - 12 = 0$,

$2x = 12$, $x = 6$. *Ans. The x-intercept is 6, y-intercept is 3.*

(b) The line passes through (0, 3) and (6, 0), $\therefore$ slope = $\frac{3-0}{0-6}=\frac{3}{-6}=-\frac{1}{2}$

27. (a) $AB=\sqrt{(-3-0)^2+(6-4)^2}=\sqrt{(-3)^2+2^2}=\sqrt{13}$,

$BC=\sqrt{(6-0)^2+(0-4)^2}=\sqrt{6^2+(-4)^2}=\sqrt{52}=2\sqrt{13}$,

$AC=\sqrt{[6-(-3)]^2+(0-6)^2}=\sqrt{9^2+(-6)^2}=\sqrt{117}=3\sqrt{13}$.

(b) AB + BC = $\sqrt{13}+2\sqrt{13}=3\sqrt{13}=$ AC. $\therefore$ A, B C are collinear.

28. $\sqrt{(m-4)^2+(2-7)^2}=\sqrt{(n-4)^2+(2-7)^2}$, $m^2-8m+16=n^2-8n+16$,

$m^2-n^2-8m+8n=0$, $(m-n)(m+n)-8(m-n)=0$, $(m-n)(m+n-8)=0$,

$\therefore$ $m+n-8=0$ ($\because$ $m\neq n$), $m+n=8$.

29. (a) $AB=\sqrt{[3-(-3)]^2+[2-(-6)]^2}=\sqrt{6^2+8^2}=\sqrt{100}=10$,

$BC=\sqrt{(5-3)^2+(-2-2)^2}=\sqrt{2^2+(-4)^2}=\sqrt{20}=2\sqrt{5}$,

$AC=\sqrt{[5-(-3)]^2+[-2-(-6)]^2}=\sqrt{8^2+4^2}=\sqrt{80}=4\sqrt{5}$,

slope of $AB=\dfrac{2-(-6)}{3-(-3)}=\dfrac{8}{6}=\dfrac{4}{3}$, slope of $BC=\dfrac{-2-2}{5-3}=\dfrac{-4}{2}=-2$,

slope of $AC=\dfrac{-2-(-6)}{5-(-3)}=\dfrac{4}{8}=\dfrac{1}{2}$.

(b) slope of BC × slope of $AC=-2\times\dfrac{1}{2}=-1$, $\therefore$ BC⊥AC

Area of $\Delta ABC=\dfrac{1}{2}(2\sqrt{5})(4\sqrt{5})=20$ sq. units.

(c) $\sin\angle A=\dfrac{BC}{AB}=\dfrac{2\sqrt{5}}{10}$, $\therefore$ $\angle A=26.6^\circ$, $\angle B=90^\circ-26.6^\circ=63.4^\circ$

(d) Slope of the altitude × slope of AB = –1, $\therefore$ $\dfrac{y-(-2)}{-7-5}\times\dfrac{4}{3}=-1$, $4(y+2)=36$, $y=7$

30. (a) $m_{AM}=\dfrac{(m-4)-2}{m-(-1)}==\dfrac{m-6}{m+1}$, $m_{BC}=\dfrac{1-(-5)}{6-(-2)}=\dfrac{3}{4}$, $\because$ AM ⊥ BC,

$\therefore$ $\dfrac{m-6}{m+1}\times\dfrac{3}{4}=-1$, $3m-18=-4m-4$, $7m=14$, $\therefore$ $m=2$, $m-4=-2$.

Ans. Coordinates of M are (2, –2).

(b) $BC=\sqrt{[6-(-2)]^2+[1-(-5)]^2}=\sqrt{100}=10$, $AM=\sqrt{(-2-2)^2+[2-(-1)]^2}=\sqrt{25}=5$

$\therefore$ Area of the $\Delta ABC=\dfrac{1}{2}(BC)(AM)=\dfrac{1}{2}(10)(5)=25$ sq. units

(c) $m_{AB}=\dfrac{2-(-5)}{-1-(2)}=\dfrac{7}{1}$, $AB=\sqrt{[2-(-5)]^2+[-1-(-2)]^2}=\sqrt{50}=5\sqrt{2}$

Slope of CN = $-1\div m_{AB}=-1\div 7=-\dfrac{1}{7}$. Area of $\Delta ABC=\dfrac{1}{2}$(AB) (CN) = 25,

$\therefore$ $\dfrac{1}{2}(5\sqrt{2})(CN)=25$, $\therefore$ $CN=\dfrac{25\times 2}{5\sqrt{2}}=\dfrac{10}{\sqrt{2}}=\dfrac{10\sqrt{2}}{2}=5\sqrt{2}$.

31. $m_{L_2} = \frac{3-0}{0-4} = -\frac{3}{4}$, $m_{L_1} = \frac{b-0}{a-0} = \frac{b}{a}$. $\because$ $L_1 \perp L_2$, $\therefore$ $m_{L_1} \times m_{L_2} = -1$, $\frac{b}{a} \times (-\frac{3}{4}) = -1$,

$\therefore$ $b = \frac{4}{3}a$ ….(i) $m_{L_2} = \frac{b-3}{a-0} = -\frac{3}{4}$, $4b - 12 = -3a$….(ii) Sub. (i) into (ii),

$4(\frac{4}{3}a) - 12 = -3a$, $16a - 36 = -9a$, $25a = 36$, $\therefore$ $a = \frac{36}{25}$, $b = \frac{4}{3}(\frac{36}{25}) = \frac{48}{25}$.

Ans. Coordinates of P are $(\frac{36}{25}, \frac{48}{25})$.

32. $m_{PQ} = \frac{-5-1}{-1-2} = 2$, $m_{SR} = \frac{k-3}{h+3}$, $\because$ PQ//SR, $\therefore$ $2 = \frac{k-3}{h+3}$, $k = 2h + 9$ …(i)

$m_{PR} = \frac{3-1}{-3-2} = -\frac{2}{5}$, $m_{SQ} = \frac{k-(-5)}{h-(-1)} = \frac{k+5}{h+1}$; $\because$ PR $\perp$ SQ, $\therefore$ $(-\frac{2}{5})(\frac{k+5}{h+1}) = -1$,

$2k + 10 = 5h + 5$, $2k = 5h - 5$ …(ii) Sub. (i) into (ii), $2(2h + 9) = 5h - 5$, $\therefore$ $h = 23$,

$k = 2(23) + 9 = 55$. *Ans. Coordinates of S are (23, 55).*

33. (a) $\because$ AM // DC, $\therefore$ $\frac{0-2}{-6-0} = \frac{t-(-3)}{-2-1}$, $\frac{1}{3} = \frac{t+3}{-3}$, $\therefore$ $t = -4$

$\because$ $\tan\angle BAO$ = slope of AB $= \frac{1}{3}$, $\therefore$ $\angle BAO = 18.43°$

$\tan\angle OAD = \frac{\text{vertical distance between D, A}}{\text{horizontal distance between D, A}} = \frac{0-4}{-2-(-6)} = \frac{4}{4} = 1$, $\therefore$ $\angle OAD = 45°$

(b) $AB = \sqrt{[(-6)-0]^2 + (0-2)^2} = \sqrt{40} = 4\sqrt{10}$

$CD = \sqrt{[-2-1]^2 + [-4-(-3)]^2} = \sqrt{10}$. $AD = \sqrt{[(-6)-(2)]^2 + [0-(-4)]^2} = \sqrt{32} = 4\sqrt{2}$

Let h be the perpendicular distance from D to AB.

$\angle DAB = \angle DAO + \angle BAO = 45° + 18.43° = 63.43°$

$\therefore$ $\sin\angle DAB = \frac{h}{AD}$, $\sin 63.43° = \frac{h}{4\sqrt{2}}$, $h = 4\sqrt{2} \times \sin 63.43° = 5.06$

(c) Area of trapezium ABCD = $\frac{1}{2}(AB+CD) \times h$

$= \frac{1}{2}(2\sqrt{10} + \sqrt{10}) \times 5.06 = 24.0$ sq. units.

34. (a) $m_{OR} = \frac{4-0}{-6-0} = -\frac{2}{3}$. $\because$ OR$\perp$PQ, $\therefore$ slope of PQ = $-1 \div (-\frac{2}{3}) = \frac{3}{2}$

(b) Let the coordinates of P and Q be $(p, 0)$ and $(0, q)$ respectively.

$\because$ P,R,Q are collinear, $\therefore$ $m_{PR} = m_{RQ} = m_{PQ} = \frac{3}{2}$. $\frac{4-0}{-6-p} = \frac{3}{2}$, $8 = -18 - 3p$,

$3p = -26$, $\therefore$ $p = -\frac{26}{3}$, $\frac{q-4}{0-(-6)} = \frac{3}{2}$, $2q - 8 = 18$, $2q = 26$, $\therefore$ $q = 13$

$\therefore$ Area of $\Delta OPQ = \frac{1}{2}(OP)(OQ) = \frac{1}{2}(\frac{26}{3})(13) = 56.3$ sq. units.

35. (a) Let the coordinates of E be (x, y). $m_{BC} = \frac{6-4}{2-10} = \frac{2}{-8} = -\frac{1}{4}$

$\because$ AM$\perp$BC, $\therefore m_{AE} \times m_{BC} = -1$, $\frac{y-1}{x-1} = -1 \div (-\frac{1}{4}) = 4$,

$\therefore y-1 = 4x-4$, $y = 4x - 3$…(i) $m_{AC} = \frac{4-1}{10-1} = \frac{1}{3}$, $\because$ BN$\perp$AC, $\therefore m_{BE} \times m_{AC} = -1$,

$\frac{y-6}{x-2} = -1 \div \frac{1}{3} = -3$, $y - 6 = 6 - 3x$, $y = 12 - 3x$ …(ii)

Sub (i) into (ii), $4x - 3 = 12 - 3x$, $\therefore x = \frac{15}{7}$, $\therefore y = 4(\frac{15}{7}) - 3 = \frac{39}{7}$

Ans. The coordinates of E are $(\frac{15}{7}, \frac{39}{7})$.

(b) $m_{BA} = \frac{6-1}{2-1} = \frac{5}{1} = 5$; $\therefore$ slope of the altitude through C = $-1 \div 5 = -\frac{1}{5}$.

But $m_{EC} = (\frac{39}{7} - 4) \div (\frac{15}{7} - 10) = \frac{11}{-55} = -\frac{1}{5}$. $\because m_{EC}$ = slope of the altitude through C,

$\therefore$ the altitude from C to AB passes through E.

36. (a) Coordinates of A = $(-r, 0)$, coordinates of B = $(r, 0)$.

(b) slope of AP = $\frac{y-0}{x-(-r)} = \frac{y}{x+r}$, slope of PB = $\frac{y-0}{x-r} = \frac{y}{x-r}$.

$\therefore$ slope of AP × slope of PB = $\frac{y}{x+r} \cdot \frac{y}{x-r} = \frac{y^2}{x^2 - r^2}$.

However, $OP^2 = r^2$, $\therefore x^2 + y^2 = r^2$, $x^2 - r^2 = -y^2$,

$\therefore$ slope of AP × slope of PB = $\frac{y^2}{-y^2} = -1$. $\therefore$ AP $\perp$ PD, i.e. $\angle APB = 90°$.

Unit 22 Coordinates – Point of division

1. (a) $P = (\frac{-8\times2+2\times3}{2+3}, \frac{5\times2+(-1)\times3}{2+3}) = (\frac{-10}{5}, \frac{7}{5}) = (-2, \frac{7}{5})$

(b) $P = (\frac{-3\times1+(-12)\times2}{1+2}, \frac{4\times1+0\times2}{1+2}) = (\frac{-27}{3}, \frac{4}{3}) = (-9, \frac{4}{3})$

(c) $P = (\frac{2\times5+\left(-\frac{1}{4}\right)\times4}{5+4}, \frac{\left(\frac{3}{5}\right)\times5+6\times4}{5+4}) = (\frac{9}{9}, \frac{27}{9}) = (1, 3)$

(d) AP : PB = $1 : \frac{3}{7}$ = 7 : 3,

$\therefore P = (\frac{-7\times3+3\times7}{3+7}, \frac{-7\times3+(-2)\times7}{3+7}) = (\frac{0}{10}, \frac{-35}{10}) = (0, -3.5)$

2. (a) $M=(\frac{7+(-5)}{2},\frac{-2+(-8)}{2})=(\frac{2}{2},\frac{-10}{2})=(1,-5)$

(b) $M=(\frac{-3+0}{2},\frac{2.5+(-5.5)}{2})=(\frac{-3}{2},\frac{-3}{2})$

(c) $M=(\frac{\frac{4}{3}+4}{2},\frac{-\frac{13}{6}+\frac{2}{3}}{2})=(\frac{16}{3}\times\frac{1}{2},-\frac{9}{6}\times\frac{1}{2})=(\frac{8}{3},-\frac{3}{4})$

3. Let the coordinates of A be (x, y). $0=\frac{x\times4+8\times3}{3+4}$, $0=4x+24$, $\therefore\ x=-6$

$1=\frac{y\times4+11\times3}{3+4}$, $7=4y+33$, $-26=4y$, $\therefore\ y=-6.5$

Ans. The coordinates of A are (–6, –6.5).

4. Let the coordinates of N be (x, y). $\frac{x+5.5}{2}=1$, $x+5.5=2$, $x=-3.5$.

$\frac{y+(-3)}{2}=\frac{1}{2}$, $y-3=1$, $y=4$. *Ans. The coordinates of N are (–3.5, 4).*

5. (a) PQ : QR = (3 – 1) : (9 – 3) = 2 : 6 = 1 : 3

(b) $k=\frac{7\times3+(-9)\times1}{3+1}=\frac{21-9}{4}=\frac{12}{4}=3$

6. Let the coordinates of the points be (x_1, y_1) and (x_2, y_2).

The ratio = 2 : 1, $\therefore\ x_1=\frac{-2\times2+7\times1}{2+1}=\frac{3}{3}=1$, $y_1=\frac{8\times2+(-2)\times1}{2+1}=\frac{14}{3}$.

The ratio = 1 : 2, $\therefore\ x_2=\frac{-2\times1+7\times2}{2+1}=\frac{12}{3}=4$, $y_2=\frac{8\times1+(-2)\times2}{2+1}=\frac{4}{3}$.

Ans. The coordinates of the points are $(1,\frac{14}{3})$ *and* $(4,\frac{4}{3})$.

7. (a) Let the ratio be $r : s$. The y-coordinate of the point of division = 0,

$\therefore\ s(-\frac{7}{3})+r(\frac{28}{5})=0$, $\frac{28}{5}r=\frac{7}{3}s$, $\frac{r}{s}=\frac{7}{3}\times\frac{5}{28}=\frac{5}{12}$. *Ans. The ratio is 5 : 12.*

(b) The x-coordinate of the point of division = 0,

$\therefore$ The ratio = $(5\frac{5}{6}-0)\div[0-(-1\frac{1}{4})]=\frac{35}{6}\div\frac{5}{4}=\frac{35}{6}\times\frac{4}{5}=\frac{14}{3}$

8. $\because$ AB : AC = 3 : 5, $\therefore$ AB : BC = 3 : 2.

Let the coordinates of C be (x, y). $\frac{x\times3+8\times2}{3+2}=-1$, $3x+16=-5$, $x=-7$.

$\frac{y\times3+(-4)\times2}{3+2}=-7$, $3y-8=-35$, $3y=-27$, $y=-9$.

Ans. The coordinates of C are (–7, –9).

9. (a) The mid-point of PR $=(\frac{6+3}{2},\frac{9+(-12)}{2})=(4.5,-1.5)$

(b) Let the coordinates of S be (x, y). $\because$ diagonals bisect each other,

$\therefore\ 4.5=\frac{12+x}{2},\ x=-3;\ -1.5=\frac{-30+y}{2},\quad y=-3+30=27.$

Ans. The coordinates of S are (–3, 27).

10. $\because$ ΔPHK~ΔPQR (A.A.A), $\therefore\ \frac{PH}{PQ}=\frac{HK}{QR}=\frac{1}{4}$,

PH : PQ = 1 : 4, PH : HQ = 1 : 3 $\therefore$ QH : HP = 3 : 1.

Let the coordinates of H be (x, y), $x=\frac{-7\times3+9}{3+1}=-3,\ y=\frac{2\times3+12}{3+1}=4.5.$

Ans. The coordinates of H are (–3, 4.5).

11. Sub. $x = 0$ and $y = 0$ into the equation, we have $4y = 12$, $y = 3$ and $x = 12$,

$\therefore$ coordinates of A and B are (0, 3) and (12, 0) respectively.

Let the coordinates of P be (x, y). $x=\frac{0\times3+12\times1}{3+1}=3,\ y=\frac{3\times3+0\times1}{3+1}=2.25.$

Ans. The coordinates of P are (3, 2.25).

12. $\because$ AC : CB = 3 : 2, $\therefore$ AB : BC = (3 – 2) : 2 = 1 : 2

Let the coordinates of C be (x, y). $2=\frac{-2\times2+x\times1}{2+1},\ 6=-4+x,\ x=10.\ 5=\frac{3\times2+y\times1}{2+1},$

$15=6+y,\ y=9.$ *Ans. The coordinates of C are (10, 9).*

13. (a) Let the coordinates of P and Q be (x_P, y_P) and (x_Q, y_Q) respectively.

$x_P=\frac{7\times1+(-2)\times3}{1+3}=\frac{7-6}{4}=0.25,\ y_P=\frac{6\times1+0\times3}{1+3}=\frac{6}{4}=1.5.$

$x_Q=\frac{7\times1+10\times3}{1+3}=\frac{37}{4}=9.25,\ y_Q=\frac{6\times1+(-6)\times3}{1+3}=\frac{-12}{4}=-3.$

Ans. The coordinates of P and Q are (0.25, 1.5) and (9.25, –3) respectively.

(b) Slope of AB $=\frac{0-(-6)}{-2-10}=\frac{6}{-12}=-\frac{1}{2}$, slope of PQ $=\frac{-3-1.5}{9.25-0.25}=\frac{-4.5}{9}=-\frac{1}{2}$,

$\because$ slope of PQ = slope of AB, $\therefore$ PQ//AB

14. Let P(x, y) be the intersection point of the diagonals.

$\because$ Diagonals bisect each other, $\therefore\ x=\frac{-3+5}{2}=1,\ y=\frac{2+(-4)}{2}=-1.$

$\because$ P is a point on BD, $\therefore$ slope of BD = slope of BP $=\frac{7-(-1)}{8-1}=\frac{8}{7}$

15. $a-b=\frac{2\times4+5\times3}{4+3}-\frac{-9\times4+5\times3}{4+3}=\frac{8+15}{7}-\frac{-36+15}{7}=\frac{44}{7}$

16. $m=\frac{a\times a+(-b)\times b}{a+b}=\frac{a^2-b^2}{a+b}=\frac{(a+b)(a-b)}{a+b}=a-b$

$n=\frac{(-a)\times a+b\times b}{a+b}=\frac{b^2-a^2}{a+b}=\frac{(b+a)(b-a)}{a+b}=b-a$

17. Let the coordinates of P and Q be $(0, y)$ and $(x, 0)$ respectively.

$a=\frac{0+x}{2}$, $\therefore\ x=2a$; $b=\frac{y+0}{2}$, $\therefore\ y=2b$

$\therefore$ PQ $=\sqrt{x^2+y^2}=\sqrt{(2a)^2+(2b)^2}=\sqrt{4a^2+4b^2}=2\sqrt{a^2+b^2}$

18. Let AB = x. $\therefore$ CD = $3x$, AD = $5x$, BC = $5x - x - 3x = x$, AC = $x + x = 2x$

$\therefore$ AB : BC = 1 : 1, AC : CD = 2 : 3.

Coordinates of B $=(\frac{-2+8}{1+1},\frac{-3+7}{1+1})=(\frac{6}{2},\frac{4}{2})$ = (3, 2).

Let the coordinates of D be (x_2, y_2). $\frac{x_2\times 2+(-2)\times 3}{2+3}=8$, $2x_2=46$, $x_2=23$;

$\frac{y_2\times 2+(-3)\times 3}{2+3}=7$, $\frac{2x_2-9}{5}=7$, $2y_2=44$, $y_2=22$.

Ans. The coordinates of B and D are (3, 2) and (23, 22) respectively.

19. Coordinates of P $=(\frac{1\times 2+6\times 1}{2+1},\frac{2\times 2+(-10)\times 1}{2+1})=(\frac{8}{3},\frac{-6}{3})=(\frac{8}{3},-2)$.

Sub. the coordinates into $3x - y + k = 0$, we have $3(\frac{8}{3})-(-2)+k=0$, $8 + 2 + k = 0$,

$\therefore\ k=-10$

20. Coordinates of the mid-point $=(\frac{3a-1+(-3)}{2},\frac{4+5-a}{2})=(\frac{3a-4}{2},\frac{9-a}{2})$.

Sub. the coordinates into the equation $x - 2y + 6 = 0$, we have

$(\frac{3a-4}{2})-2(\frac{9-a}{2})+6=0$, $3a-4-2(9-a)+12=0$, $5a-10=0$, $\therefore\ a=2$

21. (a) PC = CR and PA = AQ, $\therefore$ AC//QR//BR and AC $=\frac{1}{2}$ QR = BR (mid-pt thm),

$\therefore$ ABRC is a parallelogram (opp. sides equal and //).

(b) Let the coordinates of R be (x, y). $\because$ mid-point of BC = mid-point of AR,

$\therefore\ \frac{x+(-4)}{2}=\frac{1+(-1)}{2}$, $x=4$; $\frac{y+6}{2}=\frac{7+3}{2}$, $y=4$.

Ans. The coordinates of R are (4, 4).

22. $-2\frac{3}{5}=\frac{(n-9)\times 3+(m+4)\times 2}{3+2}$, $\frac{-13}{5}=\frac{3n-27+2m+8}{5}$, $6=3n+2m$ …(i)

$-6\frac{4}{5}=\frac{(5m+1)\times 3+n\times 2}{3+2}$, $-\frac{34}{5}=\frac{15m+3+2n}{5}$, $-37=15m+2n$ …(ii)

(i)×2 – (ii)×3, $\therefore\ 12-(-111)=4m-45m$, $41m=-123$, $m=-3$

Sub $m=-3$ into (i), $6=3n+2(-3)$, $12=3n$, $n=4$.

Ans. Coordinates of A = (4–9, –3×5+1) = (–5, –14); Coordinates of B are (–3+ 4, 4) = (1, 4).

23. Consider $3x + 5y - 30 = 0$: when $x = 0$, $5y = 30$, $y = 6$; when $y = 0$, $3x = 30$, $x = 10$, coordinates of P and Q are (0, 6) and (10, 0) respectively.

$\because$ ΔOPR and ΔORQ have the same heights, $\therefore$ ratio of areas = PR : RQ = 1: 3

$\therefore$ Coordinates of R $=(\frac{0\times3+10\times1}{3+1},\frac{6\times3+0\times1}{3+1})=(\frac{10}{4},\frac{18}{4})=(2.5, 4.5)$

24. (a) Area of ΔOPB $=\frac{1}{3}\times$ area of ΔOAB, $\frac{1}{2}nr=\frac{1}{3}\times\frac{1}{2}mn,\quad\therefore\ r=\frac{m}{3}$

(b) Area of ΔOPA $=\frac{1}{3}\times$ area of ΔOAB, $\frac{1}{2}ms=\frac{1}{3}\times\frac{1}{2}mn,\quad\therefore\ s=\frac{n}{3}$

(c) BP : PQ $=(n-s):(s-0)=(n-\frac{n}{3}):(\frac{n}{3}-0)=\frac{2n}{3}:\frac{n}{3}=2:1$

(d) $r=\frac{9\times2+0\times1}{2+1}=\frac{18}{3}=6$, $m=3(6)=18$. *Ans. The coordinates of A are (18, 0).*

25. (a) Coordinates of M $=\left(\frac{a+0}{2},\frac{b+0}{2}\right)=\left(\frac{a}{2},\frac{b}{2}\right)$. Coordinates of N $=\left(\frac{c+0}{2},0\right)=\left(\frac{c}{2},0\right)$

(b) Coordinates of G $=\left(\frac{1(a)+2(\frac{c}{2})}{1+2},\frac{1(b)+2(0)}{1+2}\right)=\left(\frac{a+c}{3},\frac{b}{3}\right)$

(c) Coordinates of H $=\left(\frac{1(c)+2(\frac{a}{2})}{1+2},\frac{1(0)+2(\frac{b}{2})}{1+2}\right)=\left(\frac{a+c}{3},\frac{b}{3}\right)$

(d) Coordinates of R $=\left(\frac{a+c}{2},\frac{b+0}{2}\right)=\left(\frac{a+c}{2},\frac{b}{2}\right)$

$\therefore$ Coordinates of K $=\left(\frac{1(0)+2(\frac{a+c}{2})}{1+2},\frac{1(0)+2(\frac{b}{2})}{1+2}\right)=\left(\frac{a+c}{3},\frac{b}{3}\right)$

(e) PN, QM and OR are the medians of ΔOPQ. $\because$ Coordinates of G, H and K are the same, $\therefore$ the medians are concurrent, and their point of intersection, centroid, divides each of them in the ratio 2 : 1.

26. (a) AB = OC = c (prop. of rhombus), $\therefore$ Coordinates of B = $(a + c, b)$

(b) mid-point of AC = $(\frac{a+c}{2},\frac{b+0}{2})=(\frac{a+c}{2},\frac{b}{2})$

mid-point of OB = $(\frac{0+(a+c)}{2},\frac{0+b}{2})=(\frac{a+c}{2},\frac{b}{2})$

$\because$ mid-point of AC = mid-point of OB, $\therefore$ AC and OB bisect each other, i.e. the diagonals of a rhombus bisect each other.

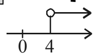
(c) Slope of OB $=\frac{b-0}{(a+c)-0}=\frac{b}{a+c}$, slope of AC $=\frac{b-c}{a-s}=\frac{b}{a-c}$

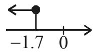
$OA^2=OC^2$, $\therefore (a-0)^2+(b-0)^2=c^2$, $\therefore\ a^2+b^2=c^2$, $a^2-c^2=-b^2$

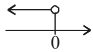
slope of OB × slope of AC $= \frac{b}{a+c} \times \frac{b}{a-c} = \frac{b^2}{a^2-c^2} = \frac{b^2}{-b^2} = -1$,

$\therefore$ OB $\perp$ AC, i.e. diagonals of a rhombus are perpendicular to each other.

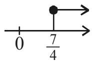
Unit 23 Inequalities

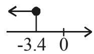
1. (a) 

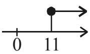
(b) 

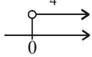
(c) 

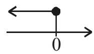
2. (a) $x > -2$ (b) $x \le 1.5$ (c) $x \ge 0$ (d) $x < -3\frac{1}{7}$

3. (a) $x \ge \frac{7}{4}$ 

(b) $x \le -3.4$ 

(c) $x \ge 11$ 

   (d) $x > 0$ 

(e) $x \le 0$ 

4. (a) $x \le 13$ (b) $y \le 7$ (c) $5x + 3 > 13$ (d) $\frac{y}{2} - 2 \le 7$

   (e) $\frac{x+10}{4} \ge 8$ (f) $\frac{2}{5}m - 1 \le 11$

5. (a) $-9x < 9$, $x > -1$ (b) $4x \ge -14$, $\therefore$ $x \ge -\frac{7}{2}$

   (c) $-8x \le 0$, $\therefore$ $x \ge 0$ (d) $7 - x < 7$, $-x < 0$, $\therefore$ $x > 0$

   (e) $6x - 9 \le 4x$, $2x \le 9$, $\therefore$ $x \le \frac{9}{2}$

   (f) $-20x + 5 > 2x - 6$, $-22x > -11$, $\therefore$ $x < \frac{1}{2}$

   (g) $12x + 8 \ge 3x + 14 - 7x$, $16x \ge 6$, $\therefore$ $x \ge \frac{3}{8}$

   (h) $5 - 4x < 8 - 6x + 3$, $2x < 6$, $\therefore$ $x < 3$

   (i) $2(5x - 7x - 14) > 12 - 3x$, $2(-2x - 14) > 12 - 3x$,
   $-4x - 28 > 12 - 3x$, $-x > 40$, $\therefore$ $x < -40$

6. (a) $3x + 4 < 54 - 12x$, $15x < 50$, $\therefore$ $x < \frac{10}{3}$

   (b) $3(3x+4) \ge 5(2-x)$, $9x + 12 \ge 10 - 5x$, $14x \ge -2$, $\therefore$ $x \ge -\frac{1}{7}$

   (c) $(5x - 1) - 9 \le 0$, $5x \le 10$, $\therefore$ $x \le 2$

   (d) $(8x + 12) - 5 > 50x$, $-42x > -7$, $\therefore$ $x < \frac{1}{6}$

   (e) $4(2x+11) + 3(6-x) \ge 12$, $8x + 44 + 18 - 3x \ge 12$, $5x \ge -50$, $\therefore$ $x \ge -10$

   (f) $-3(x-5) + 24 < 2(2x-3)$, $-3x + 15 + 24 < 4x - 6$, $-7x < -45$, $\therefore$ $x > \frac{45}{7}$

   (g) $x + 6 - 2(x-3) \le 20$, $x + 6 - 2x + 6 \le 20$, $-x \le 8$, $\therefore$ $x \ge -8$

   (h) $6(1+2x) - 2(4x+7) < 3(9-x)$, $6 + 12x - 8x - 14 < 27 - 3x$,

$4x-8<27-3x,\ \ 7x<35,\ \ \therefore\ \ x<5$

7. $2(15+x)\le 18,\ \ 15+x\le 9,\ \ x\le -6$. *Ans. The greatest value of x is –6.*

8. $\frac{y}{3}+13\le y,\ \ y+39\le 3y,\ \ -2y\le -39,\ \ y\ge 19.5$. *Ans. The least value of y is 20.*

9. Let x be the smallest integer. $x+(x+1)+(x+2)<15,\ \ 3x<12, x<4$.
   Ans. The maximum value of the smallest number is 3.

10. Let x be the larger odd number. $x+(x-2)>28,\ \ 2x>30,\ \ x>15$
    Ans. The least value of the larger odd number is 16.

11. Let h cm be David's height. $h+(h-14)\ge 280,\ \ 2h\ge 294,\ \ \therefore h\ge 147$.
    Ans. The height of David is at least 147 cm.

12. Let x be the number of hotdogs. $16x+8.4\times 5\le 150,\ \ 16x\le 108, x\le 6.75$.
    Ans. She can buy 6 hotdogs at most.

13. Let x be the number of \$2 coins. $2x+0.5(x-8)<56,\ \ 2x+0.5x-4<56,$
    $2.5x<60, x<24$. *Ans. The maximum number of \$2 coins is 23.*

14. $2(y+15)>3y,\ 2y+30>3y,\ \ 30>y,\ \ y<30$. Besides, y must be a positive number.
    Ans. y must be a positive number smaller than 30.

15. Let x be the number of incorrect answers, $3(20-x)-2x>50,$
    $60-3x-2x>50,\ \ -5x>-10,\ \ x<2$.
    Ans. The maximum number of incorrect answers is 1.

16. $2(9+a)\ge 40,\ 9+a\ge 20,\ a\ge 11$
    Minimum area = Minimum width $\times 9 = 11\times 9 = 99$ cm^2

17. Let x be the smaller number. $x>\frac{x+4}{2},\ 2x>x+4,\ \ x>4$; and x must be a multiple of 4.
    Ans. The least value of the smaller number is 8.

18. Let x be the largest number. $x+(x-3)+(x-6)\le 30,\ 3x\le 39,\ x\le 13$; and x must be a multiple of 3. *Ans. The greatest value of the largest number is 12.*

19. (a) Let x be the smaller number. $x+(x+7)<19,\ \ 2x<12,\ \ \therefore\ x<6$.
    Ans. The smaller number is smaller than 6.
    (b) $\because\ 0<x<6,\ \ \therefore\ 7<x+7<13$, and x is an integer.
    Ans. The possible values of the larger number are 8, 9, 10, 11 and 12.

20. $a<-3,\ \ \therefore\ a+b<b-3$……(i); $b<15,\ \ b-3<12$……….(ii);
    From (i) and (ii), $a+b<b-3<12,\ \ \therefore\ a+b<12$

21. (a) Let $a=\frac{1}{2}$, $a^2=(\frac{1}{2})^2=\frac{1}{4}$, $a^2<a$. $\therefore$ The statement is not correct.
    (b) If $a=-5,\ \ b=-3,\ \ a<b,$ but $a^2=(-5)^2=25,\ \ b^2=(-3)^2=9,\ \ a^2<b^2$
    $\therefore$ The statement is not correct.
    (c) If $c=2,\ \ d=-2,\ \ c<d,$ but $\frac{1}{c}=\frac{1}{2},\ \ \frac{1}{d}=-\frac{1}{2}$, $\frac{1}{c}>\frac{1}{d}$
    $\therefore$ The statement is not correct.
    (d) If $a=-4,\ \ b=3,\ \ c=-6,\ \ d=-2,\ \ a<b,$ and $c<d,$

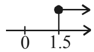
but ac = (–4) (–6) = 24, $bd = 3(-2) = -6$, $a \times c > b \times d$.

∴ The statement is not correct.

22. (a) $6x + 9 - 1 - 5x > x - 5,\ x + 8 > x - 5,\ 8 > -5,$ *Ans. x can be any real numbers.*

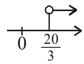
(b) $-\dfrac{3-4x}{2} > 2x - 1,\ -3 + 4x > 4x - 2, -3 > -2$ *Ans. There is no solution.*

23. (a) $0.81 - 3.24x + 6.97x \le 1.05(3x + 1.6x - 0.8),$
$0.81 + 3.73x \le 1.05(4.6x - 0.8)$



$0.81 + 3.73x \le 4.83x - 0.84,\ -1.1x \le -1.65,\ x \ge \dfrac{1.65}{1.1},$ ∴ $x \ge 1.5$

(b) $60 \times \dfrac{1}{3}[-\dfrac{9}{4} + \dfrac{8x}{5} + \dfrac{1}{4}(9x-1)] > 60 \times (\dfrac{6x-1}{5} + \dfrac{6-x}{6} + \dfrac{1}{30})$



$20(-\dfrac{9}{4} + \dfrac{8x}{5} + \dfrac{9x}{4} - \dfrac{1}{4}) > 12(6x-1) + 10(6-x) + 2,$

$-45 + 32x + 45x - 5 > 72x - 12 + 60 - 10x + 2$

$77x - 50 > 62x + 50,\quad 15x > 100,$ ∴ $x > \dfrac{20}{3}$

24. (a) Not true. The product of two negative numbers must be positive.

(b) True. ∵ a < b, ∴ a + 5 < b + 5, but b + 5 < b + 6, ∴ a + 5 < b + 6.

(c) True. $kx + h^2 < hx + k^2,\ h^2 - k^2 < hx - kx,\ (h+k)(h-k) < (h-k)x,$
∵ $h - k > 0$, ∴ $h + k < x,\ x > h + k$

25. (a) Not true. For example, when $p = -1$, $q = -2$, $(-1) + (-2) = -3 < 0$.

(b) True. ∵ $q < p$, ∴ $q - p < p - p$, i.e. $q - p < 0$.

(c) True. ∵ $8 > p > q > -6$, ∴ $8 > p$ and $q > -6$, i.e. $0 > (p - 8)$ and
$(q + 6) > 0$. Since $(p - 8)$ is negative and $(q + 6)$ is positive,
∴ their product must be negative.

26. In a triangle, the sum of the lengths of any two sides must be greater than that of the third side.
∴ $x < 6 + 3$, $x < 9$ …..(i); $6 < x + 3$, $x > 3$ …..(ii); $3 < 6 + x$, $x > -3$ …..(iii);
∴ x must be integers from 3 to 9. *Ans. The possible values of x are 4, 5, 6, 7 and 8.*

27. $6x < -y,\ \dfrac{6x}{y} > -1$ (∵ $y < 0$), ∴ $\dfrac{x}{y} > -\dfrac{1}{6}$

28. $k > 5$, i.e. $5 - k < 0$. $5y + k - ky \le 8 - 3k$, $(5 - k)y \le 8 - 4k$, ∴ $y \ge \dfrac{8-4k}{5-k}$ (∵ 5 – k < 0)

29. (a) $m = 1 - n$, and $m > -8$, ∴ $1 - n > -8$, $-n > -9$, $n < 9$

(b) $n = 2m + 4$, $\dfrac{n-4}{2} = m$, ∴ $\dfrac{n-4}{2} > -8$, $n - 4 > -16$, $n > -12$

(c) $2n = 7 - 3m$, $m = \dfrac{7-2n}{3}$, ∴ $\dfrac{7-2n}{3} > -8$, $7 - 2n > -24$, $-2n > -31$, $n < \dfrac{31}{2}$

30. (a) $24 + y = 3x$, $x = \dfrac{24+y}{3}$, ∵ $x > 0$, ∴ $\dfrac{24+y}{3} > 0$, $24 + y > 0$, $y > -24$

(b) $y = 3x - 24$, ∵ $y \le -15$, ∴ $3x - 24 \le -15$, $3x \le 9$, $x \le 3$

(c) $\because\ x>7,\quad \therefore\ \dfrac{24+y}{3}>7,\quad 24+y>21,\quad y>-3,\quad \therefore\ y$ can be 0.

31. $\dfrac{1}{x}>\dfrac{1}{y},\quad \dfrac{1}{x}-\dfrac{1}{y}>0,\quad \therefore\ \dfrac{y-x}{xy}>0.$ But $x>y$, $0>y-x$, i.e. $(y-x)$ is a negative number. Since $\dfrac{y-x}{xy}$ is positive but $(y-x)$ is negative, $\therefore$ xy must be negative, that is, x and y must be of opposite signs. But $x>y$, $\therefore\ x>0$ and $y<0$.

32. (a) Greatest value = 3(6) + 2(–2) = 18 – 4 = 14

(b) The smallest value of $x^2=0^2=0$, the smallest value of $y^2=(-2)^2=4$,
$\therefore$ the smallest value $x^2+y^2=0+4=4$.
The greatest value of $x^2=6^2=36$, the greatest value of $y^2=(-12)^2=144$,
$\therefore$ the greatest value $x^2+y^2=36+144=180.\quad \therefore\ 4\le x^2+y^2\le 180$

(c) The smallest value of $y-x=y_{smallest}-x_{biggest}=-12-6=-18$.
The greatest value of $y-x=y_{biggest}-x_{smallest}=-2-(-3)=1.\quad \therefore\ -18\le y-x\le 1$

(d) Least value $=\dfrac{6}{-2}=-3$, greatest value $=\dfrac{-3}{-2}=\dfrac{3}{2},\quad \therefore\ -3\le\dfrac{x}{y}\le\dfrac{3}{2}$

33. (a) $12-3(2x+1)\ge 4(6x-4),\quad 12-6x-3\ge 24x-16,\quad -30x\ge -25,\quad \therefore\ x\le\dfrac{5}{6}$

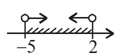
(b) Let $x=\dfrac{6y+1}{3}$. The inequality becomes: $1-\dfrac{1}{4}(2x+1)\ge\dfrac{1}{3}(6x-4)$

From (a), $x\le\dfrac{5}{6},\quad \therefore\ \dfrac{6y+1}{3}\le\dfrac{5}{6},\quad 12y+2\le 5,\quad y\le\dfrac{1}{4}$

34. (a) $4x+1>-19,\quad 4x>-20,\quad \therefore\ x>-5$

(b) $4-x<18-8x,\quad 7x<14,\quad \therefore\ x<2$

(c) $\because\ k>-5$ and $k<2$, $\therefore$ k are numbers from –5 to 2.



35. (a) Let n be the number of sides. $3n\le 45,\quad n\le 15$;
but the smallest number of sides is 3, $\therefore\ n\ge 3$.

Each interior angle $=\dfrac{(n-2)\times 180^\circ}{n}=\dfrac{180^\circ n-360^\circ}{n}=180^\circ-\dfrac{360^\circ}{n}$,

it is greatest when $n=15$, and smallest when $n=3$.

$\therefore$ The greatest angle $=180^\circ-\dfrac{360^\circ}{15}=156^\circ$,

and the smallest angle $=180^\circ-\dfrac{360^\circ}{3}=60^\circ$.

(b) Least possible size of an interior angle = 60° (equilateral triangle);
Let x be the size of an exterior angle, $180^\circ-172^\circ\le x\le 180^\circ-60^\circ,\quad \therefore\ 8^\circ\le x\le 120^\circ$

36. (a) Let x be the number of copies. For Shop A, $1500+1.2x\le 2000$,

$1.2x\le 500,\quad x\le 416\dfrac{2}{3},\quad \therefore$ its maximum number of copies is 416.

For Shop B, $900 + 1.5x \le 2000$, $1.5x \le 1100$, $x \le 733\frac{1}{3}$,

$\therefore$ its maximum number of copies is 733. *Ans. Print Shop B should be chosen.*

(b) $1500 + 1.2x < 900 + 1.5x$, $-0.3x < -600$, $\therefore$ $x > 2000$

Ans. It is cheaper to choose A when the number of copies is more than 2000.

37. (a) Let x be the no. of \$2 coins. $\therefore$ no. of \$5 coins = $\frac{140-2x}{5} = 28 - \frac{2}{5}x$.

$\because$ the no. of coins must be an integer, $\therefore$ $\frac{2}{5}x$ must be an integer,

$\therefore$ x must be a multiple of 5, i.e. the no. of \$2 coins must be a multiple of 5.

(b) $\frac{140-2x}{5} - x > 3$, $140 - 2x - 5x \ge 15$, $-7x \ge -125$, $x \le 17.9$;

$\because$ x must be a multiple of 5, $\therefore$ $x = 15$. *Ans. The maximum number of \$2 coins is 15.*