ANSWERS

Unit 1 Errors in measurement

1.

	(a)	(b)	(c)	(d)
lower limit	419.5 cm	98.5	256.75 g	6995
upper limit	420.5 cm	99.5	256.85 g	7005

2. (a) The speed = $\frac{31.225+31.235}{2}$ = 31.23 km/h

   (b) 4 sig. fig., or 2 decimal places, or 0.01 km/h

3. The weight lies between 13.95 kg and 14.05 kg.

4. The lower limit = 420 000 − $\frac{100}{2}$ = \$419 950,

   the upper limit = 420 000 + $\frac{100}{2}$ = \$420 050

5. The lower limit = (38.4 − $\frac{0.1}{2}$) ÷ 58 = 0.6612 minute,

   the upper limit = (38.4 + $\frac{0.1}{2}$) ÷ 58 = 0.6629 minute

6. 30 m: lower limit = 30 − $\frac{1}{2}$ = 29.5 m, upper limit = 30 + $\frac{1}{2}$ = 30.5 m

   16 m: lower limit = 16 − $\frac{1}{2}$ = 15.5 m, upper limit = 16 + $\frac{1}{2}$ = 16.5 m

   ∴ lower limit of the perimeter = 2 × (29.5+15.5) = 90 m,
   upper limit of the perimeter = 2 × (30.5+16.5) = 94 m

7. 12 cm: lower limit = 12 − $\frac{0.5}{2}$ = 11.75 cm, upper limit = 12 + $\frac{0.5}{2}$ = 12.25 cm

   8 cm: lower limit = 8 − $\frac{0.5}{2}$ = 7.75 cm, upper limit = 8 + $\frac{0.5}{2}$ = 8.25 cm

   ∴ lower limit of the area = 11.75 × 7.75 = 91.1 cm^2 (3 sig. fig.),
   upper limit of the area = 12.25 × 8.25 = 101 cm^2 (3 sig. fig.)

8.

	Max absolute error	Relative error
(a)	0.5 cm^3	$=\frac{0.5}{336}=\frac{1}{672}$
(b)	0.005 kg	$=\frac{0.005}{0.84}=\frac{1}{168}$
(c)	0.05° C	$=\frac{0.05}{19}=\frac{1}{380}$
(d)	\$ 5	$=\frac{5}{780}=\frac{1}{156}$

9.

	Max absolute error	% error (3 sig. fig.)
(a)	$=\frac{5}{2}=2.5$ g	$=\frac{2.5}{165}\times100\%=1.52\%$
(b)	$=\frac{2}{2}=1$ mm	$=\frac{1}{5.8}\times100\%=17.2\%$
(c)	$=\frac{0.01}{2}=0.005$ L	$=\frac{0.005}{22}\times100\%=0.0227\%$
(d)	$=\frac{10}{2}=5$ s	$=\frac{5}{960}\times100\%=0.521\%$

10. (a) The maximum absolute error = $\frac{10}{2}$ = 5 g, relative error = $\frac{5}{540}=\frac{1}{108}$,

    % error = $\frac{1}{108}$ × 100% = 0.926% (3 sig. fig.)

    (b) The maximum absolute error = $\frac{5}{2}$ = 2.5 cm^2, relative error = $\frac{2.5}{240}=\frac{1}{96}$,

$\%$ error = $\frac{1}{96} \times 100\% = 1.04\%$ (3 sig. fig.)

11. The maximum absolute error = $66° \times \frac{1}{12} = 5.5°$,

$\therefore$ the true value lies between (66 ± 5.5°), that is between 60.5° and 71.5° .

12. (a) The maximum error = 49 × 5% = 2.45 kg

(b) The range of actual weight = (49 ± 2.45) kg,
that is between 46.55 kg and 51.45 kg.

13. (a) The maximum absolute error = 0.5 m/s,

$\therefore$ the % error = $\frac{0.5}{26} \times 100\% = 1.92\%$ (3 sig. fig.)

(b) The maximum error in 1 s = 0.5 m,

$\therefore$ the max error in 20 minutes = 0.5 × 20 × 60 = 600 m

14. The height of the boy = $\frac{154.5 + 165.5}{2} = 165$ cm,

the maximum absolute error = 165.5 – 165 = 0.5 cm,

$\therefore$ the % error = $\frac{0.5}{165} \times 100\% = 0.303\%$ (3 sig. fig.)

15. (a) The maximum % error = $\frac{0.2}{13} \times 100\% = 1.54\%$ (3 sig. fig.)

(b) The maximum % error = $\frac{15}{185} \times 100\% = 8.11\%$ (3 sig. fig.)

16. Lower limit of the speed = 8.5m/s, upper limit of the speed = 9.5 m/s.

$\therefore$ The least possible distance travelled in 1 day
= 8.5 × (24 × 60 × 60) = 734 400 m =734.4 km
The greatest possible distance travelled in 1 day
= 9.5 × (24 × 60 × 60) = 820 800 m =820.8 km

17. 8.3 m lies between (8.3 ± 0.05)m; 17.4 m lies between (17.4 ± 0.05)m.

$\therefore$ The shortest length of the remaining rope = 17.35 – 8.35 = 9 m
The longest length of the remaining rope = 17.45 – 8.25 = 9.2 m

18. The area lies between (64 ± 0.5) cm^2; the base lies between (12 ± 0.5) cm.

Area = $\frac{1}{2}$ × base × height, $\therefore$ height = $\frac{2 \times \text{area}}{\text{base}}$

$\therefore$ Lower limit of the height = $\frac{2 \times 63.5}{12.5}$ = 10.16 cm

Upper limit of the height = $\frac{2 \times 64.5}{11.5}$ = 11.22 cm

19. The shortest time = $\frac{45 \times 24}{80 + 4}$ min + 45 × (9 – 1)s = 18 min 51 s (to the nearest s)

The longest time = $\frac{45 \times 24}{80 - 4}$ min + 45 × (9+1)s = 21 min 43 s (to the nearest s)

20. (a) The lower limit = 85 × (1 – 4%) = 81.6 g.

$\because$ 82 g is greater than the lower limit, $\therefore$ it will be accepted.

(b) The upper limit of the weight a packet of noodles = 85 × (1 + 4%) = 88.4 g.

$\therefore$ The no. of packets = $\frac{2\text{kg}}{88.4\text{g}} = \frac{2\,000}{88.4} \approx 22.6$

Ans. The maximum no. of packets is 22.

21. The max absolute error = $\dfrac{0.5}{2}$ = 0.25 cm.

Let the length of the measured object be x cm when the % error is 1%.

$\therefore\ \dfrac{0.25}{x}\times 100\% = 1\%$, $x = 0.25 \times 100 = 25$

If the object is shorter than x, the error will be greater than 1%.

Ans. He can measure objects with length between 25 cm and 1 m.

22. (a) 3 min 20 s = 3 × 60 + 20 = 200s; the max error = $0.5x$

$\therefore\ \dfrac{0.5x}{200}\times 100\% = 1.25\%$, $x = \dfrac{1.25\times 200}{100\times 0.5} = 5$

(b) The max. error = $\dfrac{x}{2} = \dfrac{5}{2} = 2.5$ s, $\therefore$ the range = 3 min 20 s ± 2.5 s.

$\therefore$ It lies between 3 min 17.5 s and 3 min 22.5 s.

23. (a) 6.5 cm

(b) Maximum absolute error $= \dfrac{0.5}{2} = 0.25$ cm, relative error $= \dfrac{0.25}{6.5} = \dfrac{1}{26}$

(c) Relative error $= \dfrac{\text{maximum absolute error}}{\text{measured value}} = \dfrac{0.25}{\text{measured value}}$

If the other crayon is shorter, the measured value is smaller.

The denominator of the relative error is smaller while the numerator remains unchanged, thus the relative error becomes larger.

24. (a) (i) $(8 + x)(3 + x) = 24 + 11x + x^2$

(ii) $(8 - x)(3 - x) = 24 - 11x + x^2$

(b) Upper limit of area of triangle $= \dfrac{(8+k)(3+k)}{2} = \dfrac{24+11k+k^2}{2}$

Lower limit of area of triangle $= \dfrac{(8-k)(3-k)}{2} = \dfrac{24-11k+k^2}{2}$

Difference $= \dfrac{24+11k+k^2}{2} - \dfrac{24-11k+k^2}{2} = 11k$

25. (a) 0.5 cm

(b) Pentagon = trapezium + rectangle, the required lower limit

$= \dfrac{(27.5+19.5)(16.5-13.5)}{2} + (27.5)(13.5) = 441.75\ \text{cm}^2$

Alternative method:

Pentagon = rectangle – triangle

The required lower limit = $27.5\times 16.5 - \dfrac{1}{2}(27.5-19.5)(16.5-13.5) = 441.75\ \text{cm}^2$

(c) Upper limit of the area of the paper

$= \dfrac{(28.5+20.5)(17.5-14.5)}{2} + (28.5)(14.5) = 486.75\ \text{cm}^2 > 483\ \text{cm}^2$ $\therefore$ It is possible.

26. (a) The least possible weight = $\left(500 - \dfrac{1}{2}\right)$ g = 499.5 g

(b) (i) The upper limit of the weight of the bag of flour

$= \left(22.7 + \dfrac{0.1}{2}\right)$ kg = 22.75 kg = 22 750 g.

Actual weight of the bag of flour < 22 750 g. $\therefore$ It is impossible.

(ii) The greatest number of bags $= \dfrac{22\,750}{499.5} = 45.545... < 46$. $\therefore$ It is impossible.

Unit 2 Simple factorization & grouping terms

1. (a) $= 6(3y^2 + 1)$ (b) $= 7x(x - 7)$
(c) $= 3x(3y - 4t + 5ty)$ (d) $= 4x(4x^2 - 2x + 1)$
(e) $= 12bc(3a^2 + 2ac + 5)$ (f) $= 15r^3s^2(-3r - 4s + 5s^2)$
2. (a) $= (x + y)(1 - k)$ (b) $= 10(a^2 + b^2)(3p + 5q)$
(c) $= (m + n)^3\,[8(m + n) - 1] = (m + n)^3\,(8m + 8n - 1)$
(d) $= (a + b)(c + d)(1 + c + d)$
3. (a) $= (y^2 - y) + (xy - x) = y(y-1) + x(y-1) = (y-1)(x+y)$
(b) $= (na + ny) + (a + y) = n(a + y) + (a + y) = (a + y)(n + 1)$
(c) $= (3k^3 + 21k^2) - (k + 7) = 3k^2(k + 7) - (k + 7) = (k + 7)(3k^2 - 1)$
(d) $= (mn - 2nr) - (my - 2ry) = n(m - 2r) - y(m - 2r) = (m - 2r)(n - y)$
(e) $= (2ab^2c + 6a^2b) - (5bc^2 + 15ac) = 2ab(bc + 3a) - 5c(bc + 3a)$
$= (bc + 3a)(2ab - 5c)$
(f) $= (e + f)^2 - (e + f) = (e + f)(e + f - 1)$
4. (a) $= r(x - y) + s(x - y) = (x - y)(r + s)$
(b) $= 8x^2(4a - 3b) - 4x(4a - 3b) = 4x(4a - 3b)(2x - 1)$
5. (a) $= (8ay + 4a^2) + (6xy + 3ax) = 4a(2y + a) + 3x(2y + a) = (2y + a)(4a + 3x)$
(b) $= (rs - 3r) - (5s - 15) = r(s - 3) - 5(s - 3) = (s - 3)(r - 5)$
(c) $= (p^2 - p) - (pq - q) = p(p - 1) - q(p - 1) = (p - 1)(p - q)$
(d) $= (c + b) - (ac^2 + abc) = (c + b) - ac(c + b) = (c + b)(1 - ac)$
(e) $= (pq + r) + (r^2pq + rp^2q^2) = (pq + r) + rpq(r + pq) = (r + pq)(1 + rpq)$
(f) $= (xz - 2x^2z) - (3yz^2 - 6xyz^2) = xz(1 - 2x) - 3yz^2(1 - 2x)$
$= z(1 - 2x)(x - 3yz)$
6. (a) $= (6x^2 + 9) + (2ax^3 + 3ax) + (2x^2y + 3y)$
$= 3(2x^2 + 3) + ax(2x^2 + 3) + y(2x^2 + 3) = (2x^2 + 3)(3 + ax + y)$
(b) $= (ah^2 + bhn - hp) - (ahn + bn^2 - np) = h(ah + bn - p) - n(ah + bn - p)$
$= (ah + bn - p)(h - n)$
(c) $= (x^3 - x^2y + x^2) - (3x - 3y + 3) = x^2(x - y + 1) - 3(x - y + 1)$
$= (x - y + 1)(x^2 - 3)$
(d) $= (pt^2 - p^2t + 3rp^2t) + (6rt - 6rp + 18r^2p) = pt(t - p + 3rp) + 6r(t - p + 3rp)$
$= (t - p + 3rp)(pt + 6r)$
7. (a) $-x + 1 = 1 - x$; $\therefore$ $x - 1$ is different.
(b) $-a + b = -(a - b)$; $\therefore$ $a + b$ is different.
(c) $(x - y)(a - b) = [-(y - x)][-(b - a)] = (y - x)(b - a)$; $\therefore$ $(x + y)(a + b)$ is different.
(d) $(m - n)^2 = [-(n - m)]^2 = (n - m)^2$; $\therefore$ $-(n - m)^2$ is different.
(e) $(x-y)^3 = [-(y-x)]^3 = (-1)^3(y-x)^3 = -(y-x)^3$; $\therefore$ $(y - x)^3$ is different.
8. (a) $= 2(a + b)(p - q) - 4(p - q)(x + y) = 2(p - q)(a + b - 2x - 2y)$
(b) $= 18(p - q)^3 - 12(p - q)^2 = 6(p - q)^2(3p - 3q - 2)$ (c) $= (y + 3)^2\,(5y + 1)$
(d) $= (y - x)^2(m - n)^3 + (y - x)^3(m - n)^4 = (y - x)^2(m - n)^3\,[1 + (y - x)(m - n)]$
9. (a) $= (8p + 8q) + (p + q)^2 = 8(p + q) + (p + q)^2 = (p + q)(8 + p + q)$
(b) Cannot be factorized. (c) Cannot be factorized. (d) Cannot be factorized.

(e) $= 4rs^2 - 4t^2 + st - 16rst = (4rs^2 - 16rst) + (st - 4t^2)$
$= 4rs(s-4t) + t(s-4t) = (s-4t)(4rs+t)$

(f) Cannot be factorized.

(g) $= (3n + 4mn) - (6p + 8mp) + (18m + 24m^2)$
$= n(3+4m) - 2p(3+4m) + 6m(3+4m) = (3+4m)(n-2p+6m)$

10. $= 4[-(a+b)]^2 - (a+b) = 4(a+b)^2 - (a+b) = (a+b)[4(a+b)-1] = (a+b)(4a+4b-1)$

11. $= (x-y)^3 + 9(x-y)^2 - (x-y) = (x-y)[(x-y)^2 + 9(x-y) - 1]$
$= (x-y)(x^2 - 2xy + y^2 + 9x - 9y - 1)$

12. $= \ell(s^2t + \ell s^2t + s + \ell s - st^2 - t) = \ell[(s^2t + s) + (\ell s^2t + \ell s) - (t + st^2)]$
$= \ell[s(st+1) + \ell s(st+1) - t(st+1)] = \ell(st+1)(s + \ell s - t)$

13. $= (1 + x^2 + x^4 + x^6) + x(1 + x^2 + x^4 + x^6) = (1 + x^2 + x^4 + x^6)(1+x)$
$= [(1+x^2) + x^4(1+x^2)](1+x) = (1+x^4)(1+x^2)(1+x)$
Ans. Its 3 factors are $(1+x)$, $(1+x^2)$, *and* $(1+x^4)$.

14. $= 1 + x + 2x + 2x^2 - x^2 - x^3 = (1+x) + 2x(1+x) - x^2(1+x) = (1+x)(1+2x-x^2)$

15. The area of David's rectangle $= 15(2y^3 + y^2 + 2y + 1) = 15[\, y^2(2y+1) + (2y+1)\,]$
$= 15(2y+1)(y^2+1)$ cm^2
The width of Donna's rectangle $= (30y + 15) = 15(2y+1)$ cm
∵ the two rectangles have the same area,
∴ the length of Donna's rectangle $= \dfrac{15(2y+1)(y^2+1)}{15(2y+1)} = (y^2+1)$ cm

16. (a) When $n = 1$, S $= 1^2 + 1 + 41 = 43$; When $n = 2$, S $= 2^2 + 2 + 41 = 47$;
When $n = 3$, S $= 3^2 + 3 + 41 = 53$; When $n = 4$, S $= 4^2 + 4 + 41 = 61$;
When $n = 5$, S $= 5^2 + 5 + 41 = 71$

(b) $S = n^2 + n + 41 = n(n+1) + 41$
When $n = 41$, $S = 41(42) + 41 = 41(42 + 1)$ which is a composite number.
When $n = 40$, $S = 40(41) + 41 = 41(40 + 1)$ which is also a composite number.
Ans. The two values of n are 40 and 41.

Unit 3 Algebraic identities & factorization

1. $x^2 + 6x + p \equiv x^2 + 2qx + q^2$, ∴ $2q = 6$ and $p = q^2$, $q = 3$ and $p = 3^2 = 9$

2. $x^2 + 4x + 4 - x^2 + x + 2 \equiv \text{A}x + (\text{B} - \text{A})$, $5x + 6 \equiv \text{A}x + (\text{B} - \text{A})$,
∴ A = 5 and B – A = 6; ∴ B – 5 = 6, B = 11. *Ans. A = 5 and B = 11.*

3. $x^2 - 6x + 9 + \text{A}x - 3\text{A} \equiv x^2 + \text{B}$, $x^2 + (\text{A} - 6)x + (9 - 3\text{A}) \equiv x^2 + \text{B}$, ∴ A – 6 = 0
and 9 – 3A = B. A = 6 and 9 – 3(6) = B, B = –9. *Ans. A = 6 and B = –9.*

4. $3\text{A}x^2 + (3\text{B} + \text{A})x + \text{B} \equiv 6x^2 + \text{C}x - 2$, ∴ 3A = 6, 3B + A = C and B = –2,
A = 2 and 3(–2) + 2 = C, C = –4. *Ans. A = 2, B = –2 and C = –4.*

5. L.H.S.$= [(x + 3y) + (3y - x)][(x + 3y) - (3y - x)] = (6y)(2x) = 12xy =$ R.H.S.
∴ It is an identity.

6. (a) L.H.S. $= 3x + 18 + 2x - 16 = 5x + 2$, R.H.S.$= 7x + 14 - 5x - 9 = 2x + 5 \neq$ L.H.S.
∴ It is not an identity.

(b) L.H.S. $= x^2 - 3xy$, R.H.S.$= x^2 - 3xy - 4y^2 + 4y^2 = x^2 - 3xy =$ L.H.S.
∴ It is an identity.

(c) L.H.S.$= 8x - 3y - 2y - 3z - 3x + 2z = 5x - 5y - z$,
R.H.S.$= 5x - 5y - 5z + 4z = 5x - 5y - z =$ L.H.S., ∴ It is an identity.

(d) L.H.S. $= y^2 + 6y + 9 - 10 = y^2 + 6y - 1$, R.H.S. $= y^2 - 1 \neq$ L.H.S.
$\therefore$ It is not an identity.

(e) R.H.S. $= 2x^2 - (2x^2 - 3x - 9) = 3x + 9 = 3(x + 3) =$ L.H.S., $\therefore$ It is an identity.

7. L.H.S. $= 2[36x^2 + 12x + 1 - (36x^2 + 9x - 10)] = 2(3x + 11) = 6x + 22$,
R.H.S. $= 10x - 18 + 55x + 40 - 60x + x = 6x + 22 =$ L.H.S.
$\therefore$ It is an identity.

8. (a) $= 25x^2 + 40xy + 16y^2$ (b) $= 49m^2 - 4n^2$ (c) $= 9x^2 - 48x + 64$
(d) $= 1 - 16x^2$ (e) $= 36a^2b^2 - 25$
(f) $= (4t - 9)^2 = 16t^2 - 72t + 81$ (g) $= 16p^4 - 56p^2 + 49$

9. (a) $= (10 + 2x)^2 = 100 + 40x + 4x^2$ (b) $= 4x^2 - 2(2x)(\frac{1}{x}) + \frac{1}{x^2} = 4x^2 - 4 + \frac{1}{x^2}$
(c) $= 2(36x^2 + 60xy + 25y^2) = 72x^2 + 120xy + 50y^2$
(d) $= (\frac{2}{m})^2 + 2(\frac{2}{m})(\frac{3}{n}) + (\frac{3}{n})^2 = \frac{4}{m^2} + \frac{12}{mn} + \frac{9}{n^2}$
(e) $= (7a + \frac{3}{a})(7a - \frac{3}{a}) = 49a^2 - \frac{9}{a^2}$
(f) $= 3(16 - y^4) = 48 - 3y^4$ (g) $= (-6x)^2 - (13y)^2 = 36x^2 - 169y^2$

10. (a) $= (3 - 5y)^2$ (b) $= (2n - 7)^2$ (c) $= (x + 8)^2$
(d) $= (3y + 1)(3y - 1)$ (e) $= (5m + 2n)^2$ (f) $= (7xy + 2)^2$
(g) $= (9a - 10b)(9a + 10b)$ (h) $= 121b^2 - 16a^2 = (4a + 11b)(11b - 4a)$

11. (a) $= 3(x^2 - 25) = 3(x - 5)(x + 5)$ (b) $= 5(y^2 - 6y + 9) = 5(y - 3)^2$
(c) $= (ab + 6pq)^2$ (d) $= 1 - (4y^2)^2 = (1+4y^2)(1-4y^2) = (1+4y^2)(1+2y)(1-2y)$
(e) $= 3(x^4 - 9y^4) = 3(x^2 - 3y^2)(x^2 + 3y^2)$ (f) $= 9(16 - 9k^2) = 9(4 - 3k)(4 + 3k)$
(g) $= 4(4m^2 - 9) = 4(2m - 3)(2m + 3)$
(h) $= [3y - (5y + x)][3y + (5y + x)] = -(2y + x)(8y + x)$
(i) $= p(p^2 - 10pq + 25q^2) = p(p - 5q)^2$
(j) $= (x^4 - 1)(x^4 + 1) = (x^2 - 1)(x^2 + 1)(x^4 + 1) = (x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$
(k) $= [(x - 2y) + 7]^2 = (x - 2y + 7)^2$
(l) $= [2(a - b)]^2 - [5(a + b)]^2 = [(2a - 2b) - (5a + 5b)][(2a - 2b) + (5a + 5b)]$
$= -(3a + 7b)(7a + 3b)$

12. (a) not possible $(not: 4x^2 + 4x + 1)$ (b) not possible $(not: y^2 + 14y + 49)$
(c) $(1 - 3a)^2$ (d) not possible $(not: n^2 - 12n + 36)$
(e) not possible $(not: x^2 - 9y^2)$ (f) $= 5(4k^2 - 1) = 5(2k + 1)(2k - 1)$
(g) not possible $(not: 16y^2 + 8y + 1)$ (h) $= m^2 + 6m + 9 = (m + 3)^2$
(i) $= (\frac{a}{3})^2 - 2(\frac{a}{3})(\frac{b}{2}) + (\frac{b}{2})^2 = (\frac{a}{3} - \frac{b}{2})^2$ (j) $= (x)^2 + 2(x)(\frac{1}{x}) + (\frac{1}{x})^2 = (x + \frac{1}{x})^2$

13. (a) $= [(a + b) + c]^2 = (a + b)^2 + 2c(a + b) + c^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
(b) $= [(x + 3) - 4y][(x + 3) + 4y] = (x + 3)^2 - (4y)^2 = x^2 + 6x + 9 - 16y^2$
(c) $= [(2x - y) + 6]^2 = (2x - y)^2 + 12(2x - y) + 36$
$= 4x^2 + y^2 - 4xy + 24x - 12y + 36$
(d) $= [p + (3q - r)][p - (3q - r)] = p^2 - (3q - r)^2$
$= p^2 - (9q^2 - 6qr + r^2) = p^2 - 9q^2 + 6qr - r^2$

14. $= (\frac{m-1}{m} + \frac{m+1}{m})(\frac{m-1}{m} - \frac{m+1}{m}) = (\frac{2m}{m})(\frac{-2}{m}) = \frac{-4}{m}$

15. $= (10001 + 9999)(10001 - 9999) = 20000 \times 2 = 40000$

16. $= (x^2 + 2x + 1) + (y + xy) = (x + 1)^2 + y(x + 1) = (x + 1)(x + y + 1)$

17. $= (m^2 + 2mn + n^2) + (m^2n + mn^2) = (m+n)^2 + mn(m+n) = (m+n)(m+n+mn)$

18. $= y^2 - 4 + \dfrac{4}{y^2} = y^2 - 2(y)(\dfrac{2}{y}) + (\dfrac{2}{y})^2 = (y - \dfrac{2}{y})^2$

19. $= mn + 4 - 2m - 2n = (mn - 2m) - (2n - 4) = m(n-2) - 2(n-2) = (n-2)(m-2)$

20. $= (s^2 - 4rs + 4r^2) - \dfrac{s^2}{4} = (s-2r)^2 - (\dfrac{s}{2})^2$

$= [(s-2r) + \dfrac{s}{2})]\,[(s-2r) - \dfrac{s}{2}] = (\dfrac{3s}{2} - 2r)(\dfrac{s}{2} - 2r)$

21. $= y(x^2 - y^2 - 2x + 1) = y[(x^2 - 2x + 1) - y^2] = y[(x-1)^2 - y^2] = y(x-1+y)(x-1-y)$

22. $= (2x+1+x^2)(2x+1-x^2) = (x+1)^2(2x+1-x^2)$

23. $= (x^2+9+6x)(x^2+9-6x) = (x+3)^2(x-3)^2$

24. $= x^2 - y^2 - 6x + 6y = (x+y)(x-y) - 6(x-y) = (x-y)(x+y-6)$

25. $= x^2 + y^2 + 2xy + 2xy - 1 - x^2y^2 = (x^2 + 2xy + y^2) - (1 - 2xy + x^2y^2) = (x+y)^2 - (1-xy)^2$
$= [(x+y) + (1-xy)][(x+y) - (1-xy)] = (x+y+1-xy)(x+y-1+xy)$

26. $(a + \dfrac{1}{a})^2 = 2^2$, $\quad a^2 + 2a(\dfrac{1}{a}) + \dfrac{1}{a^2} = 4$, $\quad a^2 + 2 + \dfrac{1}{a^2} = 4$, $\quad a^2 + \dfrac{1}{a^2} = 2$

27. $= h^2 - k^2 - h + k = (h^2 - k^2) - (h-k) = (h-k)(h+k) - (h-k) = (h-k)(h+k-1)$

28. $= [(a^2 + 6a) + 9]^2 = [(a+3)^2]^2 = (a+3)^4$

29. (a) $(p^2 + 8q^2)^2$ (b) $= (p^4 + 16p^2q^2 + 64q^4) - 16p^2q^2$
$= (p^2 + 8q^2)^2 - (4pq)^2 = (p^2 + 8q^2 - 4pq)(p^2 + 8q^2 + 4pq)$

30. (a) $(a^2 + b^2)^2$ (b) $= (a^4 + 2a^2b^2 + b^4) - a^2b^2 = (a^2+b^2)^2 - (ab)^2$
$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

31. (a) $= \dfrac{1}{3}(x^2 + 6x + 9) = \dfrac{1}{3}(x+3)^2$

(b) $= \dfrac{1}{3}(p+q)^2 + 2p + 2q + 3 = \dfrac{1}{3}(p+q)^2 + 2(p+q) + 3$

Put $p + q = x$ in part (a), the result $= \dfrac{1}{3}[(p+q)+3]^2 = \dfrac{1}{3}(p+q+3)^2$

32. (a) $= [(\dfrac{3}{x} + \dfrac{x}{3}) + (\dfrac{3}{x} - \dfrac{x}{3})][(\dfrac{3}{x} + \dfrac{x}{3}) - (\dfrac{3}{x} - \dfrac{x}{3})] = (\dfrac{6}{x})(\dfrac{2x}{3}) = 4$

(b) From (a), $(\dfrac{3}{x} + \dfrac{x}{3})^2 - (\dfrac{3}{x} - \dfrac{x}{3})^2 = 4$,

$\therefore\ (\dfrac{3}{x} + \dfrac{x}{3})^2 - (\sqrt{5})^2 = 4$, $\quad (\dfrac{3}{x} + \dfrac{x}{3})^2 = 9$, $\quad \dfrac{3}{x} + \dfrac{x}{3} = 3$ or -3

33. (a) R.H.S. $= (x+y)(x^2 - xy + y^2) = x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3 =$ L.H.S.
$\therefore$ It is an identity.

(b) R.H.S. $= (x-y)(x^2 + xy + y^2) = x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 = x^3 - y^3 =$ L.H.S.
$\therefore$ It is an identity.

(c) $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x^3 + y^3)(x-y)(x^2 + xy + y^2)$ [from (b)]
$= (x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$ [from (a)]

Ans. Its factors are $(x+y)$, $(x-y)$, $(x^2 - xy + y^2)$ *and* $(x^2 + xy + y^2)$.

34. Dividend $= 16x^2 + kx + 4$, divisor $= 2x + 5$, quotient $= mx + n$, remainder $= -1$.
$\because$ Dividend $\equiv$ divisor $\times$ quotient + remainder
$\therefore\ 16x^2 + kx + 4 \equiv (2x+5)(mx+n) - 1$, $\quad 16x^2 + kx + 4 \equiv 2mx^2 + 2nx + 5mx + 5n - 1$

Comparing the coefficients, we get:
$16 = 2m$, $4 = 5n - 1$ and $k = 2n + 5m$,
$\therefore\ m = \frac{16}{2} = 8$, $n = \frac{4+1}{5} = 1$, $k = 2(1) + 5(8) = 42$.
Ans. $m = 8$, $n = 1$, $k = 42$.

35. (a) $x^2 - 33 = 16y^2$, $x^2 - 16y^2 = 33$, $(x+4y)(x-4y) = 33$;
but $1 + 4y = x$, $1 = x - 4y$; $\therefore\ (x+4y)(1) = 33$, $x + 4y = 33$
(b) $x = 33 - 4y$ ….. (i), and $x = 1 + 4y$ ….. (ii).
From (i) and (ii), $1 + 4y = 33 - 4y$, $8y = 32, y = 4$
$\therefore\ x = 4(4) + 1 = 17$ *Ans.* $x = 17$, $y = 4$.

36. $x^2 - 2x - y^2 + 4y - 3 = (x^2 - 2x + 1) - (y^2 - 4y + 4) = (x-1)^2 - (y-2)^2$
$= [(x-1) + (y-2)]\,[(x-1) - (y-2)] = (x + y - 3)(x - y + 1)$

37. $= x^4 + 18^2 - 100x^2 = [x^4 - 2(18)x^2 + 18^2] + 2(18)x^2 - 100x^2$
$= (x^2 - 18)^2 - 64x^2 = (x^2 - 18)^2 - (8x)^2 = (x^2 - 18 + 8x)(x^2 - 18 - 8x)$

38. (a) (i) $(x - A)^2 = x^2 - 2Ax + A^2$
(ii) $(4 + 5x)(5x - 4) = (5x + 4)(5x - 4) = (5x)^2 - 4^2 = 25x^2 - 16$
(b) L.H.S. $= x^2 - 2Ax + A^2 + Bx(3 - x)$
$= x^2 - 2Ax + A^2 + 3Bx - Bx^2 = (1 - B)x^2 + (3B - 2A)x + A^2$
R.H.S. $= 5C - (4 + 5x)(5x - 4)$
$= 5C - (25x^2 - 16) = 5C - 25x^2 + 16 = -25x^2 + (5C + 16)$
By comparing coefficients,
$1 - B = -25$ ..... (1), $3B - 2A = 0$ ..... (2), $5C + 16 = A^2$ ..... (3)
From (1), $B = 26$;
From (2), $3(26) - 2A = 0$, $A = 39$;
From (3), $5C + 16 = 39^2$, $C = 301$

39. (a) (i) $= (4x)^2 - 2(4x)(1) + 1^2 = (4x - 1)^2$
(ii) $= (4x - 1)^2 - (5y)^2 = (4x - 1 - 5y)(4x - 1 + 5y)$
(b) $= 16x^2 - 8x - 4x - 25y^2 - 5y + 1 + 1$
$= (16x^2 - 8x + 1 - 25y^2) - 4x + 1 - 5y$
$= (4x - 1 - 5y)(4x - 1 + 5y) - (4x - 1 + 5y) = (4x - 1 + 5y)[(4x - 1 - 5y) - 1]$
$= (4x - 1 + 5y)(4x - 2 - 5y)$

40. (a) L.H.S. $= k^2 + x(x + 3k) = k^2 + x^2 + 3kx$
R.H.S. $= (x + k)^2 + kx = x^2 + 2kx + k^2 + kx = k^2 + x^2 + 3kx =$ L.H.S.
$\therefore$ It is an identity.
(b) (i) $x = y - 2k$,
$\therefore\ x + k = y - 2k + k = y - k$, $(x + k)^2 = (y - k)^2$
(ii) L.H.S. of the identity in (a)
$= k^2 + x(x + 3k)$
$= k^2 + (y - 2k)[\,(y - 2k) + 3k]$
$= k^2 + (y - 2k)\,(y + k)$

R.H.S. of the idenity in (a)
$= (x + k)^2 + kx$
$= (y - k)^2 + k(y - 2k)$ [From (b(i))]

$\therefore$ From(a), $k^2 + (y - 2k)(y + k) \equiv (y - k)^2 + k(y - 2k)$.
$\therefore$ The claim is agreed.

41. (a) Area of P $= (9a^2 + 42a + 49) = [(3a)^2 + 2(3a)(7) + 7^2] = (3a + 7)^2$
$\therefore$ The length of each side of P is $(3a + 7)$.
(b) Area of Q $= [(9a^2 + 42a + 49) - (42a + 74)]$
$= 9a^2 - 25$
$= (3a - 5)\,(3a + 5)$
(c) $\because$ Area of Q = (width)(length), $\therefore$ width of Q $= 3a + 5$.

Unit 4 Algebraic fractions

1. (a) $\dfrac{3}{5y^4}$ (b) $\dfrac{3x^4}{2}$ (c) $\dfrac{3b^2}{7}$ (d) $\dfrac{3(x+2y)}{x-2y}$

   (e) $\dfrac{5x^2(z-4)}{z}$ (f) $\dfrac{4}{3(2a+b)}$

2. (a) $=\dfrac{9x^2(6x-1)}{9x}=x(6x-1)$ (b) $=\dfrac{2(3m-n)}{10x(3m-n)}=\dfrac{1}{5x}$

   (c) $=\dfrac{a-b}{(a-b)(a+b)}=\dfrac{1}{a+b}$

   (d) $=\dfrac{4(x^2-2x+1)}{x(x-a)-(x-a)}=\dfrac{4(x-1)^2}{(x-a)(x-1)}=\dfrac{4(x-1)}{x-a}$

   (e) $=\dfrac{3x(x^2-a)+(x^2-a)}{7x(3x+1)}=\dfrac{(3x+1)(x^2-a)}{7x(3x+1)}=\dfrac{x^2-a}{7x}$

3. (a) $=\dfrac{2(3-2a)}{-3(3-2a)}=-\dfrac{2}{3}$ (b) $=\dfrac{8(1-k)}{-k^2(1-k)}=-\dfrac{8}{k^2}$

   (c) $=\dfrac{-y(y^2+5)}{y^2+5}=-y$ (d) $=\dfrac{9(x-y)^2}{12(x-y)}=\dfrac{3}{4}(x-y)$

   (e) $=\dfrac{-3x^2(5y-2x)}{7(5y-2x)}=-\dfrac{3x^2}{7}$

4. (a) $\dfrac{5}{6}$ (b) $=\dfrac{12m}{7}\times\dfrac{21}{4m}=9$ (c) $=\dfrac{x^4}{3(x-2)}\times\dfrac{3}{x^3}=\dfrac{x}{x-2}$

   (d) $=9ab^3\times\dfrac{6}{a^2b^2}=\dfrac{54b}{a}$ (e) $=\dfrac{1}{5}ab^2\times\dfrac{b^2}{5a}\times a^2=\dfrac{a^2b^4}{25}$

   (f) $=n^2x^3\times\dfrac{3}{2nx^2}\times\dfrac{3}{2n}=\dfrac{9x}{4}$ (g) $=\dfrac{3(b+2)}{6(b-2)}\times\dfrac{4(b-2)}{5(2+b)}=\dfrac{2}{5}$

   (h) $=\dfrac{m(1-a)}{b(x-1)}\div\dfrac{c(1-a)}{n(1-x)}=\dfrac{m(1-a)}{b(x-1)}\times\dfrac{-n(x-1)}{c(1-a)}=-\dfrac{mn}{bc}$

5. (a) $=\dfrac{4(b+1)}{a-3}\times\dfrac{18(a-3)}{4(a-1)}\times\dfrac{1}{6(b+1)}=\dfrac{3}{a-1}$

   (b) $=\dfrac{(x-y)(x+y)}{x^2+y^2}\times\dfrac{1}{(x+y)^2}\times[(x^2)^2-(y^2)^2]$

   $=\dfrac{(x-y)}{(x+y)(x^2+y^2)}\times(x^2+y^2)(x^2-y^2)=\dfrac{(x-y)(x+y)(x-y)}{x+y}=(x-y)^2$

   (c) $=\dfrac{b(a+b)}{a(2a-b)}\times\dfrac{5(2a-b)}{b^2(3a+1)}\times\dfrac{1}{a+b}=\dfrac{5}{ab(3a+1)}$

   (d) $=\dfrac{x(y+1)-(y+1)}{x^2(x-1)-(x-1)}\times\dfrac{-12(x-1)}{8(y+1)}=\dfrac{(x-1)(y+1)}{(x^2-1)(x-1)}\times\dfrac{-3(x-1)}{2(y+1)}=\dfrac{-3(x-1)}{2(x-1)(x+1)}=\dfrac{-3}{2(x+1)}$

6. (a) $=\dfrac{10y-9y}{12}=\dfrac{y}{12}$ (b) $=\dfrac{12r-(2r+3)}{9}=\dfrac{10r-3}{9}$

   (c) $=\dfrac{5(a-3)+2(a+2)}{30}=\dfrac{5a-15+2a+4}{30}=\dfrac{7a-11}{30}$

(d) $= \frac{y-7+5}{5} = \frac{y-2}{5}$ (e) $= \frac{4x-(x+2)}{4} = \frac{4x-x-2}{4} = \frac{3x-2}{4}$

7. (a) $\frac{1+5x}{xy}$ (b) $= \frac{3-(4+b)}{3a} = \frac{3-4-b}{3a} = -\frac{b+1}{3a}$

(c) $= \frac{3(2x+a)-2(a-3x)-(x+a)}{6x} = \frac{6x+3a-2a+6x-x-a}{6x} = \frac{11x}{6x} = \frac{11}{6}$

(d) $\frac{15m-2}{5m}$ (e) $= \frac{8x-(1-6x)}{4x} = \frac{8x-1+6x}{4x} = \frac{14x-1}{4x}$

8. (a) $\frac{12-y}{3(y+5)}$ (b) $= \frac{14-12}{21(x-y)} = \frac{2}{21(x-y)}$

(c) $= \frac{b}{a-b} + \frac{a}{a-b} = \frac{b+a}{a-b}$ (d) $= \frac{8-x}{6-x} - \frac{2}{6-x} = \frac{8-x-2}{6-x} = \frac{6-x}{6-x} = 1$

(e) $= \frac{10y+9y-6y}{12(a-b)} = \frac{13y}{12(a-b)}$

9. (a) $= \frac{x(x-1)-2x^2}{x-1} = \frac{x^2-x-2x^2}{x-1} = \frac{x(1+x)}{1-x}$ (b) $= \frac{m+n+m-n}{m+n} = \frac{2m}{m+n}$

(c) $= \frac{3y(x+y)}{3xy} - \frac{x(x-y)}{3xy} = \frac{3xy+3y^2-x^2+xy}{3xy} = \frac{4xy+3y^2-x^2}{3xy}$

(d) $= \frac{2(k+2)-3(k-3)}{(k-3)(k+2)} = \frac{2k+4-3k+9}{(k-3)(k+2)} = \frac{13-k}{(k-3)(k+2)}$

(e) $= \frac{y(2y-x)-x(2x-y)}{(2x-y)(2y-x)} = \frac{2y^2-xy-2x^2+xy}{(2x-y)(2y-x)} = \frac{2(y+x)(y-x)}{(2x-y)(2y-x)}$

10. (a) $= \frac{x-4+3x}{x(x-1)(x-4)} = \frac{4(x-1)}{x(x-1)(x-4)} = \frac{4}{x(x-4)}$

(b) $= \frac{y-2(y+3)}{(y+3)^2} = \frac{y-2y-6}{(y+3)^2} = -\frac{y+6}{(y+3)^2}$

(c) $= \frac{4(1-x)-2(x-2)}{(3x-4)(x-2)(1-x)} = \frac{4-4x-2x+4}{(3x-4)(x-2)(1-x)} = \frac{8-6x}{(3x-4)(x-2)(1-x)}$

$= \frac{-2(3x-4)}{(3x-4)(x-2)(1-x)} = \frac{2}{(x-2)(x-1)}$

(d) $= \frac{5(a+2)-4(a+1)}{(a+1)(a+6)(a+2)} = \frac{5a+10-4a-4}{(a+1)(a+6)(a+2)}$

$= \frac{a+6}{(a+1)(a+6)(a+2)} = \frac{1}{(a+1)(a+2)}$

(e) $= \frac{2(1+k)+2(1-k)-2}{(1+k)(1-k)} = \frac{2+2k+2-2k-2}{(1+k)(1-k)} = \frac{2}{(1+k)(1-k)}$

(f) $= \frac{4(x-2)+3(x+2)-7x}{(x+2)(x-2)} = \frac{4x-8+3x+6-7x}{(x+2)(x-2)} = \frac{-2}{(x+2)(x-2)}$

11. (a) $= \frac{5}{2(a-4b)} + \frac{a-b}{b(a-4b)} = \frac{5b+2(a-b)}{2b(a-4b)} = \frac{5b+2a-2b}{2b(a-4b)} = \frac{3b+2a}{2b(a-4b)}$

(b) $= \frac{2}{(x+3)(x-4)} - \frac{1-x}{(4-x)(4+x)} = \frac{2(x+4)+(1-x)(x+3)}{(x+3)(x-4)(x+4)}$

$= \frac{2x+8+3-2x-x^2}{(x+3)(x-4)(x+4)} = \frac{11-x^2}{(x+3)(x-4)(x+4)}$

(c) $= \dfrac{2}{x(x+2)} + \dfrac{4}{(x+2)(x+6)} = \dfrac{2(x+6)+4x}{x(x+2)(x+6)} = \dfrac{2x+12+4x}{x(x+2)(x+6)}$

$= \dfrac{6(x+2)}{x(x+2)(x+6)} = \dfrac{6}{x(x+6)}$

(d) $= \dfrac{a+b}{b(b-a)} - \dfrac{a+b}{a(b-a)} = \dfrac{a+b}{b-a}(\dfrac{1}{b} - \dfrac{1}{a}) = \dfrac{a+b}{b-a}(\dfrac{a-b}{ab}) = -\dfrac{a+b}{ab}$

12. $\dfrac{x^2(x-y)+y^2(x-y)}{(x^2)^2-(y^2)^2} = \dfrac{(x^2+y^2)(x-y)}{(x^2+y^2)(x^2-y^2)} = \dfrac{x-y}{(x+y)(x-y)} = \dfrac{1}{x+y}$

13. $\dfrac{2a-b}{(3a+b)(x-y)} + \dfrac{3-x}{4x(x-y)} = \dfrac{4x(2a-b)}{4x(3a+b)(x-y)} + \dfrac{(3a+b)(3-x)}{4x(x-y)(3a+b)}$

$= \dfrac{8ax-4bx+9a-3ax+3b-bx}{4x(3a+b)(x-y)} = \dfrac{5ax-5bx+9a+3b}{4x(3a+b)(x-y)}$

14. $\dfrac{m-3n}{(3m-5n)^2} - \dfrac{m+n}{(3m-5n)(4+n)} = \dfrac{(4+n)(m-3n)}{(4+n)(3m-5n)^2} - \dfrac{(3m-5n)(m+n)}{(3m-5n)^2(4+n)}$

$= \dfrac{4m-12n+mn-3n^2-(3m^2-2mn-5n^2)}{(4+n)(3m-5n)^2} = \dfrac{4m-12n+3mn+2n^2-3m^2}{(4+n)(3m-5n)^2}$

15. $8ab^2+12+8b+16ab+4b^2+24a = 4(2ab^2+3+2b+4ab+b^2+6a)$

$= 4[(3+6a)+(2b+4ab)+(b^2+2ab^2)] = 4(1+2a)(3+2b+b^2)$

$\therefore$ The given expression $= \dfrac{5(3+2b+b^2)}{4(1+2a)(3+2b+b^2)} - \dfrac{7a-4}{3(2a-1)(2a+1)}$

$= \dfrac{5}{4(1+2a)} - \dfrac{7a-4}{3(2a-1)(2a+1)} = \dfrac{5\cdot 3(2a-1)-4(7a-4)}{12(1+2a)(2a-1)}$

$= \dfrac{30a-15-28a+16}{12(1+2a)(2a-1)} = \dfrac{2a+1}{12(2a+1)(2a-1)} = \dfrac{1}{12(2a-1)}$

16. $= (\dfrac{p^2-q^2}{pq}) \div (\dfrac{q-p}{pq}) = \dfrac{(p+q)(p-q)}{pq} \times \dfrac{pq}{q-p} = -(p+q)$

17. $= \dfrac{2ab+a^2+b^2}{ab} \div \dfrac{a^2-b^2}{ab} = \dfrac{(a+b)^2}{ab} \cdot \dfrac{ab}{(a+b)(a-b)} = \dfrac{a+b}{a-b}$

18. $= 1 \div \left(1 + \dfrac{(2y)\cdot y}{(\frac{1}{y}-y)\cdot y}\right) = 1 \div \left(1 + \dfrac{2y^2}{1-y^2}\right) = 1 \div \left(\dfrac{1-y^2+2y^2}{1-y^2}\right) = 1 \div \dfrac{1+y^2}{1-y^2} = \dfrac{1-y^2}{1+y^2}$

19. (a) The other adjacent side = area × 2 ÷ side = $2(4x^2 - 20x + 25) \div (2x - 5)$

$= 2(2x-5)^2 \div (2x-5) = 2(2x-5)$ cm

(b) $2(2x-5) > (2x-5)$, $\therefore$ $2(2x-5) = 18$, $2x-5 = 9$, $\therefore$ $x = 7$

$\therefore$ Its area $= 4(7)^2 - 20(7) + 25 = 81$ cm^2

Unit 5 Use of formulae

1. (a) $A = lw$ (b) $A = 6x^2$ (c) $T = \dfrac{D}{S}$ (d) $C = \dfrac{a}{12} \times m + b \times n$, $\therefore$ $C = \dfrac{am}{12} + bn$

2. (a) $a = \dfrac{180(12-2)}{12} = \dfrac{1800}{12} = 150$ (b) $99 = \dfrac{1}{2}(11)(4+b)$, $18 = 4+b$, $\therefore$ $b = 14$

(c) $D = (-6)^2 - 4(3)(-2) = 36 + 24 = 60$ (d) $\dfrac{1}{6} = \dfrac{1}{4} - \dfrac{1}{v}$, $\dfrac{1}{v} = \dfrac{1}{4} - \dfrac{1}{6} = \dfrac{1}{12}$, $\therefore$ $v = 12$

(e) $7600 = 4000(1+0.15t)$, $1.9 = 1+0.15t$, $0.15t = 0.9$, $\therefore\ t = 6$

3. (a) $s = (10)(3) + \frac{1}{2}(8)(3)^2 = 30 + 36 = 66$

(b) $84 = (6)(4) + \frac{1}{2}a(4)^2$, $60 = 8a$, $\therefore\ a = 7.5$

4. (a) $p - b = 2ax$, $\therefore\ x = \frac{p-b}{2a}$

(b) $ax + bx = c$, $x(a+b) = c$, $\therefore\ x = \frac{c}{a+b}$

(c) $\frac{x}{a} = c - b$, $\therefore\ x = a(c-b)$

(d) $m - x = n^2$, $\therefore\ x = m - n^2$

(e) $2rx = q - k$, $\therefore\ x = \frac{q-k}{2r}$

(f) $2x = 5b - e$, $\therefore\ x = \frac{5b-e}{2}$

(g) $8m = h - 4x$, $4x = h - 8m$, $\therefore\ x = \frac{h-8m}{4}$

(h) $3y + 3 = x + u$, $\therefore\ x = 3y + 3 - u$

(i) $\frac{f}{x} = g - 1$, $\therefore\ x = \frac{f}{g-1}$

(j) $ax + b = ct$, $ax = ct - b$, $x = \frac{ct-b}{a}$

5. (a) $m + n = x - ax$, $m + n = x(1-a)$, $\therefore\ x = \frac{m+n}{1-a}$

(b) $ax + 2a = bx - 3b$, $2a + 3b = bx - ax$, $2a + 3b = x(b-a)$, $\therefore\ x = \frac{2a+3b}{b-a}$

(c) $nx + mx = mns$, $x(n+m) = mns$, $\therefore\ x = \frac{mns}{n+m}$

(d) $xy + a = b$, $xy = b - a$, $\therefore\ x = \frac{b-a}{y}$

(e) $sh + sx = kr - rx$, $sx + rx = kr - sh$, $x(s+r) = kr - sh$, $\therefore\ x = \frac{kr-sh}{s+r}$

(f) $x^2 - cx = x^2 - ax + bx - ab$, $ab = cx - ax + bx$, $ab = x(c - a + b)$,
$\therefore\ x = \frac{ab}{c-a+b}$

(g) $mqx - 3pq = 4pnx$, $mqx - 4pnx = 3pq$, $x(mq - 4pn) = 3pq$,
$\therefore\ x = \frac{3pq}{mq-4pn}$

(h) $\frac{6nx+3m}{2k} = -x$, $6nx + 3m = -2kx$, $6nx + 2kx = -3m$, $x(6n+2k) = -3m$,
$\therefore\ x = \frac{-3m}{6n+2k}$

6. (a) $\frac{s}{180} = n - 2$, $\therefore\ n = \frac{s}{180} + 2$

(b) $2A = bh$, $\therefore\ b = \frac{2A}{h}$

(c) $y - c = mx$, $\therefore\ m = \frac{y-c}{x}$

(d) $A - 2\pi r^2 = 2\pi rh$, $\therefore\ h = \frac{A-2\pi r^2}{2\pi r}$

(e) $3v = \pi r^2 h$, $\therefore\ h = \frac{3v}{\pi r^2}$

(f) $x - a = (n-1)d$, $\frac{x-a}{d} = n - 1$, $\therefore\ n = \frac{x-a}{d} + 1$

7. (a) $b^2 = c^2 - a^2$, $\therefore\ b = \sqrt{c^2 - a^2}$ (b) $\frac{2T}{n} = a + b$, $\therefore\ a = \frac{2T}{n} - b$

(c) $s - ut = \frac{1}{2}at^2$, $\therefore\ a = \frac{2(s - ut)}{t^2}$ (d) $\frac{v + u}{uv} = \frac{1}{f}$, $\therefore f = \frac{uv}{u + v}$

(e) $\frac{1}{u} - \frac{1}{f} = \frac{1}{v}$, $\frac{f - u}{uf} = \frac{1}{v}$, $\therefore v = \frac{uf}{f - u}$

(f) $bx + ay = ab$, $ay = ab - bx$, $ay = b(a - x)$, $\therefore\ b = \frac{ay}{a - x}$

(g) $s(R - 1) = a(R^n - 1)$, $\therefore\ a = \frac{s(R - 1)}{R^n - 1}$

(h) $aH = bH - bh$, $bh = bH - aH$, $bh = H(b - a)$, $\therefore\ H = \frac{bh}{b - a}$

(i) $kx - ky = mn + p - y$, $y - ky = mn + p - kx$, $y(1 - k) = mn + p - kx$,

$\therefore\ y = \frac{mn + p - kx}{1 - k}$

(j) $40as - 40bs = abc$, $40as = abc + 40bs$, $40as = b(ac + 40s)$,

$\therefore\ b = \frac{40as}{ac + 40s}$

(k) $2s = 2an + n(n - 1)d$, $2s - 2an = n(n - 1)d$, $\therefore\ d = \frac{2s - 2an}{n(n - 1)}$

8. (a) $C = \frac{5}{9}(F - 32)$, $\frac{9C}{5} = F - 32$, $\therefore\ F = \frac{9C}{5} + 32$

(b) Water boils at 100°C. When $C = 100$, $F = \frac{9(100)}{5} + 32 = 212$.

Ans. Water boils at 100°C and 212°F.

(c) Water freezes at 0°C. When $C = 0$, $F = \frac{9(0)}{5} + 32 = 32$.

Ans. Water freezes at 0°C and 32°F.

(d) When $F = 99.5$, $C = \frac{5}{9}(99.5 - 32) = 37.5$.

Ans. The normal body temperature is 37.5°C.

(e) When $C = 40.5$, $F = \frac{9(40.5)}{5} + 32 = 104.9$.

Ans. His body temperature is 104.9°F.

9. When $n = 1$, $S = 9$. When $n = 2$, $S = 13 = 9 + 4(1)$.
When $n = 3$, $S = 17 = 9 + 4(2)$. $\therefore\ S = 9 + 4(n - 1) = 4n + 5$

10. When $w = 0$, $l = 15$. When $w = 10$, $l = 18 = 15 + \frac{3}{10}(10)$.

When $w = 20$, $l = 21 = 15 + \frac{3}{10}(20)$. When $w = 30$, $l = 24 = 15 + \frac{3}{10}(30)$.

$\therefore\ l = 15 + \frac{3}{10}w$

11. $\mathrm{B} = \frac{\mathrm{A} + \mathrm{C}}{2}$, $\therefore\ \mathrm{C} = 2\mathrm{B} - \mathrm{A}$

12. (a) $\because$ $(n - 1)$ cards should be sent by each of the n students. $\therefore\ T = n(n - 1)$.
(b) When $n = 36$, $T = 36(36 - 1) = 1260$. *Ans. The number of cards sent is 1260.*

13. (a) $k = an + b$ (b) $k = an + b$, $k - b = an$, $\therefore\ n = \dfrac{k-b}{a}$

(c) $k = c(n-1) + d$, $k = c(\dfrac{k-b}{a} - 1) + d$, $k = \dfrac{kc - bc}{a} - c + d$,

$ak = kc - bc - ac + ad$, $ak - kc = ad - bc - ac$,

$k(a-c) = ad - bc - ac$, $\therefore\ k = \dfrac{ad - bc - ac}{a - c}$

14. (a) $1 + 2 + 3 + ... + 1000 = \dfrac{1000(1000+1)}{2} = 500500$

(b) $2 + 4 + 6 + ... + 1000 = 2(1 + 2 + 3 + ... + 500) = 2 \times \dfrac{500(500+1)}{2} = 250500$

(c) $1 + 3 + 5 + ... + 999$
$= (1 + 2 + 3 + ... + 1000) - (2 + 4 + 6 + ... + 1000) = 500500 - 250500 = 250000$

(d) $500 + 501 + 502 + ... + 1000$
$= (1 + 2 + 3 + ... + 1000) - (1 + 2 + 3 + ... + 499)$
$= 500500 - \dfrac{499(499+1)}{2} = 500500 - 124750 = 375750$

(e) Sum $= 3 + 6 + 9 + ... + 999 = 3(1 + 2 + 3 + ... + 333) = 3 \times \dfrac{333(333+1)}{2} = 166833$

15. (a) When $t = 0$, $s = -5(0-3)^2 + 80 = -45 + 80 = 35$
Ans. *The height of the building is 35 m.*

(b) $\because\ (t-3)^2 \geq 0$, $\therefore\ -5(t-3)^2 \leq 0$, $-5(t-3)^2 + 80 \leq 80$, $s \leq 80$
Ans. *The maximum height the stone can reach is 80 m.*

(c) The height is maximum when $(t-3)^2 = 0$, i.e. $t = 3$
Ans. *The stone takes 3 s to reach the maximum height.*

16. (a) $\dfrac{1}{a}$ of the hall will be painted. (b) $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{x}$

(c) When $a = 24$, $x = 8$, $\dfrac{1}{24} + \dfrac{1}{b} = \dfrac{1}{8}$, $\dfrac{1}{b} = \dfrac{1}{12}$, $b = 12$,
Ans. *It takes 12 hours for Simon to complete the work alone.*

17. (a) By Formula A, when $P = 81$, $N = 10\sqrt{81} = 10(9) = 90$
By Formula B, when $P = 81$, $N = 0.85(81) + 15 = 83.85$

(b) By Formula A, when $N = 50$, $50 = 10\sqrt{P}$, $5 = \sqrt{P}$, $P = 5^2 = 25$
By Formula B, when $N = 50$, $50 = 0.85P + 15$, $35 = 0.85P$, $P = 41.2$
Ans. *Jane's original score is 25, and Susan's original score is 41.2.*

(c) From the result of (b), original scores of 25 and 41.2 will become 50 using Formula A and B respectively. Therefore, more students will pass the examination if Formula A is adopted.

18. (a) $\dfrac{24}{24-h} = \dfrac{8}{x}$, $3x = 24 - h$, $\therefore\ h = 24 - 3x$

(b) $V = \dfrac{(x+8)h}{2} \times 18 = 9(x+8)(24-3x) = 9(192 - 3x^2)$, $\therefore\ V = 1728 - 27x^2$

(c) When $x = 6$, $V = 1728 - 27(6)^2 = 1728 - 972 = 756$.
Ans. *The volume of water is 756 cm^3.*

(d) Let H cm be the height of the container, $\frac{2}{3}H = 24 - 3(4) = 12$, $H = 18$.

When the container is full of water, $h = 18$ and $x = a$,

$\therefore\ 18 = 24 - 3a$, $3a = 6$, $a = 2$.

(e) When $x = 2$, $V = 1728 - 27(2)^2 = 1728 - 108 = 1620$

Ans. The capacity of the container is 1620 cm³.

19. (a) Height of the larger pyramid = $(h + y)$, base area = b^2

Height of the smaller pyramid = h, base area = a^2

$\therefore$ volume of the frustum = $\frac{1}{3}b^2(h+y) - \frac{1}{3}a^2h = \frac{1}{3}(b^2h - a^2h + b^2y)$

(b) $\frac{a}{b} = \frac{h}{h+y}$, $\therefore\ bh = ah + ay$, $bh - ah = ay$, $\therefore\ h = \frac{ay}{b-a}$

(c) Combining (a) and (b), volume of the frustum

$= \frac{1}{3}[(b^2 - a^2)h + b^2y] = \frac{1}{3}[(b+a)(b-a)\frac{ay}{b-a} + b^2y] = \frac{1}{3}[(b+a)ay + b^2y]$

$= \frac{1}{3}(aby + a^2y + b^2y) = \frac{1}{3}y(a^2 + ab + b^2)$

20. (a) $3 + 12s - 4r = 7 - 12s$, $-4r = 4 - 24s$, $r = -1 + 6s$

(b) (i) $r = -1 + 6\left(\frac{4t}{3}\right)$, $r = -1 + 8t$, $t = \frac{r+1}{8}$

(ii) If $r = 15$, $t = \frac{15+1}{8} = 2$

21. (a) (i) RHS = $\frac{(a+b)(a-b)}{ab} = \frac{a^2 - b^2}{ab} = \frac{a^2}{ab} - \frac{b^2}{ab} = \frac{a}{b} - \frac{b}{a}$ = LHS

(ii) $\frac{\frac{a}{b} - \frac{b}{a}}{a-b} = \left(\frac{a}{b} - \frac{b}{a}\right) \times \frac{1}{a-b} = \frac{(a+b)(a-b)}{ab} \times \frac{1}{a-b} = \frac{a+b}{ab}$

(b) (i) $c = \frac{\frac{a}{b} - \frac{b}{a}}{a-b}$, from the result of (a)(ii), $c = \frac{a+b}{ab}$,

$\therefore\ abc = a + b$, $abc - a = b$, $a(bc - 1) = b$, $a = \frac{b}{bc-1}$

(ii) If $b = 4$ and $c = -\frac{3}{2}$, $a = \frac{4}{4\left(-\frac{3}{2}\right) - 1} = -\frac{4}{7}$

22. (a) When $r = 3$ and $h = \frac{5}{10} = 0.5$, $A = 2\pi(3)^2 + 2\pi(3)(0.5) = 21\pi$

Ans. Total surface area = 21π cm².

(b) (i) $A = 2\pi r^2 + 2\pi rh$, $2\pi rh = A - 2\pi r^2$, $h = \frac{A - 2\pi r^2}{2\pi r}$

(ii) $h = \frac{396 - 2\left(\frac{22}{7}\right)(7)^2}{2\left(\frac{22}{7}\right)(7)} = \frac{88}{44} = 2$

(c) When $h = 1$, $A = 2\pi r^2 + 2\pi r = 2\pi r(r + 1)$.
Let A' be the new *area*.
$A' = 2\pi(r - 1)[(r - 1) + 1] = 2\pi r(r - 1)$,
$A - A' = 2\pi r(r + 1) - 2\pi r(r - 1) = 2\pi r[(r + 1) - (r - 1)] = 2\pi r(2) = 4\pi r$
Ans. The required decrease is $4\pi r$ *cm*^{2}.

23. (a) $C = P + Q = 5k + \frac{440}{n} + 4k + \frac{520}{n} = 9k + \frac{960}{n}$

(b) When $n = 80$, $C = 129$. $129 = 9k + \frac{960}{80}$, $k = 13$

(c) When $n = 48$, $C = 9(13) + \frac{960}{48} = 137$.

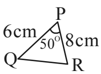
Profit percentage $= \frac{10550 - (137)(48)}{(137)(48)} \times 100\% = 60.4\%$ (3 sig. fig.)

(d) When $C = 120$, $9(13) + \frac{960}{n} = 120$, $n = 320$.

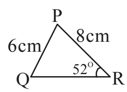
Ans. At least 320 suits.

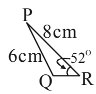
Unit 6 Congruent triangles

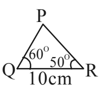
1. (a) $\Delta ABC \cong \Delta PRQ$ (b) $\Delta XYZ \cong \Delta LMN$
2. (a) $a = 55°$, $b = 11$ (b) $x = 32°$, $y = 5$
   (c) $x = 90°$, $y = 20$ (d) $a = 180° - 35° - 28° = 117°$, $b = 8.5$
3. (a) z = NM = QO (corr. sides, ≅Δs), QO = QP = 5 cm (corr. sides, ≅Δs),
   ∴ z = 5cm. ∠QOP = 53° (base ∠s, isos. Δ);
   ∠Q = 180° – 53° × 2 = 74° (∠ sum of isos. Δ)
   x = ∠NMO = ∠QOP = 53° (corr. ∠s, ≅Δs); y = ∠Q = 74° (corr. ∠s, ≅Δs)
   (b) AB = CD (corr. sides, ≅Δs), ∴ $2a - 1 = \frac{3}{2}a + 4$, $\frac{1}{2}a = 5$, $a = 10$
   BC = DA (corr. sides, ≅Δs), ∴ $4b + 1 = 15$, $4b = 14$, $b = 3.5$
   (c) BE = BD (corr. sides, ≅Δs), ∴ $x = 12$
   AB = CB (corr. sides, ≅Δs), ∴ $y + 12 = 9 + x = 9 + 12 = 21$, $y = 21 - 12 = 9$
4. (a) 14cm (b) 17cm (c) 8cm
5. (a) 53º (b) ∠X = 180° – 53° – 66° = 61° (c) 61°
6. (a) ΔPQR ≅ ΔLMN (SAS) (b) ΔABC ≅ ΔYXZ (RHS)
   (c) ΔPQR ≅ ΔFED (AAS) (d) ΔMNL ≅ ΔYXZ (ASA)
7. (a) yes, ΔABC ≅ ΔZXY (SAS) (b) yes, ΔLMN ≅ ΔZXY (SSS)
   (c) no (d) yes, ΔXYZ ≅ ΔLMN (AAS)
   (e) yes, ΔABC ≅ ΔEFD (RHS) (f) no
8. (a) ΔABC ≅ ΔZYX (RHS) (b) ΔPQR ≅ ΔXYZ (or: ΔYXZ) (SAS)
   (c) ΔPQR ≅ ΔZYX (AAS)
9. ΔDAB ≅ ΔDEC (RHS)

10. ∠CDE = ∠FED (∵ ∠CDF + ∠FDE = ∠FEC + ∠CED),
    ∠CED = ∠FDE, DE = ED, ∴ ΔCED ≅ ΔFDE (ASA)
11. (a) In ΔADB and ΔBCA, AB = BA (common), AD = BC and BD = AC (given),
    ∴ ΔADB ≅ ΔBCA (SSS)
    (b) ∠DAB = ∠CBA (corr. ∠s, ≅Δs)
12. (a) In ΔDAB and ΔDEC, AB = EC and AD = ED (given), ∠BAD = ∠CED = 90° (given).
    ∴ ΔDBA ≅ ΔDCE (RHS)
    (b) ∠ECD = ∠ABD (corr. ∠s, ≅Δs)
    ∠ABD = 180° – 90° – 68° = 22° (∠ sum of Δ), ∴ ∠ECD = 22°
13. (a) In ΔPQN and ΔPRM,
    PN = PM (given), ∠PNQ = ∠PMR = 90° (given),
    ∠QPN = ∠RPM (common), ∴ ΔQPN ≅ ΔRPM (ASA).
    (b) QN = RM (corr. sides, ≅Δs)
14. ∠EGF = 180° – 60° – 70° = 50° (∠ sum of Δ)
    In ΔEFG and ΔHGF, FG = GF (common), ∠EFG =∠HGF (given),
    ∠EGF = ∠HFG (proved), ∴ ΔEFG ≅ ΔHGF (ASA)
15. (a) In ΔCEF and ΔCDH, ∠C = ∠C (common), CF = CH (given),
    ∠CFE = 90° = ∠CHD (given), ∴ ΔCEF ≅ ΔCDH (ASA).
    (b) CD = CE (corr. sides, ≅ Δs)
16. (a) 

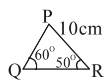
No, the two triangles will be congruent (∵ SAS)

    (b) 

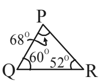
or 

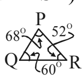
    (c) 

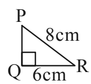
No, the two triangles will be congruent (∵ ASA)

    (d) 

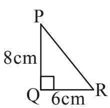
No, the two triangles will be congruent (∵ AAS)

    (e) 



    (f) 

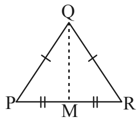
No, the two triangles will be congruent (∵ RHS)

    (g) 

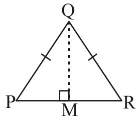
No, the two triangles will be congruent (∵ SAS)

17. ∠RPQ = ∠SPQ and ∠PQR = ∠PQS (given), PQ = PQ (common),
    ∴ ΔRPQ ≅ ΔSPQ (ASA), ∴ RP = SP (corr. sides, ≅Δs)
18. (a) ΔQNP. In ΔRMP and ΔQNP, PM = PN (given), PR = 2PN = PQ (given),
    ∠RPM = ∠QPN (common), ∴ ΔRMP ≅ ΔQNP (SAS)
    (b) ∠PNQ = ∠PMR (corr. ∠s, ≅Δs)
19. Join AC. In ΔABC and ΔADC, AB = AD and BC = DC (given),
    AC = AC (common), ∴ ΔABC ≅ ΔADC (SSS), ∴ ∠B = ∠D (corr. ∠s, ≅Δs)
20. ∠CED + ∠FEC = ∠FDE + ∠CDF (given), i.e. ∠FED = ∠CDE
    In ΔEDC and ΔDEF, ∠CED = ∠FDE (given), ∠FED = ∠CDE (proved),
    ED = DE (common), ∴ ΔEDC ≅ ΔDEF (ASA), ∴ EC = DF (corr. sides, ≅Δs)

21. (a) In ΔPMQ and ΔRMQ,
$PM = RM$ (given), $PQ = RQ$ (given), $QM = QM$ (common)
$\therefore$ $\Delta PMQ \cong \Delta RMQ$ (SSS),
$\therefore$ $\angle PMQ = \angle RMQ$ (corr. $\angle$s, $\cong\Delta$s),
$\because$ $\angle PMQ + \angle RMQ = 180°$ (adj. $\angle$s on st. line),
$\therefore$ $\angle PMQ = \angle RMQ = 90°$, i.e. $QM \perp PR$



(b) In ΔPMQ and ΔRMQ,
$QM \perp PR$ (given), $PQ = RQ$ (given), $QM = QM$ (common)
$\therefore$ $\Delta PMQ \cong \Delta RMQ$ (RHS), $\therefore$ $PM = RM$ (corr. sides, $\cong\Delta$s)



22. Join QS. In ΔQPS and ΔSRQ,
$PS = RQ$ and $PQ = RS$ (given), $QS = SQ$ (common).
$\therefore$ $\Delta QPS \cong \Delta SRQ$ (SSS), $\therefore$ $\angle QSP = \angle SQR$ (corr. $\angle$s, $\cong\Delta$s)
23. $\angle EGF = 180° - 73° - 59° = 48°$
In ΔFGE and ΔGFH, $FG = GF$ (common),
$\angle EFG = \angle HGF = 59°$ (given), $\angle EGF = \angle HFG = 48°$ (proved),
$\therefore$ $\Delta FGE \cong \Delta GFH$ (ASA), $\therefore$ $EF = HG$ (corr. sides, $\cong\Delta$s)
24. In ΔQLM and ΔQNM, $QL = QN$ (given), $QM = QM$ (common),
$\angle QLM = \angle QNM = 90°$ (given), $\therefore$ $\Delta QLM \cong \Delta QNM$ (RHS),
$\therefore$ $LM = NM$ (corr. sides, $\cong\Delta$s).
In ΔLPM and ΔNRM, $LM = NM$ (proved), $\angle MLP = \angle MNR = 90°$ (given),
$PM = RM$ (given), $\therefore$ $\Delta LPM \cong \Delta NRM$ (RHS)
25. (a) In ΔABC and ΔEDC, $\angle A = \angle E$ (given), $\angle C = \angle C$ (common),
$BC = DC$ (given), $\therefore$ $\Delta ABC \cong \Delta EDC$ (AAS),
$\therefore$ $AB = ED$ (corr. sides, $\cong\Delta$s)
(b) $\because$ $\Delta ABC \cong \Delta EDC$ (proved), $\therefore$ $AC = EC$ (corr. sides, $\cong\Delta$s),
but $BC = DC$ (given),
$\therefore$ $AC - DC = EC - BC$, i.e. $AD = EB$
26. In ΔAEB and ΔDEC, $AE = ED$ and $BE = EC$ (given),
$\angle AEB = \angle DEC$ (vert. opp. $\angle$s), $\therefore$ $\Delta AEB \cong \Delta DEC$ (S.A.S.),
$\therefore$ $\angle ABE = \angle DCE$ (corr. $\angle$s, $\cong\Delta$s), $\therefore$ $AB \parallel CD$ (alt. $\angle$s equal)
27. (a) $\angle CDE + \angle CDF = 90°$, $\angle CDE + \angle ADE = 90°$, $\therefore \angle CDF = \angle ADE$
(b) $AD = CD$, $\angle EAD = 90° = \angle FCD$, $\angle ADE = \angle CDF$ [from (a)],
$\therefore$ $\angle AED \cong \Delta CFD$ (ASA)
28. $\angle BAD = 60° = \angle CAE$, $BA = CA$, $AD = AE$, $\therefore$ $\Delta BAD \cong \Delta CAE$ (SAS)
29. In ΔABE and ΔCBD, $AB = CB$ and $BE = BD$ (equilateral Δs),
$\angle ABE = \angle CBD = 60°$ (equilateral Δs), $\therefore$ $\Delta ABE \cong \Delta CBD$ (SAS),
$\therefore$ $\angle BAE = \angle BCD$ (corr. $\angle$s, $\cong\Delta$s)
30. In ΔABD and ΔCBE, $AB = CB$ and $BD = BE$ (equilateral Δs),
$\angle ABD = \angle ABC - \angle DBF = 60° - \angle DBF$ (equilateral Δ),
$\angle CBE = \angle DBE - \angle DBF = 60° - \angle DBF$ (equilateral Δ), $\therefore$ $\angle ABD = \angle CBE$,
$\therefore$ $\Delta ABD \cong \Delta CBE$ (SAS), $\therefore$ $AD = CE$ (corr. sides, $\cong\Delta$s)
31. (a) In ΔABF and ΔCBE, $AB = CB$ and $BF = BE$ (equilateral Δs),
$\angle ABF = \angle ABC - \angle FBC = 60° - \angle FBC$ (equilateral Δ),

$\angle CBE = \angle FBE - \angle FBC = 60° - \angle FBC$ (equilateral Δ), ∴ $\angle ABF = \angle CBE$,
∴ ΔABF ≅ ΔCBE (SAS), ∴ AF = CE (corr. sides, ≅Δs)

(b) ΔBCE. In ΔFDE and ΔBCE, DE = CE and FE = BE (equilateral Δs),
$\angle DEF = \angle DEC - \angle FEC = 60° - \angle FEC$ (equilateral Δ),
$\angle CEB = \angle FEB - \angle FEC = 60° - \angle FEC$ (equilateral Δ),
∴ $\angle DEF = \angle CEB$, ∴ ΔFDE ≅ ΔBCE (SAS).
∵ ΔFDE ≅ ΔBCE (proved), ∴ FD = BC (corr. sides, ≅Δs),
but AB = BC (corr. sides, ≅Δs), ∴ FD = AB

32. (a) In ΔABD and ΔACE, AB = AC and AD = AE (equilateral Δs),
$\angle BAD = \angle CAE = 60°$ (equilateral Δs), ∴ ΔABD ≅ ΔACE (SAS),
∴ $\angle ABD = \angle ACE$ (corr. ∠s, ≅Δs)

(b) In ΔCDF and ΔBDA, $\angle ACE = \angle ABD$ (proved),
$\angle CDF = \angle BDA$ (vert. opp. ∠s), ∴ $\angle DFC = \angle DAB$ (the 3rd ∠ of Δ).
$\angle DAB = 60°$ (equilateral Δ), but $\angle DFC = x + y$ (ext. ∠ of Δ),
∴ $x + y = 60°$

33. (a) In ΔACE and ΔDBF, AE = DF and CE = BF (given),
AC = AB + BC and DB = DC + BC, but AB = DC (given), ∴ AC = DB,
∴ ΔACE ≅ ΔDBF (SSS), ∴ $\angle A = \angle D$ (corr. ∠s, ≅Δs),
∴ AE // DF (alt. ∠s eq.)

(b) In ΔBAE and ΔCDF, $\angle A = \angle D$ (proved), AE = DF and
AB = DC (given), ∴ ΔBAE ≅ ΔCDF (SAS),
∴ EB = FC (corr. sides, ≅ Δs)

34. (a) In ΔBAE and ΔCDE, BA = CD (given), BE = CE (given),
$AE = DE = \frac{1}{2}AD$ (given),
∴ ΔBAE ≅ ΔCDE (SSS), ∴ $\angle BAD = \angle CDA$ (corr. ∠s, ≅Δs)

(b) In ΔBAD and ΔCDA, AB = DC (given), AD = DA (common),
$\angle BAD = \angle CDA$ (proved), ∴ ΔBAD ≅ ΔCDA (SAS),
∴ BD = CA (corr. sides, ≅Δs)

35. (a) $\angle BAE = \angle BAC - \angle CAE$, $\angle ACD = \angle FDC - \angle CAE$ (ext. ∠ of Δ),
but $\angle BAC = \angle FDC$ (given), ∴ $\angle BAE = \angle ACD$

(b) In ΔABE and ΔCAD, AB = CA (given), $\angle BAE = \angle ACD$ (proved),
$\angle AEB = 180° - \angle BEF$ and $\angle CDA = 180° - \angle FDC$ (adj. ∠s on st. line),
but $\angle BEF = \angle FDC$ (given), ∴ $\angle AEB = \angle CDA$,
∴ ΔABE ≅ ΔCAD (AAS), ∴ BE = AD (corr. sides, ≅Δs)

36. In ΔBCD and ΔACE, BC = AC and CD = CE (equilateral Δs),
$\angle ACB = \angle ECD = 60°$ (equilateral Δs), $\angle ACD = 60° - 50° = 10°$,
∴ $\angle ECA = 60° - 10° = 50° = \angle DCB$, ∴ ΔBCD ≅ ΔACE (SAS),
∴ $\theta = \angle CBD$ (corr. ∠s, ≅Δs), ∴ $\theta = 180° - 100° - 50° = 30°$ (∠ sum of Δ)

37. (a) In ΔABP and ΔCBS, PB = SB (given), $\angle APB = \angle CSB = 90°$ (square),

∠ABP =∠CBS (vert. opp. ∠s), ∴ ΔABP ≅ ΔCBS (ASA),
∴ AB = CB (corr. sides, ≅Δs)

(b) In ΔABR and ΔCBR, BR = BR (common), ∠ABR = ∠CBR (given),
AB = CB (proved), ∴ ΔABR ≅ ΔCBR (SAS), ∴ AR = CR (corr. sides, ≅Δs).
∵ ΔABR ≅ ΔCBR (proved),
∴ the ⊥ distance from B to AR = the ⊥ distance from B to RC

$$= BS = \frac{1}{2}\times 9 = 4.5 \text{ cm}$$

38. In ΔQBP and ΔRCQ, $a_1 = a_2$ (given), QB = RC = 6 cm (given),
∠BQP + $a_3 = a_2$ + ∠CRQ (ext. ∠ of Δ), ∴ ∠BQP = ∠CRQ ($a_3 = a_2$),
∴ ΔQBP ≅ ΔRCQ (ASA), ∴ BP = CQ = 3 cm (corr. sides, ≅Δs)

39. (a) PQ = 2PA = 2AQ and PR = 2PB = 2BR (mid-points)
∵ PQ = PR, ∴ 2PA = 2AQ = 2PB = 2BR, PA = AQ = PB = BR
In ΔBPQ and ΔAPR, PQ = PR (given), PB = PA (proved),
∠BPQ = ∠APR (common ∠), ∴ ΔBPQ ≅ ΔAPR (SAS), ∴∠Q = ∠R (corr. ∠s, ≅Δs)

(b) In ΔACQ and ΔBCR, ∠ACQ = ∠BCR (vert. opp. ∠s), AQ = BR (proved),
∠Q = ∠R (proved), ∴ ΔACQ ≅ ΔBCR (ASA)

40. (a) ∠ABC = 60° (equil. Δ), ∠EBF = 60° – 18° = 42°,
∠BED = 180° – 90° – 42° = 48° (∠ sum of Δ)

(b) In ΔBEF and ΔBDF, ∠BDF = 48° (given), ∠BEF = 48°(from (a)),
∴∠BEF = ∠BDF, ∠BFE = ∠BFD = 90° (given), BF = BF (common side),
∴ΔBEF ≅ ΔBDF (AAS)

(c) (i) In ΔAEF and ΔADF, EF = DF (corr. sides, ≅Δs), ∠AFE = ∠AFD = 90° (given),
AF = AF (common side), ∴ ΔAEF ≅ ΔADF (SAS)

(ii) ∠EAF = 60° (property of equil. Δ), ∠DAF = ∠EAF (corr. ∠s, ≅Δs) = 60°
∠EBF = 42° ≠ ∠DAF, ∴ AD is not parallel to EB (alt. ∠s not equal)

41. (a) (i) ∠BAD = ∠EAC (given), ∠BAD – ∠CAD = ∠EAC – ∠CAD, ∠BAC = ∠EAD,
BA = EA (given), ∠B = ∠E (given), ∴ ΔABC ≅ ΔAED (ASA)

(ii) AC = AD (corr. sides, ≅Δs), ∴∠ADC = ∠ACD (base ∠s, isos. Δ) = 80°,
∠CAD = 180° – 2(80°) = 20° (∠ sum of Δ)

(b) Let ∠BAC = 3*k*, ∠CBA = 4*k*. ∠EAD = ∠BAC (corr. ∠s, ≅Δs) = 3*k*
∠ACB = ∠CAE (alt. ∠s, BC // AE)
= 20° + 3*k*,
∠BAC + ∠CBA + ∠ACB = 180° (∠ sum of Δ)
3*k* + 4*k* + (20° + 3*k*) = 180°, *k* = 16°
∠ADE = ∠ACB (corr. ∠s, ≅Δs)
= 20° + 3(16°) = 68°

42. (a) AS = BQ (given), PS = PQ (given), ∠ASP = ∠BQP = 90° (given),
∴ ΔSPA ≅ ΔQPB (SAS)

(b) (i) ∠BPC : ∠CPS : ∠SPA = 3 : 1 : 2.
Let ∠BPC = 3*k*, ∠CPS = *k* and ∠SPA= 2*k*.
∠QPB = ∠SPA (corr. ∠s, ≅Δs) = 2*k*
∠QPB + ∠BPC + ∠CPS = ∠QPS = 90°, 2*k* + 3*k* + *k* = 90°, *k* = 15°,
∴ ∠SPA = 2(15°) = 30°

(ii) ∠APC = ∠APS + ∠SPC = 2*k* + *k* = 3*k* = ∠BPC, PC = PC (common), PA = PB (corr. sides, ≅Δs), ∴ΔPAC ≅ ΔPBC (SAS)

(iii) ∠CPS : ∠SPA = 1 : 2, ∴ ∠CPS = $\frac{1}{2}(30°) = 15°$

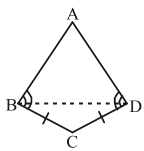
∠PCS = 180° − ∠SPC − ∠ PSC = 180° − 15° − 90° = 75°
∠PCB = ∠PCS (corr. ∠s, ≅Δs) = 75°
∠RCB = 180° − ∠PCS − ∠PCB (adj. ∠s on a st. line)
= 180° − 75° − 75° = 30°

Unit 7 Isosceles triangles

1. (a) $y = \frac{180° - 90°}{2} = 45°$
   (b) $2(2y - 8)° + 3y° = 180°$, $7y° = 196°$, ∴ $y = 28$
   (c) $(180° - 2y°) \times 2 + y° = 180°$, $360° - 4y° + y° = 180°$, ∴ $y = 60$
   (d) $(y° + y°) \times 2 + 4y° = 180°$, $8y° = 180°$, ∴ $y = 22.5$
   (e) $2(y + 5)° = (3y - 18)°$, $2y + 10 = 3y - 18$, ∴ $y = 28$
   (f) $180° - 2(2y°) = 5y°$, $9y° = 180°$, ∴ $y = 20$
2. (a) The third angle = 180° − 32° − 74° = 74° (∠ sum of Δ),
   ∴ 4*y* – 21 = *y* + 12 (sides opp., eq. ∠ s), 3*y* = 33, ∴ *y* = 11
   (b) ∠ R = ∠ S = *y* (base ∠s, isos.Δ); *y* + *y* = 72° (ext. ∠ of Δ), 2*y* = 72°, ∴ *y* = 36°
   (c) ∠ EGF = 180° – 120° = 60° (adj. ∠s on st. line);
   m + 60° + 60° = 180° (∠ sum of isos. Δ), ∴ *m* = 60°
   (d) ∠ A = ∠ ABD = 35° (base ∠s, isos. Δ); ∠ DBC = *n* (base ∠s, isos. Δ);
   n + *n* + 35° + 35° = 180° (∠ sum of Δ), 2*n* = 110°, ∴ *n* = 55°
   (e) ∠ CDE = 76° (alt.∠s, AB // CD); ∠ ABE = 115° − 76° = 39° (ext. ∠ of Δ);
   ∠C = ∠ ABE = 39° (alt.∠s,, AB // CD), ∴ *x* = ∠C = 39° (base ∠s, isos. Δ)
   (f) ∠LMN = 180° − 90° − 63° = 27° (∠ sum of Δ);
   ∠KLN = 27° (alt. ∠s, LK // NM); ∠KML = ∠KLN = 27° (base ∠s, isos. Δ);
   b + 27° + 27° = 180° (∠ sum of Δ), ∴ *b* = 126°
   (g) ∠BCA = ∠A = 50° (base ∠s, isos. Δ); ∠CBD = *x* (base ∠s, isos. Δ);
   x + *x* = 50° (ext. ∠ of Δ), 2*x* = 50°, ∴ *x* = 25°
   (h) ∠CEB = 180° – 25° – 85° = 7 ∠s, isos. Δ);
   y + 70° = 85° (alt. ∠s, AB // DF), ∴ *y* = 15°
   (i) ∠S = ∠SQR (base ∠s, isos. Δ); 2 ∠SQR + 46° = 180° (∠ sum of Δ),
   ∠SQR = 67°; ∠QRP = ∠SQR = 67° (alt. ∠s, SQ//RP);
   ∵ PR = PQ (given), ∴ ∠QRP = ∠PQR = 67°;
   a + 67° + 67° = 180° (∠ sum of Δ), ∴ *a* = 46°
3. ∠C = ∠ABC (base ∠s, isos. Δ);
   ∴ 2 ∠C + 38° = 180° (∠ sum of Δ),
   ∠BDC = ∠C = 71° (base ∠s, isos. Δ); ∠C = 71°
   ∴ ∠ABD = 71° – 38° = 33° (ext. ∠of Δ)
4. ∠R = ∠PQR (base∠s, isos. Δ);
   ∴ 2 ∠R + *y* = 180° (∠ sum of Δ), R $= 90° - \frac{y}{2}$

$x + \angle R = 90°$ (ext. $\angle$ of Δ), $x = 90° - \left(90° - \frac{y}{2}\right) = \frac{y}{2}$, $\therefore\ y = 2x$

5. $\angle CBD = a$ (base $\angle$s, isos. Δ); $\angle A = \angle ACB$ (base $\angle$s, isos. Δ);
$\angle A + \angle ACB = a$ (ext. $\angle$ of Δ), $2\angle A = a$, $\angle A = \frac{a}{2}$
$b = \angle D + \angle A = a + \frac{a}{2} = \frac{3a}{2}$ (ext. $\angle$ of Δ)
6. (a) In ΔACD, $\angle D = x$ (base $\angle$s, isos. Δ).
In ΔBCD, $\angle D = y$ (base $\angle$s, isos. Δ); $\therefore\ x = y$.
(b) $\angle ABC = \angle D + y = 2x$ (ext. $\angle$ of Δ)
$\angle ACB = \angle ABC = 2x$ (base $\angle$s, isos. Δ)
$x + 2x + 2x = 180°$ ($\angle$ sum of Δ), $5x = 180°$, $\therefore\ x = 36°$
7. In ΔABC, $\angle BAC = \theta$ (base $\angle$s, isos. Δ), $\therefore\ \angle B = 180° - 2\theta$ ($\angle$ sum of Δ).
In ΔADC, $\angle ADC = \theta$ (base $\angle$s, isos. Δ), $\therefore\ \angle DAB = \angle B = 180° - 2\theta$.
$\angle B + \angle DAB = \angle ADC$ (ext. $\angle$ of Δ), $2(180° - 2\theta) = \theta$, $5\theta = 360°$, $\therefore\ \theta = 72°$
8. $\angle ABC = 18°$ (base $\angle$s, isos. Δ)
$\angle BCD = 18° + 18° = 36°$ (ext. $\angle$ of Δ), $\therefore\ \angle BDC = 36°$ (base $\angle$s, isos. Δ)
$\angle EBD = 36° + 18° = 54°$ (ext. $\angle$ of Δ).
$\because$ BD = DE (given), $\therefore\ \angle BED = 54°$ (base $\angle$s, isos. Δ);
$\angle EDF = 54° + 18° = 72°$ (ext. $\angle$ of Δ)
$\because$ DE = EF (given), $\therefore\ \angle EFD = 72°$ (base $\angle$s, isos. Δ),
$\therefore\ \theta = 72° + 18° = 90°$ (ext. $\angle$ of Δ)
9. $\angle DBC = 20° + 28° = 48°$ (ext. $\angle$ of Δ);
$\angle C = 180° - 28° - 20° - 84° = 48°$ ($\angle$ sum of Δ).
$\because \angle DBC = \angle C = 48°$, $\therefore$ DB = DC (sides opp., eq. $\angle$s), $\therefore\ \Delta BCD$ is isosceles.
10. Join BD.
$\angle CBD = \angle CDB$ (base $\angle$s, isos. Δ), but $\angle ABC = \angle ADC$ (given),
$\therefore \angle ABC - \angle CBD = \angle ADC - \angle CDB$,
i.e. $\angle ABD = \angle ADB$, $\therefore$ AB = AD (sides opp., eq. $\angle$s)

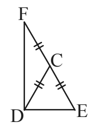


11. $\angle XYZ = \angle XZY$ (base $\angle$s, isos. Δ),
but $\angle XYA = \angle AYZ$ and $\angle XZA = \angle AZY$ (given)
$\therefore\ 2\angle AYZ = 2\angle AZY$, $\angle AYZ = \angle AZY$, $\therefore$ AY = AZ (sides opp., eq. $\angle$s)
$\therefore\ \Delta AYZ$ is also isosceles.
12. In ΔABD and ΔFEC, AB = FE (given), $\angle B = \angle E$ (given),
BD = BC + CD = DE + CD = EC ($\because$ BC = DE), $\therefore\ \Delta ABD \cong \Delta FEC$ (SAS),
$\therefore\ \angle GCD = \angle GDC$ (corr. $\angle$s, $\cong\Delta$s), $\therefore$ GC = GD (sides opp., eq. $\angle$s),
$\therefore\ \Delta CDG$ is isosceles.
13. (a) In ΔABC and ΔCDA, AC = CA (common),
$\angle ACB = a_2 = a_1 = \angle CAD$ (given),
$\angle CAB = a_1 + b_2 = a_2 + b_2 = \angle ACD$ (given), $\therefore\ \Delta ABC \cong \Delta CDA$ (ASA)
(b) $\because\ \Delta ABC \cong \Delta CDA$ (proved), $\therefore$ BC = DA (corr. sides, $\cong\Delta$s);
$\because\ a_1 = a_2$ (given), $\therefore$ EA = EC (sides opp., eq. $\angle$s);
BE = BC – EC = DA – EA = DE, $\therefore\ \Delta BDE$ is isosceles.
14. $\angle BPQ = \angle BQP$ (base $\angle$s, isos. Δ); $\angle BPQ = x + z$ (ext. $\angle$ of Δ),
$\therefore\ \angle BQP = \angle BQP = x + z$ (base $\angle$s, isos. Δ); $\angle CQR = y - z$ (ext. $\angle$ of Δ);
$\because\ \angle BQP = \angle CQR$ (vert. opp. $\angle$s),

$\therefore\ x+z=y-z,\quad 2z=y-x,\ \therefore\ z=\dfrac{1}{2}(y-x)$

15. $\because$ QR = RS (given), $\therefore$∠RSQ = 40° (base ∠s, isos. Δ);
∠SRQ = 180° – 40° – 40° = 100° (∠ sum of Δ). ∠PQR = 60° (equi. Δ),
∠PRS = ∠PRQ + ∠SRQ = 60° + 100° = 160°.
$\because$ PR = RS (given), $\therefore$∠PSR = θ (base ∠s, isos. Δ)
2θ + 160° = 180° (∠ sum of Δ), 2θ = 20°, $\therefore$ θ = 10°

16. $\because$ CD = CE (given), $\therefore$∠CDE = ∠E (base ∠s, isos. Δ);
$\because$ CD = CF (given), $\therefore$ ∠CDF = ∠F (base ∠s, isos. Δ);
∠CDE + ∠E + ∠CDF + ∠F = 180° (∠ sum of Δ),
2 ∠CDE + 2 ∠CDF = 180°, ∠CDE + ∠CDF = 90°,
$\therefore$ ∠FDE = 90°



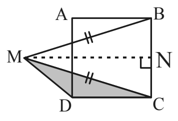
17. ∠A = ∠C (base ∠s, isos. Δ); ∠BQC = θ + ∠A (ext. ∠of Δ);
∠BQC + ∠C = 3θ (ext. ∠of Δ); $\therefore$ θ + ∠A + ∠A = 3θ, 2 ∠A = 2θ, ∠A = θ,
$\therefore$ ΔAQB is isosceles (sides opp., eq. ∠s).

18. (a) In ΔABC, ∠C = 2*a* (base ∠s, isos. Δ),
$\therefore$ 2*a* + 2*a* + 100° =180° (∠ sum of Δ), 4*a* = 80°, *a* = 20°.
2*a* + *a* + 2*b* = 180° (ext. ∠of Δ), 100° + *a* = 2*b*, 2*b* =120°, *b* = 60°.
$\therefore$ ∠DEA = 2*a* + *b* = 2(20°) + 60° =100° (ext. ∠of Δ)
(b) In ΔABD and ΔAED, AD = AD (common), ∠BAD = ∠DAE (given),
∠DEA = ∠B = 100° (proved), $\therefore$ ΔABD ≅ ΔAED (AAS),
$\therefore$ DB = DE (corr. sides, ≅Δs)

19. (a) $\because$ QP = QR (given), $\therefore$ ∠P = ∠QRP (base ∠s, isos. Δ);
$\because$ AP = AB (given), $\therefore$ ∠P = ∠ABP (base ∠s, isos. Δ),
$\therefore$ AB // QD (corr. ∠s eq.)
(b) In ΔABC and ΔDRC, AB = DR (common), ∠ABC = ∠DRC and
∠BAC = ∠RDC (alt. ∠s, AB // QD), $\therefore$ ΔABC ≅ ΔDRC (ASA),
$\therefore$ BC = RC (corr. sides, ≅Δs), i.e. C is the mid-point of BR.

20. $\because$ ΔABC ≅ ΔEDC (given), $\therefore$ *x* = 68° (corr. ∠s, ≅Δs);
$\because$ BC = DC (corr. sides, ≅Δs), $\therefore$ ∠BDC = *x* = 68° (base ∠s, isos. Δ),
y + 68° + 68° = 180° (∠ sum of Δ), $\therefore$ *y* = 44°

21. Draw MN ⊥ BC.
In ΔBMN and ΔCMN, MN = MN (common),
MB = MC (given), ∠MNB = ∠MNC = 90° (construction),
$\therefore$ ΔBMN ≅ ΔCMN (R.H.S.)
$\therefore$ BN = CN $=\dfrac{1}{2}$BC (corr. sides, ≅Δs),
$\therefore$ Area of ΔDMC $=\dfrac{1}{2}\times DC\times CN=\dfrac{1}{2}(\sqrt{18})(\dfrac{1}{2}\sqrt{18})=\dfrac{18}{4}=4.5\ \text{cm}^2$



22. (a) $\because$ ∠ACB = *a* (base ∠s, isos. Δ), $\therefore$ ∠CBD = 2*a* (ext. ∠of Δ),
$\therefore$ ∠FDE = 2*a* (corr. ∠s, BC // DF).
In ΔDEF, ∠DFE = *b* (base ∠s, isos. Δ),
$\therefore$ *b* + *b* + 2*a* = 180° (∠ sum of Δ), 2*a* + 2*b* = 180°, $\therefore$ *a* + *b* = 90°
(b) *a* + *b* + ∠AGE = 180° (∠sum of Δ), 90° + ∠AGE = 180°, $\therefore$ ∠AGE = 90°

23. Let ∠PHQ = ∠PQH = *a* (base ∠s, isos. Δ); $\therefore$ ∠PQK = *a* – θ
Let ∠RKQ = ∠RQK = *b* (base ∠s, isos. Δ).

$\angle PQK + RQK = 100°$, i.e. $(a - \theta) + b = 100°$, $a + b = 100° + \theta$
In ΔHKQ, $a + b + \theta = 180°$ ($\angle$ sum of Δ), $\therefore$ $a + b = 180° - \theta$.
$\therefore$ $100° + \theta = 180° - \theta$, $2\theta = 80°$, $\therefore$ $\theta = 40°$

24. (a) $\angle PLM = a$ (base $\angle$s, isos. Δ), $\therefore$ $\angle P = 180° - 2a$ ($\angle$ sum of Δ);
$\angle RNM = b$ (base $\angle$s, isos. Δ), $\therefore$ $\angle R = 180° - 2b$ ($\angle$ sum of Δ);
$\angle P + \angle R + \theta = 180°$ ($\angle$ sum of Δ),
$(180° - 2a) + (180° - 2b) + \theta = 180°$, $360° - 2a - 2b + \theta = 180°$, $\therefore$ $\theta = 2a + 2b - 180°$
(b) $\theta = 2a + 2b - 180°$, $2a + 2b = \theta + 180°$, $\therefore$ $a + b = \frac{\theta}{2} + 90°$
$a + b + \angle LMN = 180°$ (adj. $\angle$s on st. line),
$\frac{\theta}{2} + 90° + \angle LMN = 180°$, $\therefore$ $\angle LMN = 90° - \frac{\theta}{2}$

25. (a) In ΔABC, $\angle A = \angle ACB$ (base $\angle$s, isos. Δ),
$\therefore$ $2\angle A + 20° = 180°$ ($\angle$ sum of Δ), $\angle A = 80°$ and $\angle ACB = 80°$.
In ΔACD, $\angle A = \angle ADC$ (base $\angle$s, isos. Δ),
$2\angle A + \angle ACD = 180°$ ($\angle$ sum of Δ),
$2(80°) + \angle ACD = 180°$, $\angle ACD = 20°$, $\therefore$ $\angle DCE = 80° - 20° = 60°$.
In ΔDCE, $\angle DEC = \angle DCE$ (base $\angle$s, isos. Δ),
$\angle CDE + 2(60°) = 180°$ ($\angle$ sum of Δ), $\therefore$ $\angle CDE = 60°$
(b) $\angle EDB = 60° - 20° = 40°$ (ext. $\angle$of Δ); $\therefore \angle EFD = 40°$ (base $\angle$s, isos. Δ);
$\angle FEB = 40° - 20° = 20°$ (ext. $\angle$of Δ); $\because$ $\angle FEB = \angle B = 20°$,
$\therefore$ BF = EF (sides opp., eq. $\angle$s), $\therefore$ AC = EF = BF = 6 cm

26. (a) In ΔABC and ΔCDE, BC = DE and AB = CD (given),
$\angle ABC = \angle CDE = 180° - 94° = 86°$ (adj. $\angle$s on st. line),
$\therefore$ $\Delta ABC \cong \Delta CDE$ (SAS), $\therefore$ AC = CE (corr. sides, $\cong\Delta$s),
$\therefore$ ΔACE is isosceles.
(b) $\because$ $\Delta ABC \cong \Delta CDE$ (proved), $\therefore \angle BAC = y$ (corr. $\angle$s, $\cong\Delta$s),
$x + y = 94°$ (ext. $\angle$of Δ); but $35° + x + \theta + y = 180°$ (adj. $\angle$s on st. line),
$\therefore$ $\theta + 94° + 35° = 180°$, $\therefore$ $\theta = 51°$
(c) In ΔACE, AC = CE (proved), $\therefore$ $\angle CAE = \angle CEA$ (base $\angle$s, isos. Δ),
$\therefore$ $2\angle CEA + 51° = 180°$ ($\angle$ sum of Δ), $\angle CEA = 64.5°$,
$\therefore \angle CED = 180° - 52° - 64.5° = 63.5°$ (adj. $\angle$s on st. line);
But $x = \angle CED$ (corr. $\angle$s, $\cong\Delta$s),
$\therefore$ $x = 63.5°$ and $y = 94° - x = 94° - 63.5° = 30.5°$

27. (a) $AC = \sqrt{AB^2 + BC^2} = \sqrt{(4+3)^2 + 24^2} = \sqrt{625} = 25$ cm;
Area of $\Delta ABC = \frac{7 \times 24}{2} = \frac{BD \times 25}{2}$, $\therefore$ BD = 6.72 cm
(b) Let $a = \angle BCE = \angle DCF$. $\angle BEC = 180° - 90° - a = 90° - a$ ($\angle$ sum of Δ);
$\angle DFC = 180° - 90° - a = 90° - a$ ($\angle$ sum of Δ);
But $\angle BFE = \angle DFC$ (vert. opp. $\angle$s) , $\therefore$ $\angle BFE = 90° - a = \angle BEC$,
$\therefore$ ΔBEF is isosceles (sides opp., eq. $\angle$s)
(c) BF = BE = 4 cm (proved), $\therefore$ FD = BD – BF = 6.72 – 4 = 3.72 cm

28. (a) Let $\angle AQR = x$ and $\angle APB = y$.
$\because$ BP = BA (given), $\therefore$ $\angle PAB = y$ (base $\angle$s, isos. Δ)
$\because$ RQ = RA (given), $\therefore$ $\angle RAQ = x$ (base $\angle$s, isos. Δ)

$\angle BAR = 180° - x - y$ (adj. ∠s on st. line),
$\angle PQR = 180^{v} - x - y$ (∠ sum of Δ), ∴ ∠BAR = ∠PQR

(b) In ΔABR and ΔRCQ, ∠BAR = ∠CRQ (proved),
∠ABR = ∠RCQ (corr. ∠s, QC // AB), RA = RQ (given),
∴ ΔABR ≅ ΔRCQ (A.A.S.), ∴ AB = CR (corr. sides, ≅Δs)

29. ∠BCE + ∠ECD = ∠A + ∠CBA (ext. ∠ of Δ),
∴ 90° + ∠ECD = 90° + ∠CBA, i.e. ∠ECD = ∠CBA.
In ΔECD and ΔCBA, ∠ECD = ∠CBA (proved),
CD = AB (given), ∠EDC = ∠CAB = 90° (given), ∴ ΔECD ≅ ΔCBA (AAS),
∴ EC = CB (corr. sides, ≅Δs), i.e. ΔBCE is isosceles

30. (a) ∠QRS + ∠PQR = 180° (int. ∠s, PQ // SR)
∠QRS + 123° = 180°, ∠QRS = 57°
∠QSR = ∠QRS = 57° (base ∠s, isos. Δ),
∠RQS = 180° – ∠QSR – ∠QRS (∠ sum of Δ)
= 180° – 57° – 57° = 66°

(b) ∠PQS = ∠QSR = 57° (alt. ∠s, PQ // SR)
∠PSQ = 180° – ∠PQS – ∠QPS (∠ sum of Δ)
= 180° – 57° – 66° = 57°
∵ ∠PQS = ∠PQS, ∴ PQ = PS (sides opp., eq. ∠s)

31. (a) ∠ACB = ∠ABC (base ∠s, isos. Δ)
$= \dfrac{180° - 20°}{2} = 80°$ (∠ sum of Δ)
∠BDC = ∠BCD = 80° (base ∠s, isos. Δ),
∠CBD = 180 – ∠BDC – BCD (∠ sum of Δ)
= 180° – 80° – 80° = 20°
∠DBE = ∠ABC – ∠CBD = 80° – 20° = 60°
∠BED = ∠DBE = 60° (base ∠s, isos. Δ)
∠BDE = 180° – ∠DBE – ∠BED (∠ sum of Δ)
= 180° – 60° – 60° = 60° = ∠DBE
∴ BE = DE (sides opp., eq. ∠s) , but DE = BD (given),
∴ ΔBDE is an equilateral triangle.

(b) ∠EDF = 180° – ∠BDC – ∠BDE (adj. ∠s on st. line)
= 180° – 80° – 60° = 40°
∠DFE = ∠EDF (base ∠s, isos. Δ) = 40°
20° + ∠AEF = ∠DFE (ext. ∠ of Δ) , ∠AEF = 40° – 20° = 20°
∠AEF = ∠EAF, ∴ AF = EF (sides opp., eq. ∠s),
i.e. ΔAEF is an isosceles triangle.

(c) ∠BEF = 180° – ∠AEF (adj. ∠s on st. line)
= 180° – 20° = 160°
BE = DE = EF
∴ ∠BFE = ∠EBF (base ∠s, isos. Δ)
$= \dfrac{180° - 160°}{2}$ (∠ sum of Δ) = 10°,
∠BFC = ∠DFE – ∠BFE = 40° – 10° = 30°

Unit 8 Angles related to polygons

1. (a) $\Delta GFE, \Delta GCE, \Delta DCE, \Delta DAE$ (b) $\Delta BCF, \Delta DCE$
   (c) No. (d) Yes, it is an ext. $\angle$ of ΔDCG and ΔDCE. (e) No.
2. (a) $y = 180° - 135° = 45°$ (adj.$\angle$s on st. line)
   $x + y = 128°$ (ext.$\angle$of Δ), $\therefore$ $x = 128° - 45° = 83°$
   (b) $\angle C = 120° - 35° = 85°$ (ext.$\angle$of Δ). $x = \angle C = 85°$ (alt.$\angle$s, AB // CD)
   (c) $\angle STR = 180° - 40° - 80° = 60°$ ($\angle$ sum of Δ)
   $y = \angle STR = 60°$ (vert. opp.$\angle$s).
   $x + y = 110°$ (ext.$\angle$of Δ), $\therefore$ $x = 110° - 60° = 50°$
   (d) $\angle CEF = 180° - 108° = 72°$ (adj.$\angle$s on st. line),
   $\therefore$ $\angle BCA = 72° + 35° = 107°$ (ext.$\angle$of Δ),
   $\therefore$ $\theta = 54° + 107° = 161°$ (ext.$\angle$of Δ)
   (e) $\angle BAD = y$ (base $\angle$s, isos. Δ), $\angle BDC = 32°$ (base $\angle$s, isos. Δ)
   $\angle BAD + y = \angle BDC$ (ext. $\angle$of Δ), $\therefore$ $y + y = 32°$, $y = 16°$
   (f) reflex$\angle EFG = 360° - \theta$ ($\angle$s at a pt.)
   $28° + 56° + 46° + (360° - \theta) = 360°$ ($\angle$sum of quad.), $\therefore$ $\theta = 130°$
3. (a) The angle sum = $180°(4-2) = 360°$ (b) The angle sum = $180°(5-2) = 540°$
   (c) The angle sum = $180°(6-2) = 720°$ (d) The angle sum = $180°(8-2) = 1080°$
   (e) The angle sum = $180°(15-2) = 2340°$ (f) The angle sum = $180°(18-2) = 2880°$
4. (a) $x + 2x + 118° + (x + 23°) + 151° = 180° \times 3$ ($\angle$ sum of polygon), $4x = 248°$, $x = 62°$
   (b) $y + (y - 65°) + 119° + 108° + 90° = 180° \times 3$ ($\angle$ sum of polygon), $2y = 288°$, $y = 144°$
   (c) $x + 50° + (180° - 88°) + 100° = 360°$ (sum of ext. $\angle$s), $x = 118°$
   (d) $43° + a + 130° + 39° + (360° - 172°) = 180° \times 3$ ($\angle$ sum of polygon), $a = 140°$
5. (a) Let the no. of sides be n. $180(n-2) = 2520$, $n = \frac{2520}{180} + 2 = 16$
   (b) The no. of side $= \frac{3060}{180} + 2 = 19$ (c) The no. of side $= \frac{4860}{180} + 2 = 29$
6. Let x be the no. of sides. $180(x-2) = 16 \times 90$, $x - 2 = 8$, $x = 10$
   Ans. The number of sides is 10.
7. Let n be the no. of sides. $180(n-2) = 2 \times 180(14-2)$, $n - 2 = 24$, $n = 26$
   Ans. The number of sides is 26.
8. (a) A polygon is a closed area made up of straight lines.
   A regular polygon is a polygon with all sides equal and all angles equal.
   (b) Equilateral triangle, square, regular octagon.
9. (a) Each exterior angle = $180° - 140° = 40°$, $\therefore$ no. of sides $= \frac{360}{40} = 9$
   (b) No. of sides $= \frac{360}{180-156} = \frac{360}{24} = 15$ (c) No. of sides $= \frac{360}{180-172} = \frac{360}{8} = 45$
10. $37 + 88 + 96 + 101 + 144 + x + (\frac{2}{3}x - 11) = 180 \times (7-2)$, $\frac{5}{3}x = 445$, $x = 267$
11. Let the size of the remaining angle be x. $\therefore$ Each of the equal angles = $x + 42°$.
    $\therefore$ $5(x + 42°) + x = 180°(6-2)$, $6x = 510°$, $x = 85°$.
    Ans. The remaining angle is 85°.
12. The sum of interior angles = 360°,

$\therefore$ the largest angle = $360° \times \frac{6}{2+3+5+6} = 360° \times \frac{6}{16} = 135°$

13. (a) 360° (b) 360°

14. (a) no. of sides = $\frac{360}{45} = 8$ (b) no. of sides = $\frac{360}{30} = 12$

(c) no. of sides = $\frac{360}{15} = 24$

15. Let the no. of sides be *n*. $180(n-2) = 360$, $(n-2) = 2$, $n = 4$. *Ans. It has 4 sides.*

16. Let the no. of sides be *n*. $180(n-2) = 4 \times 360$, $(n-2) = 8$, $n = 10$

$\therefore$ The size of each interior angle = $\frac{4 \times 360°}{10} = 144°$

17. Let each exterior angle be *x*. $\therefore$ Each interior angle = $6x + 12°$

$\therefore$ $x + (6x + 12°) = 180°$, $7x = 168°$, $x = 24°$. $\therefore$ The no. of sides = $\frac{360}{24} = 15$

18. (a) In ΔACI, ∠JIE = *b* + *e* (ext. ∠ of Δ). In ΔBDJ, ∠IJE = *a* + *d* (ext. ∠of Δ).
In ΔJIE, ∠JIE + ∠IJE + *c* = 180° (∠ sum of Δ), $\therefore$ $b + e + a + d + c = 180°$

(b) In ΔACE, *a* + *c* + *e* = 180° (∠sum of Δ). In ΔBDF, *b* + *d* + *f* = 180° (∠ sum of Δ).
$\therefore$ $a + b + c + d + e + f = 180° + 180° = 360°$

(c) ∠AIG = *e* + *f* (ext. ∠of Δ); ∠EGH = *a* + *b* (ext. ∠ of Δ);
∠CHI = *d* + *c* (ext. ∠of Δ).
∠AIG + ∠EGH + ∠CHI = 360° (sum of ext.∠s), $\therefore$ *a* + *b* + *c* + *d* + *e* + *f* = 360°

(d) $a + b + c + d + e + f + g + h + i = 180°(10-2) = 1440°$ (∠sum of polygon)

(e) The sum of interior angles of the hexagon = $180°(6-2) = 720°$
$a + b + c + d + e + f = 360° \times 6 - 720° = 1440°$

19. The smallest exterior angle = $360° \times \frac{1}{1+2+3+4+5} = 360° \times \frac{1}{15} = 24°$

$\therefore$ The largest interior angle = $180° - 24° = 156°$

20. ∠AFE = *a* + *b* (ext. ∠ of Δ), ∠AFE + *c* = *d* (ext. ∠ of Δ)
$\therefore$ $(a + b) + c = d$, $c = d - a - b$

21. Let the original no. of sides be *n*.
$\therefore$ The no. of sides of the new polygon is 2*n*.
$\frac{180°(n-1)}{n} : \frac{180°(n-2)}{n} = 4:3$,
$\therefore$ $\frac{n-1}{n-2} = \frac{4}{3}$, $3n - 3 = 4n - 8$, $n = 5$, $\therefore$ $2n = 10$.

22. Angles at a point = 360°; to tessellate, the interior angle must be a factor of 360°.

Each interior angle of a regular hexagon = $\frac{180° \times (6-2)}{6} = 120°$ which is a factor of 360,

$\therefore$ regular hexagons can tessellate.

Each interior angle of a regular octagon = $\frac{180°(8-2)}{8} = 135°$ which is not a factor of 360,

$\therefore$ regular octagons cannot tessellate.

23. $b = 60°$ (alt. ∠s, // lines), $2a + 60° = 2b$ (ext. ∠ of Δ),
$\therefore$ $2a + 60° = 60° \times 2$, $a = 30°$

24. Let *a* = ∠QPS = ∠SPR, *b* =∠QRS =∠SRP
$\therefore$ $2a + 54° + 2b = 180°$ (∠ sum of Δ), *a* + *b* = 63°

$\theta + a + b = 180°$ ($\angle$ sum of Δ), $\therefore$ $\theta + 63° = 180°$, $\theta = 117°$

25. Let $x = \angle ABE = \angle EBC$, $y = \angle ACE = \angle ECD$
$\angle ACD = 80° + 2x$ (ext. $\angle$ of Δ), $\therefore$ $2y = 80° + 2x$, $y = 40° + x$, $y - x = 40°$
EBC + $\angle BEC = \angle ECD$ (ext. $\angle$ of Δ), $\therefore$ $x + \angle BEC = y$, $\angle BEC = y - x = 40°$

26. $(x - 5°) + 2(x + 10°) + x + (x + 30°) + [180° - (x + 25°)] = 360°$ (sum of ext. $\angle$s)
$\therefore$ $5x - x + 200° = 360°$, $4x = 160°$, $x = 40°$

27. $\angle C = y$ (corr. $\angle$s, DE // AC), $\therefore$ $x + y + 90° = 180°$ ($\angle$ sum of Δ), $x + y = 90°$ …... (i)
$x + 2y = 125°$ (ext. $\angle$ of Δ) ……. (ii)
(ii) – (i), $\therefore$ $2y - y = 125° - 90°$, $y = 35°$; $x = 90° - 35° = 55°$

28. $2x + y + 100° + x + y = 180° \times 3$ ($\angle$ sum of polygon), $\therefore$ $3x + 2y = 440°$ ……. (i)
$60° + 100° + x + y = 360°$ ($\angle$ sum of quad.), $x + y = 200°$ ……. (ii)
(i) – (ii) × 2, $\therefore$ $3x + 2y - 2(x + y) = 440° - 2 \times 200°$, $x = 40°$
$y = 200° - x = 200° - 40° = 160°$

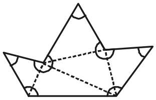
29. $\angle FAE = \angle AFE = 60°$ (equil. Δ). $\angle BAE = \dfrac{180° \times (5-2)}{5} = 108°$ ($\angle$ of regular polygon)
$\therefore$ $\angle BAF = 108° - 60° = 48°$. $\because$ AB = AE, AE = AF, $\therefore$ AB = AF
In ΔABF, $\angle AFB = \dfrac{180° - 48°}{2} = 66°$ ($\angle$ sum of isos. Δ)
Similarly, $\angle EFD = 66°$, $\angle AFB + 60° + \angle EFD + x = 360°$ ($\angle$s at a pt.)
$\therefore$ $x = 360° - 60° - 66° \times 2 = 168°$

30. Let $\angle A = \theta$. $\angle C = \angle A = \theta$ (base $\angle$s, isos. Δ), $\angle B = \angle A = \theta$ (base $\angle$s, isos. Δ)
$\angle BFC = \angle A + \angle B = \theta + \theta = 2\theta$ (ext. $\angle$ ofΔ).
$\because$ CE = CF , $\therefore$ $\angle CEF = \angle BFC = 2\theta$ (base $\angle$s, isos.Δ).
$\angle C + \angle CEF + \angle BFC = 180°$ ($\angle$ sum of Δ), $\angle\theta + 2\theta + 2\theta = 180°$, $\theta = 36°$, $\therefore$ $\angle A = 36°$

31. (a) $\angle EAB = \angle ABC = \dfrac{180° \times (5-2)}{5} = 108°$ ($\angle$ of regular polygon)
AB = BC, $\therefore$ $\angle BCA = \dfrac{180° - 108°}{2} = 36°$ ($\angle$ sum of isos.Δ)
Similarly, $\angle ABE = 36°$, $\therefore$ $\angle FBC = 108° - 36° = 72°$
$\angle EFC = \angle FBC + \angle BCA$ (ext. $\angle$of Δ), $\therefore$ $\angle EFC = 72° + 36° = 108°$
(b) $\angle AED = 108°$ (regular pentagon)
$\angle AEB = \angle ABE = 36°$ (base $\angle$s, isos. Δ), $\therefore$ $\angle FED = 108° - 36° = 72°$
$\angle EFC + \angle FED = 108° + 72° = 180°$, $\therefore$ ED // AC (int. $\angle$s supp.)

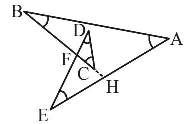
32. $\angle QPT + \angle PTS + (180° - 49°) + 103° + (180° - \theta) = 180° \times (5 - 2)$ ($\angle$ sum of polygon),
but $\angle QPT + \angle PTS = 180°$, (int. $\angle$s, QP // ST),
$\therefore$ $180° + 131° + 103° + 180° - \theta = 540°$, $\theta = 54°$

33. (a) A concave polygon has at least one interior angle greater than 180°(reflex angle).
Convex polygons have all interior angles smaller than 180°.
(b) Yes. Any n-sided concave polygons can be divided into $(n - 2)$ triangles. For example, the given 7-sided concave polygon can be divided into 5 triangles.



(c) An exterior $\angle$ = 180° – its adjacent interior $\angle$,
$\therefore$ the reflex $\angle$of a concave polygon has no adjacent exterior $\angle$,
$\therefore$ we can't find the sum of exterior angles of a concave polygon.

34. Let the equal angles be x.



Produce BC to H.
$\angle BHE = \angle B + \angle A = x + x = 2x$ (ext. ∠of Δ)
$\angle EFC = \angle D + \angle DCF = x + x = 2x$ (ext. ∠of Δ)
$\angle E + \angle BHE + \angle EFC = 180°$ (∠ sum of Δ),
∴ $x + 2x + 2x = 180°$, $5x = 180°$, $x = 36°$
Ans. The size of each of these equal angles is 36°.

35. In ABND = $a + b + d + \angle MND = 360°$ (∠ sum of quad.)
In CMN = $c + \angle CMN + \angle MNC = 180°$ (∠ sum of Δ)
In EFGM = $e + f + g + \angle LMN = 360°$ (∠ sum of quad.)
∴ $a + b + c + d + e + f + g + \angle MND + \angle MNC + \angle CMN + \angle LMN$
$= 360° + 180° + 180° + 360° = 900°$,
but $\angle MND + \angle MNC = 180°$ and $\angle CMN + \angle LMN = 180°$ (adj. ∠s on st. line),
∴ $a + b + c + d + e + f + g + 180° + 180° = 900°$
sum of the marked angles = $900° - 180° - 180° = 540°$

36. In ΔACS, $a + \frac{a}{2} + b = 180°$ (∠sum of Δ), $b = 180° - \frac{3}{2}a$ ……. (i)
In ΔBER, $\angle SRD = \frac{b}{3} + (a + 4°)$ (ext. ∠ of Δ)
In ΔSRD, $b = \angle SRD + 3a - 14°$ (ext. ∠of Δ)
∴ $b = [\frac{b}{3} + (a + 4°)] + 3a - 14°$, $\frac{2}{3}b = 4a - 10°$ ……. (ii)
Put (i) into (ii), $\frac{2}{3}(180° - \frac{3}{2}a) = 4a - 10°$,
$120° - a = 4a - 10°$, $5a = 130°$, $a = 26°$, ∴ $b = 180° - \frac{3}{2}(26°) = 141°$

37. In ABCDEJ, $a + b + c + d + e$ + reflex $\angle EJA = 180° \times 4$ (∠ sum of polygon)
In FGHIJ, $f + g + h + i + \angle FJI = 180° \times 3$ (∠ sum of polygon)
∴ $a + b + c + d + e + f + g + h + i$ + reflex $\angle EJA + \angle FJI$
$= 180° \times 4 + 180° \times 3 = 180° \times 7$
But $\angle FJI = \angle EJA$ (vert. opp.∠s),
∴ reflex ∠EJA ∠FJI = reflex ∠EJA + ∠EJA = 360° (∠s at a pt.)
∴ Sum of the marked angles = $180° \times 7 - 360° = 900°$

38. (a) BC = AB (regular polygon) = BI (given),
∴ $\angle BIC = \angle BCI$ (base ∠s, isos. Δ) = 52°, $\angle CBI = 180° - 52° - 52° = 76°$
$\angle ABC = \frac{(8-2)\times 180°}{8}$ (∠ of regular polygon) = 135°
$\angle ABI = \angle ABC - \angle CBI = 135° - 76° = 59°$,
$\angle AIB = \angle BAI$ (base ∠s, isos. Δ)
$= \frac{180° - 59°}{2}$ (∠ sum of Δ) = 60.5°

(b) Join AC. ∵ AB = BC (given),
∴ $\angle BAC = \angle ACB$ (base ∠s, isos. Δ)
$= \frac{180° - 135°}{2}$ (∠ sum of Δ) = 22.5°
$\angle CAI = \angle BAI - \angle BAC = 60.5° - 22.5° = 38°$
$\angle ACI = \angle BCI - \angle ACB = 52° - 22.5° = 29.5°$,
∵ $\angle ACI \neq \angle CAI$, ∴ AI ≠ CI. Thus, the claim is disagreed.

39. (a) $\angle CDE = \dfrac{(5-2)\times 180°}{5} = 108°$ (∠ of regular polygon)

$\angle EDI = \dfrac{(6-2)\times 180°}{6} = 120°$

$\angle JDK = \angle CDE + \angle EDI - 180° = 108° + 120° - 180° = 48°$

(b) $\angle BCD = \angle CBK = 108°$,

$\angle BKD + 48° + 108° + 108° = (4 - 2) \times 180°$ (∠ sum of polygon),

$\angle BKD = 96°$, $\angle DJK = 180° - 48° - 96° = 36°$ (∠ sum of Δ)

(c) When ∠CDI is an interior angle of an *n*-sided regular polygon, its corresponding exterior angle is ∠JDK which is 48° [from part (a)].

$n = \dfrac{360}{48} = 7.5$ which is not an integer. Thus, no.

40. (a) $\angle ABC = \dfrac{(5-2)\times 180°}{5} = 108°$ (∠ of regular polygon)

(b) Let *n* be the number of sides of Y. $\angle CBF = \dfrac{108°}{2}\times 3 = 162°$

$(n - 2) \times 180° = 162° \times n$, $180n - 360 = 162n$, $n = 20$.

Ans. The number of sides of Y is 20.

(c) $\angle ABF = 360° - \angle ABC - \angle CBF$ (∠s at a pt.) $= 360° - 108° - 162° = 90°$.

AB = BF (regular polygon),

∴ ∠BAF = ∠AFB (base ∠s, isos. Δ)

$= \dfrac{180° - 90°}{2}$ (∠ sum of Δ) = 45°

41. (a) Size of an interior angle of ABCDE $= \dfrac{(5-2)\times 180°}{5} = 108°$

∴ ∠ABC = ∠BCD = ∠CDE = 108°

∵ CD = DE (regular polygon),

∴ ∠DCE = ∠CED (base ∠s, isos. Δ)

$= \dfrac{180° - 108°}{2}$ (∠ sum of Δ) = 36°

$\angle BCE = \angle BCD - \angle DCE = 108° - 36° = 72°$

$\angle ABC + \angle BCE = 108° + 72° = 180°$, ∴ AB // CE (int. ∠s supp.)

(b) Note that ∠ADE = ∠DAE = 36°.

$\angle CGD = \angle ADE + \angle CED = 36° + 36° = 72°$.

$\angle CDG = 180° - 72° - 36°$ (∠ sum of Δ)

$= 72°$

$= \angle CGD$

∴ CG = CD (sides opp., eq. ∠s), and CD = BC (regular polygon)

∴ BC = CG

(c) BC = CG, ∴ ∠BGC = ∠CBG (base ∠s, isos. Δ),

$\angle BGC = \dfrac{180° - 72°}{2}$ (∠ sum of Δ) = 54°

But ∠ CGD = 72° ≠ ∠ BGC, ∴ the claim is disagreed.

Unit 9 Rate, ratio & proportion

1. (a) $= \frac{1200}{25} = 48$ (b) $= \frac{128}{32} = 4$ (c) $= \frac{48}{30} = 1.6$

2. 90 km/day $= \frac{90 \times 1\,000 \text{ m}}{24 \times 60 \text{ min}} = 62.5$ m/min, $\therefore$ 70 m/min is the higher rate

3. Speed $= \frac{15}{6} = 2.5$ m/s, $\therefore$ Time taken $= \frac{100}{2.5} = 40$ s

4. (a) Time taken $= \frac{100}{80} = 1.25$ h

   (b) Distance travelled $= 80 \times \frac{3}{4} = 60$ km

5. (a) $= \frac{96}{24} : \frac{360}{24} = 4 : 15$

   (b) $= 9.8 \times 10 : 4.2 \times 10 = \frac{98}{14} : \frac{42}{14} = 7 : 3$

   (c) $= \frac{3}{10} : \frac{32}{15} = \frac{3}{10} \times 30 : \frac{32}{15} \times 30 = 9 : 64$

   (d) $= (3 \times 60 \times 60)$ seconds : 200 seconds $= \frac{10\,800}{200} : \frac{200}{200} = 54 : 1$

   (e) $= (2.5 \times 100)$ ¢ : 15 ¢ $= \frac{250}{5} : \frac{15}{5} = 50 : 3$

   (f) $= 240$ g : (1.2×1000) g $= \frac{240}{240} : \frac{1\,200}{240} = 1 : 5$

   (g) $= (3 \times 1000)$ m : 800 m $= \frac{3\,000}{200} : \frac{800}{200} = 15 : 4$

   (h) $= (0.4 \times 10000)$ cm^2 : 500 cm^2 $= \frac{4\,000}{500} : \frac{500}{500} = 8 : 1$

6. (a) $\frac{x}{3} = \frac{7}{2}$, $2x = 21$, $\therefore x = 10\frac{1}{2}$

   (b) $\frac{6}{y} = \frac{5}{4}$, $24 = 5y$, $\therefore y = 4\frac{4}{5}$

7. (a) $\frac{800\text{g}}{1\,000\text{g}} = \frac{\$a}{\$120}$, $\frac{4}{5} = \frac{a}{120}$, $480 = 5a$, $\therefore a = 96$

   (b) $\frac{(7 \times 100\,000) \text{ cm}}{3 \text{ cm}} = \frac{(2 \times 100\,000) \text{ cm}}{k \text{ cm}}$, $7k = 6$, $\therefore k = \frac{6}{7}$

8. Let x be the original number, new number : original number

   $= x(1 + \frac{1}{8}) : x = \frac{9x}{8} : x = \frac{9}{8} \times 8 : 1 \times 8 = 9 : 8$

9. Paul's weight : his brother's weight $= 48 : (48 - 16) = \frac{48}{16} : \frac{32}{16} = 3 : 2$

10. (a) Speed of car $= 33 \div \frac{1}{3} = 99$ km/h; speed of train $= \frac{270}{2} = 135$ km/h

    (b) Speed of car : speed of train $= 99 : 135 = \frac{99}{9} : \frac{135}{9} = 11 : 15$

11. Let x be the number of boys in the group, $\frac{x}{x+10}=\frac{4}{5}$, $5x = 4x + 40$, $x = 40$.
    Ans. Number of boys in the group is 40.
12. Let \$$x$ be his monthly salary, $x(\frac{3}{5+3})=4\,800$, $3x = 4\,800 \times 8$, $x = 12\,800$.
    Ans. His monthly salary is \$12800.
13. Let x litres be the volume of water added, $\frac{6}{4+x}=\frac{2}{3}$, $18 = 2(4 + x)$, $10 = 2x$, $x = 5$.
    Ans. 5 litres of water must be added.
14. New salary rate = \$$\frac{276}{10}$ /h = \$27.6 / h,
    $\therefore$ New salary : original salary = 27.6 : 24 = $\frac{276}{12}:\frac{240}{12}$ = 23 : 20
15. (a) $x : y = 5 \times 5 : 3 \times 5 = 25 : 15$, $y : z = 5 \times 3 : 2 \times 3 = 15 : 6$,
    $\therefore x : y : z = 25 : 15 : 6$
    (b) $y : x = 4 : 3$, $x : z = 1 \times 3 : 2 \times 3 = 3 : 6$, $\therefore x : y : z = 3 : 4 : 6$
16. (a) $4a = 3b$, $\frac{a}{b}=\frac{3}{4}$, $\therefore a : b = 3 : 4$
    (b) $\frac{4}{5}=\frac{a}{b}$, $\therefore a : b = 4 : 5$
    (c) $7b = a$, $\frac{7}{1}=\frac{a}{b}$, $\therefore a : b = 7 : 1$
    (d) $4a + 2b = 21a - 7b$, $9b = 17a$, $\frac{9}{17}=\frac{a}{b}$, $\therefore a : b = 9 : 17$
17. $x : 12 = 4 : y$, $\frac{x}{12}=\frac{4}{y}$, $\therefore xy = 48$
18. A : B = 1 : 3, B : C = 1 : 2 = 1 × 3 : 2 × 3 = 3 : 6, $\therefore$ A : B : C = 1 : 3 : 6
    $\therefore$ The amount B gets = $500\times\frac{3}{1+3+6}$ = $500\times\frac{3}{10}$ = \$150
19. The scale = 4 cm : 2 km = 4 cm : 200 000 cm = 1 : 50 000
20. Map distance of a runway = $2.5\text{ km}\times\frac{1}{100\,000}$ = $\frac{250\,000\text{ cm}}{100\,000}$ = 2.5 cm
21. Actual distance = 3 mm × 2 500 000 = 7 500 000 mm = 7.5 km
22. Length of the hall = 5 cm × 400 = 2 000 cm = 20 m
    Width of the hall = 4 cm × 400 = 1 600 cm = 16 m
    $\therefore$ Actual area of the hall = 20 × 16 = 320 m^2
23. Let \$$x$ be Samuel's pocket money for 28 days. By direct proportion:
    $\frac{x}{165}=\frac{28}{3}$, $x=\frac{28}{3}\times 165=1540$ *Ans. The required amount is \$1540.*
24. His original average daily expenditure = \$3 500 ÷ 7 = \$500
    Let \$$x$ be his new average daily expenditure. By inverse proportion:
    $\frac{x}{500}=\frac{21}{21-6}$, $x=\frac{21}{15}\times 500=700$
    Ans. His greatest average daily expenditure will be \$700.
25. (a) We check all pairs of $\frac{x}{y}$: $\frac{4}{18}=\frac{2}{9}$; $\frac{6}{27}=\frac{2}{9}$; $\frac{10}{45}=\frac{2}{9}$
    $\because \frac{x_1}{y_1}=\frac{x_2}{y_2}$ = constant, $\therefore \frac{x_1}{x_2}=\frac{y_1}{y_2}$ for all pairs of x, y
    $\therefore$ x and y are in direct proportion.

(b) We check all pairs of $\frac{x}{y}$: $\frac{5}{15}=\frac{1}{3}$; $\frac{15}{35}=\frac{3}{7}\neq\frac{1}{3}$

$\because$ $\frac{y}{x}$ is a not constant, $\therefore$ x and y are in not direct proportion.

When x increases, y does not decrease, $\therefore$ x and y are not in inverse proportion.

Ans. x and y are neither in direct proportion nor in inverse proportion.

(c) We check all pairs of xy:

$9 \times 28 = 252$; $12 \times 21 = 252$; $24 \times 10.5 = 252$

$\because$ $x_1y_1 = x_2y_2$ = constant, $\therefore$ $\frac{x_1}{x_2}=\frac{y_2}{y_1}$ for all pairs of x, y.

$\therefore$ x and y are in inverse proportion.

26. Distance travelled = $60\times\frac{90}{60}$ = 90 km,

$\therefore$ Volume of petrol used = $\frac{90}{6}$ = 15 litres

27. No. of \$1 coins = $240\times\frac{8}{8+3+4}$ = 128, no. of 50¢ coins = $240\times\frac{3}{8+3+4}$ = 48,

no. of 20¢ coints = $240\times\frac{4}{8+3+4}$ = 64

$\therefore$ Total amount = 128 × 1 + 48 × 0.5 + 64 × 0.2 = \$164.8

28. Average speed = $\frac{5}{1000}\text{km}\div\frac{3}{3\,600}\text{h} = \frac{5}{1000}\times\frac{3\,600}{3}$ km/h = 6 km/h

29. Let $2k$ be the present age of Mary, then the present age of Lily is $3k$.

$\frac{2k+4}{3k+4}=\frac{5}{7}$, $14k + 28 = 15k + 20$, $k = 8$, $\therefore$ $2k = 16$, $3k = 24$.

Ans. The present ages of Mary and Lily are 16 and 24 respectively.

30. Let $3k$ be the length of the smaller part, then the length of the larger part is $7k$.

$\therefore$ Ratio of the three parts = $3k : 7k\times\frac{2}{2+3} : 7k\times\frac{3}{2+3}$

$= 3k\times 5 : \frac{14k}{5}\times 5 : \frac{21k}{5}\times 5$ = 15 : 14 : 21

31. $\frac{4}{a+1}\div\frac{3}{a}=\frac{7}{6}$, $\frac{4a}{3(a+1)}=\frac{7}{6}$, $24a = 21a + 21$, $3a = 21$, $\therefore$ $a = 7$

32. $a : b = 3 \times 25 : 2 \times 25 = 75 : 50$, $a : c = \frac{5}{2}\times 30 : \frac{2}{5}\times 30$ = 75 : 12,

$\therefore$ $a : b : c = 75 : 50 : 12$, $b : c = 50 : 12 = 25 : 6$; but $b : d = 6 : 7$,

$\therefore$ $b : c : d = 25 \times 6 : 6 \times 6 : 7 \times 25 = 150 : 36 : 175$, $\therefore$ $c : d = 36 : 175$

33. $2p = 3q$, $p : q = 3 : 2 = 3 \times 9 : 2 \times 9 = 27 : 18$,

$8q = 9r$, $q : r = 9 : 8 = 9 \times 2 : 8 \times 2 = 18 : 16$, $\therefore$ $p : q : r = 27 : 18 : 16$

34. $\frac{1}{a}:\frac{1}{b}=3:4$, $\frac{1}{a}\div\frac{1}{b}=\frac{3}{4}$, $\frac{b}{a}=\frac{3}{4}$, $\therefore$ $a : b = 4 : 3$, $\frac{1}{b}:\frac{1}{c}=4:5$, $\frac{1}{b}\div\frac{1}{c}=\frac{4}{5}$,

$\frac{c}{b}=\frac{4}{5}$, $\therefore$ $b : c = 5 : 4$, $\therefore$ $a : b : c = 4 \times 5 : 3 \times 5 : 4 \times 3 = 20 : 15 : 12$

(OR: $\frac{1}{a}$ = $3k$, $\frac{1}{b}$ = $4k$, $\frac{1}{c}$ = $5k$,

$\therefore\ a : b : c = \frac{1}{3k} : \frac{1}{4k} : \frac{1}{5k} = \frac{1}{3k} \times 60k : \frac{1}{4k} \times 60k : \frac{1}{5k} \times 60k = 20 : 15 : 12$

35. Length scale of map = 1 mm : 0.25 m = 0.1 cm : 0.25 m = 1 cm : 2.5 m
    Area scale of map = $(1\times1)\text{ cm}^2 : (2.5\times2.5)\text{ m}^2 = 1\text{ cm}^2 : 6.25\text{ m}^2$
    $\therefore$ Map area of the field = $400\times\frac{1}{6.25} = 64\text{ cm}^2$
36. Area scale of map = $(4\times4)\text{ cm}^2 : (1\times1)\text{ km}^2 = 16\text{ cm}^2 : 1\text{ km}^2$
    Map area = $(2 \times 3)\text{ cm}^2 = 6\text{ cm}^2$, $\therefore$ Actual area = $6\times\frac{1}{16} = \frac{3}{8}\text{ km}^2$
37. $x = y(1+25\%)$, $\frac{x}{y} = 1.25 = \frac{5}{4}$, $\therefore\ x : y = 5 : 4$
    $x = z(1-20\%)$, $\frac{x}{z} = 0.8 = \frac{4}{5}$, $\therefore\ x : z = 4 : 5$
    $\therefore\ x : y : z = 5 \times 4 : 4 \times 4 : 5 \times 5 = 20 : 16 : 25$
38. $2a + 3b = 3a + b$, $a = 2b$
    $\therefore\ (4a + b) : (3a - 2b) = [4(2b + b)] : [3(2b) - 2b] = 9b : 4b = 9 : 4$
39. Let $2a = 3b = 7c = k$, then $a = \frac{k}{2}$, $b = \frac{k}{3}$, $c = \frac{k}{7}$.
    $\therefore\ (a-b+c) : (a+b-c) = (\frac{k}{2}-\frac{k}{3}+\frac{k}{7}) : (\frac{k}{2}+\frac{k}{3}-\frac{k}{7}) = \frac{13}{42} : \frac{29}{42} = 13 : 29$
40. Let $\frac{a}{b} = \frac{c}{d} = k$, then $a = bk$, $c = dk$.
    L.H.S. = $\frac{a+c}{b+d} = \frac{bk+dk}{b+d} = \frac{k(b+d)}{b+d} = k$,
    R.H.S. = $\frac{a-c}{b-d} = \frac{bk-dk}{b-d} = \frac{k(b-d)}{b-d} = k$ = L.H.S., $\therefore\ \frac{a+c}{b+d} = \frac{a-c}{b-d}$
41. $\because$ The largest angle = $180°\times\frac{8}{3+8+1} = 180°\times\frac{8}{12} = 120° > 90°$
    $\therefore$ It is an obtuse-angled triangle.
42. Let $3k$ cm and $4k$ cm be the width and the length of the rectangle respectively.
    $(3k)(4k) = 432$, $12k^2 = 432$, $k^2 = 36$, $k = 6$, $\therefore\ 3k = 18$, $4k = 24$.
    Ans. Perimeter of the rectangle is 2(18+24) = 84 cm.
43. Let $r : 1$ be the ratio of the two types of tea.
    $75(\frac{r}{1+r}) + 50(\frac{1}{1+r}) = 60$, $75r + 50 = 60(1+r)$, $15r = 10$, $r = \frac{2}{3}$.
    Ans. The ratio of the mixture is $\frac{2}{3} : 1 = 2 : 3$.
44. The cost of the mixture = 60 ÷ (1 + 25%) = \$48.
    Let the cost price of coffee B be \$$y$.
    $15 \times \frac{2}{2+3} + y \times \frac{3}{2+3} = 48$, $30 + 3y = 240$, $y = 70$.
    Ans. The cost price of coffee B is \$70/kg.
45. $150\times\frac{m}{m+n} + 200\times\frac{n}{m+n} = 150(1+10\%)\times\frac{m}{m+n} + 200(1-30\%)\times\frac{n}{m+n}$,
    $150m + 200n = 165m + 140n$, $60n = 15m$, $\frac{m}{n} = \frac{60}{15} = \frac{4}{1}$, $\therefore\ m : n = 4 : 1$

46. Let \$2k and \$$k$ be the daily wages of a man and a woman respectively.

 $\therefore$ Ratio of hourly wages of a man and a woman $= \dfrac{2k}{10} : \dfrac{k}{8} = \dfrac{1}{5}\times 40 : \dfrac{1}{8}\times 40 = 8 : 5$

47. Let d m be the distance between home and school.

 Total time taken $= \dfrac{d}{x} + \dfrac{d}{y} = \dfrac{d(x+y)}{xy}$ seconds

 $\therefore$ His average speed $= 2d \div \dfrac{d(x+y)}{xy} = 2d \times \dfrac{xy}{d(x+y)} = \dfrac{2xy}{x+y}$ m/s

48. Let w g be the weight of the cube with side length 7 cm.

 By direct proportion, $\dfrac{w}{70} = \dfrac{7^3}{4}$, $w = 6002.5$. *Ans. It weighs 6002.5 g.*

49. Let t minutes be the time required to fill up the tank if 2 more pumps are used.

 By inverse proportion, $\dfrac{t}{30} = \dfrac{10}{10+2}$, $t = 25$. $\therefore$ Time saved = 30 – 25 = 5 min.

50. (a) Let x be the required number of workers.

 By inverse proportion, $\dfrac{x}{6} = \dfrac{3}{2}$, $x = 9$. *Ans. 9 workers are required,*

 (b) Let y be the required number of workers.

 By direct proportion, $\dfrac{y}{6} = \dfrac{1500\times 0.8\times 1}{800\times 1\times 0.6}$, $y = 15$. *Ans. 15 workers are required.*

 (c) Let z be the required number of workers.
 Originally work done per day = $800\times 1\times 0.6\div 3 = 160$ m^3.
 Now work done per day = $1200\times 2\times 1.8\div 6 = 720$ m^3.

 By direct proportion, $\dfrac{z}{6} = \dfrac{720}{160}$, $z = 27$. *Ans. 27 workers are required.*

51. Let x litres be the amount of water.

 Amount of water 1 man needs per day $= \dfrac{x}{6\times 8} = \dfrac{x}{48}$ litres

 Amount of water 1 boy needs per day $= \dfrac{x}{8\times 10} = \dfrac{x}{80}$ litres

 $\therefore$ Time lasting $= x \div (\dfrac{x}{48}\times 12 + \dfrac{x}{80}\times 4) = x \div \dfrac{3x}{10} = x \times \dfrac{10}{3x} = 3\dfrac{1}{3}$ days

52. Work done by A : work done by B : work done by C

 $= \dfrac{1}{10} : \dfrac{1}{20} : \dfrac{1}{30} = \dfrac{60}{10} : \dfrac{60}{20} : \dfrac{60}{30} = 6 : 3 : 2$

 $\therefore$ The amount A will receive $= 11\,000 \times \dfrac{6}{6+3+2} = \$6\,000$

53. If there are 60 chickens, the remaining food can last for 30 – 10 = 20 days.

 $\therefore$ The number of days for 50 chickens $= \dfrac{60\times 20}{50} = 24$.

 Ans. The remaining food can last for 24 – 20 = 4 days more.

54. Let BQ = x, then QC = $36 - x$.

 $\dfrac{(18+x)\times \text{AB}}{2} : \dfrac{[18+(36-x)]\times \text{AB}}{2} = 3 : 2$,

 $\dfrac{18+x}{54-x} = \dfrac{3}{2}$, $36 + 2x = 162 - 3x$, $5x = 126$, $x = 25.2$.

 Ans. The value of BQ is 25.2.

55. Let the longer side and the shorter side of the original rectangle be b cm and a cm respectively.
The longer side of the new rectangle will be a cm, and the shorter side will be $\frac{b}{2}$ cm.

$a:\frac{b}{2}=b:a$, $\frac{2a}{b}=\frac{b}{a}$, $2a^2=b^2$, $\frac{a^2}{b^2}=\frac{1}{2}$, $\frac{a}{b}=\frac{1}{\sqrt{2}}$.

Ans. *The ratio of lengths of the original rectangle is* $1:\sqrt{2}$. [OR: $\sqrt{2}:1$]

56. Let AB = x, ∴ BC = $2x$. Let PQ = $3y$, ∴ QR = $2y$.

$2(x+2x)=2(3y+2y)$, $3x=5y$, $x=\frac{5y}{3}$

∴ Area of ABCD : area of PQRS

$= x(2x) : (3y)(2y) = x^2 : 3y^2 = (\frac{5y}{3})^2 : 3y^2 = \frac{25}{9} : 3 = \frac{25}{9}\times 9 : 3\times 9 = 25 : 27$

57. (a) The exchange rate for pound to HK dollar = 49 ÷ 5 = \$9.8/£
The exchange rate for pound to yen = 18 × 9.8 = ¥176.4/£

(b) (i) The price of the Japanese scarf = 26 460 ÷ 18 = \$1 470
The price of the U.K. scarf = 165 × 9.8 = \$1 617

(ii) The new exchange rate be ¥ n /\$
26 460 ÷ n = 1617, n = 16.3.
Ans. *The new exchange rate is ¥16.3/\$.*

58. (a) Richard's average cycling speed $= \dfrac{98\times 1000}{\left(2+\frac{20}{60}\right)\times 60\times 60} = 11\frac{2}{3}$ m/s

(b) The distance he cycles in 45 minutes = 11.7 × 45 × 60 = 31 500 m < 35 km
∴ He cannot complete 35 km in 45 minutes.

(c) The time Steven required to finish 98 km

$= 98\,000 \div \left(11\frac{2}{3}+0.6\right)$ s = 7 989 s (4 sig. fig.)

2 hours 15 minutes = 8 100 s > 7989 s. *Ans.* *The claim is agreed.*

2 hours and 15 minutes = 2.25 hours > 2.2192 hours. ∴ The claim is agreed.

59. (a) (i) Actual length = 12 ÷ 4 × 3 = 9 km
Actual width = 5 ÷ 4 × 3 = 3.73 km

(ii) Area of the field on the map = 12cm × 5cm = 60cm^2
Actual area of the field
= 9 km × 3.75 km
= (9 × 1 000 × 100) × (3.75 × 1 000 × 100)cm^2 = 337 500 000 000 cm^2
The required ratio = 337 500 000 000 : 60 = 5 625 000 000 : 1

(b) Longer side of the paper : longer side of the field on the original map
= 30cm : 12cm = 2.5 :1
Shorter side of the paper : shorter side of the field on the original map
= 21cm : 5cm = 4.2 : 1
2.5 < 4.2, ∴ we only need to consider the longer side of the field.
30cm : 9km = 30 : 900 000 = 1 : 30 000, ∴ n = 30 000

60. (a) $x : y = 2 : 3 = 6 : 9$, $x : z = 3 : 1 = 6 : 2$, ∴ $x : y : z = 6 : 9 : 2$

(b) (i) From (a), apple juice : orange juice : lemon juice = 6 : 9 : 2

∴ volume of orange juice required $= \frac{9}{6+9+2}\times 5.1 = \frac{9}{17}\times 5.1$ L = 2.7 L

(ii) Let the volume of apple juice, orange juice and lemon juice be $6k$ L, $9k$ L and $2k$ L respectively.
If $6k = 7.5$, $k = 1.25$. If $9k = 12$, $k = 1.33$. If $2k = 3$, $k = 1.5$.
Each type of juice would be enough for the mixture only when we take the smallest value of k, ∴ we should take $k = 1.25$.
∴ The volume of mixed-fruit juice = (6 + 9 + 2)(1.25) L = 21.25 L
The volume of apple juice = 6 (1.25) L = 7.5 L
The volume of orange juice = 9 (1.25) L = 11.25 L
The volume of lemon juice = 2 (1.25) L = 2.5 L

61. (a) 1 h 30 min = 90 min; 2 h 40 min = 160 min
By inverse proportion, $x : y = 160 : 90 = 16 : 9$
(b) (i) Let $x = 16k$, $y = 9k$, where $k \neq 0$.
$(16k + 9k) \times \frac{36}{60} = 225$, $25k = 375$, $k = 15$.
Speed of pump A = 16(15) = 240 m^3/h
∴ Capacity of the tank = 240(1.5) = 360 m^3
(ii) Speed of filling the tank when pumps A and B are used simultanouesly
$= \frac{225}{36} = 6\frac{1}{4}$ m^3/ min > 6 m^3/ min
The speed of the new pump is slower, and thus more time is required.
∴ The claim is disagreed.

Unit 10 Similar triangles

1. (a) No. (∵∠B and ∠P are not the included angles)
(b) Yes. ΔABC ~ ΔQRP (∠A = 180° – 75° – 40° = 65°, ∴ AAA)
(c) Yes. ΔABC ~ ΔRQP (ratio of 2 sides, inc. ∠)
(d) No. (∵ 3 sides are not proportional.)
2. ∠RQP = 180° – 69° – 37° = 74° (∠ sum of Δ); ∠EDC = ∠RQP = 74° (corr. ∠s, ~Δs)
3. (a) YX = QR = 13cm (corr. sides, ~Δs)
(b) ∠YXZ = ∠QRP = 44° (corr. ∠s, ~Δs)
(c) ∠PQR = 180° – 47° – 44° = 89° (∠ sum of Δ); ∠ZYX = ∠ PQR = 89° (corr. ∠s, ~Δs)
4. $\frac{BC}{PR} = \frac{AC}{QR} = \frac{AB}{QP}$ (corr. sides, ~Δs), $\frac{x}{6} = \frac{y}{7} = \frac{10}{4}$,
∴ $x = \frac{10}{4} \times 6 = 15$; $y = \frac{10}{4} \times 7 = 17.5$; $z = \angle B = 86°$ (corr. ∠s, ~Δs)
5. (a) $\frac{m}{8} = \frac{n}{6} = \frac{15}{10}$ (corr. sides, ~Δs), ∴ $m = \frac{15}{10} \times 8 = \underline{\underline{12}}$, $n = \frac{15}{10} \times 6 = \underline{\underline{9}}$
(b) ΔABC~ΔQPR (AAA), $\frac{AC}{QR} = \frac{BC}{PR} = \frac{AB}{QP}$ (corr. sides, ~Δs), $\frac{x}{6} = \frac{6}{y} = \frac{12}{8}$
∴ $x = \frac{12}{8} \times 6 = 9$; $y = 6 \times \frac{8}{12} = 4$
(c) $\frac{x}{10} = \frac{y}{6} = \frac{12}{8}$ (corr. sides, ~Δs), ∴ $x = \frac{12}{8} \times 10 = 15$; $y = \frac{12}{8} \times 6 = 9$
(d) $\frac{7}{4} = \frac{3h+1}{5} = \frac{2k-3}{4}$ (corr. sides, ~Δs),

$\therefore\ 35 = 12h + 4,\quad h = \frac{31}{12};\quad 28 = 8k - 12,\quad k = \frac{30}{8} = \frac{15}{4}$

6. (a) ΔEFG ~ ΔKFH (AAA), $\frac{\text{EF}}{\text{KF}} = \frac{\text{EG}}{\text{KH}}$ (corr. sides, ~Δs),

$\frac{m+6}{6} = \frac{28}{16},\quad m+6 = \frac{28}{16}\times 6 = 10.5,\quad \therefore\ m = 4.5$

(b) ΔPQR ~ ΔSQT (AAA), $\frac{\text{PQ}}{\text{QR}} = \frac{\text{SQ}}{\text{QT}}$ (corr. sides, ~Δs),

$\frac{z+6}{z} = \frac{3+9}{3} = 4,\quad z+6 = 4z,\quad 3z = 6,\quad \therefore\ z = 2$

7. (a) ΔABC ~ ΔAED (AAA), $\frac{\text{AB}}{\text{AE}} = \frac{\text{AC}}{\text{AD}}$ (corr. sides, ~Δs),

$\frac{y+6}{8} = \frac{8+7}{6} = \frac{5}{2},\quad 2y + 12 = 40,\quad 2y = 28,\quad \therefore\ y = 14$

(b) ΔPQR ~ ΔNMR (AAA), $\frac{\text{NR}}{\text{PR}} = \frac{\text{NM}}{\text{PQ}}$ (corr. sides, ~Δs),

$\frac{r}{50} = \frac{8}{20},\quad \therefore\ r = \frac{8}{20}\times 50 = 20$

(c) ΔPQR ~ ΔTQS (AAA), $\frac{\text{QR}}{\text{QS}} = \frac{\text{PQ}}{\text{TQ}}$ (corr. sides, ~Δs),

$\frac{18+x}{28} = \frac{28+8}{18} = 2,\quad 18 + x = 56,\quad \therefore\ x = 38$

8. (a) In ΔEFG and ΔKFH, $\angle$F = $\angle$F (common),
$\angle$E = $\angle$HKF and $\angle$G = $\angle$KHF (corr. $\angle$s, GE // HK),
$\therefore$ ΔEFG ~ ΔKFH (AAA), $\therefore$ $\frac{\text{EF}}{\text{KF}} = \frac{\text{EG}}{\text{KH}}$ (corr. sides, ~Δs),

$\frac{2+r}{r} = \frac{20}{16} = \frac{5}{4},\quad 8 + 4r = 5r,\quad \therefore\ r = 8$

(b) In ΔADE and ΔABC, $\angle$A = $\angle$A (common),
$\angle$ADE=$\angle$ABC and $\angle$AED = $\angle$ACB (corr. $\angle$s, DE//BC),
$\therefore$ ΔADE ~ ΔABC (AAA), $\therefore$ $\frac{\text{AE}}{\text{AC}} = \frac{\text{DE}}{\text{BC}}$ (corr. sides, ~Δs)

$\frac{y}{y+15} = \frac{8}{20} = \frac{2}{5},\quad 5y = 2y + 30,\quad 3y = 30,\quad \therefore\ y = 10$

(c) In ΔPQR and ΔFGR, $\angle$R = $\angle$R (common),
$\angle$P =$\angle$RFG and $\angle$Q = $\angle$RGF (corr. $\angle$s, PQ // FG), $\therefore$ ΔPQR ~ ΔFGR (AAA)
$\frac{\text{FG}}{\text{PQ}} = \frac{\text{FR}}{\text{PR}}$ (corr. sides, ~Δs), $\frac{a}{36} = \frac{9}{9+18} = \frac{1}{3},\quad 3a = 36,\quad \therefore\ a = 12$

9. (a) $\angle$ACB = $\angle$ECD (vert. opp. $\angle$s), $\angle$A = $\angle$E and $\angle$B = $\angle$D (alt. $\angle$s, AB // DE),
$\therefore$ ΔABC ~ ΔEDC (AAA), $\frac{\text{AB}}{\text{ED}} = \frac{\text{BC}}{\text{DC}} = \frac{\text{AC}}{\text{EC}}$ (corr. sides, ~Δs),

$\frac{8}{y} = \frac{x}{6} = \frac{3}{4},\quad \therefore\ x = \frac{3}{4}\times 6 = 4.5;\quad y = 8\times\frac{4}{3} = \frac{32}{3}$

(b) $\angle$PRQ = $\angle$TRS (vert. opp. $\angle$s), $\angle$P = $\angle$T and $\angle$Q = $\angle$S (alt. $\angle$s, PQ // ST),
$\therefore$ ΔPQR ~ ΔTSR (AAA), $\frac{\text{PQ}}{\text{TS}} = \frac{\text{QR}}{\text{SR}} = \frac{\text{PR}}{\text{TR}}$ (corr. sides, ~Δs),

$\frac{5}{k}=\frac{4}{10}=\frac{h}{12}$, $\therefore$ $h=\frac{4}{10}\times12=4.8$; $k=5\times\frac{10}{4}=12.5$

10. Let y m be the actual height. $\frac{y\text{ m}}{6\text{ cm}}=\frac{60\text{ m}}{8\text{ cm}}$, $y=\frac{60}{8}\times6=45$

Ans. The actual height is 45 m.

11. (a) $\frac{XZ}{XM}=\frac{XY}{XN}$ (corr. sides, ~Δs), $\therefore$ $\frac{a}{18}=\frac{6}{12}$, $a=\frac{6}{12}\times18=9$
    (b) ∵ ∠Z = ∠M (corr. ∠s, ~Δs), ∴ YZ // MN (alt. ∠s eq.)
12. (a) No. Their corr.∠s are equal, but their sides may not be proportional.
    (b) Yes. Their corr.∠s are equal, and their sides are proportional.
    (c) No. Their sides are proportional, but their corr.∠s may not be equal.
    (d) Yes. Their corr. ∠s are equal, and their sides are proportional (1:1).
13. $\frac{\text{Perimeter of the second }\Delta}{\text{Perimeter of the first }\Delta}=\frac{96}{5+3+4}=\frac{96}{12}=8$
    ∴ the longest side = 5 × 8 = 40 cm
14. ΔAEC ~ ΔADB (AAA), $\therefore$ $\frac{AC}{1.8}=\frac{40}{2.6}$, AC = 27.7 m (3 sig. fig.)
    Ans. The distance between the tree and the building = 27.7 – 1.8 = 25.9 m.
15. ∠BAC = ∠AED = 77° (corr. ∠s, ~Δs), ∠ADE = ∠BCA = 44° (corr. ∠s, ~Δs),
    ∴ ∠DAE = 180° – 77° – 44° = 59° (∠ sum of Δ), ∴ ∠BAD = 77° – 59 ° = 18°
16. (a) ∠Q = 180° – 80° – 32° = 68° = ∠B (∠ sum of Δ),
    ∠A = 180° – 68 ° – 80° = 32° = ∠R (∠ sum of Δ), ∠P =∠C = 80° (given),
    ∴ ΔPQR ~ ΔCBA (AAA)
    (b) $\frac{AB}{RQ}=\frac{AC}{RP}=\frac{BC}{QP}$ (corr. sides, ~Δs), $\frac{m}{9}=\frac{n}{13}=\frac{3}{6}$,
    $\therefore$ $m=\frac{3}{6}\times9=4.5$, $n=\frac{3}{6}\times13=6.5$
17. (a) ∠BCA + ∠BCD = ∠CDE + ∠CED (ext. ∠ of Δ),
    ∠BCA + 90° = ∠CDE + 90°, ∴ ∠BCA = ∠CDE
    (b) ∠A = ∠E = 90° (given), ∠BCA = ∠CDE (proved),
    ∠ABC = ∠ECD (∠ sum of Δ), ∴ ΔABC ~ ΔECD (AAA)
    (c) $\frac{CE}{BA}=\frac{CD}{BC}$ (corr. sides, ~Δs), $\frac{CE}{10}=\frac{20}{26}$, $CE=\frac{20}{26}\times10=\frac{100}{13}$
18. (a) In ΔADB and ΔBDC, ∠ADB = ∠BDC = 90° (given);
    ∠ABD = 90° – ∠CBD, ∠BCD = 180° – 90° – ∠CBD = 90° – ∠CBD (∠sum of Δ),
    ∴ ∠ABD = ∠BCD; ∠A = ∠CBD (∠sum of Δ); ∴ ΔADB ~ ΔBDC (AAA)
    (b) ∵ΔADB ~ ΔBDC (proved), $\therefore$ $\frac{CD}{BD}=\frac{BD}{AD}$ (corr. sides, ~Δs), $CD=\frac{BD^2}{AD}=\frac{6^2}{4}=9$
19. (a) In ΔMNX and ΔYZX, $\frac{XM}{XY}=\frac{15}{9}=\frac{5}{3}$, $\frac{XN}{XZ}=\frac{20}{12}=\frac{5}{3}$,
    ∠MXN = ∠YXZ (vert. opp. ∠s), ∴ ΔMNX ~ ΔYZX (ratio of 2 sides, inc. ∠)
    (b) $\frac{YZ}{MN}=\frac{3}{5}$ (corr. sides, ~Δs), $\frac{y}{18}=\frac{3}{5}$, $y=\frac{3}{5}\times18=10.8$
    (c) ∠Y = ∠M and ∠Z = ∠N (corr.∠s, ~Δs)
    To claim that YZ // MN, what is required is ∠Y = ∠N or ∠Z = ∠M, but such information is not provided, ∴ the claim is disagreed.

20. (a) In ΔBAD and ΔCDB, ∠BAD = ∠BDC (given), $\frac{AD}{DB}=\frac{4}{8}=\frac{1}{2}$, $\frac{AB}{DC}=\frac{6}{12}=\frac{1}{2}$,
∴ ΔBAD ~ ΔCDB (ratio of 2 sides, inc. ∠)
(b) $\frac{DB}{BC}=\frac{1}{2}$ (corr. sides, ~Δs), $\frac{8}{BC}=\frac{1}{2}$, ∴ BC = 8 × 2 = 16

21. (a) (i) ∠PMN = ∠PQR (corr. ∠s, MN // QR), ∠PNM = ∠PRQ (corr. ∠s, MN // QR), ∠MPN = ∠QPR (common ∠), ∴ ΔPMN ~ ΔPQR (AAA)
(ii) ∠NMT = ∠QRT (alt. ∠s, MN // QR), ∠MNT = ∠RQT (alt. ∠s, MN // QR), ∠MTN = ∠RTQ (vert. opp. ∠), ∴ ΔMNT ~ ΔRQT (AAA)
(b) (i) ∵ ΔMNT ~ ΔRQT (proved), ∴ $\frac{QR}{NM}=\frac{QT}{NT}$ (corr. sides, ~Δs)
$\frac{QR}{7}=\frac{6}{4}$, ∴ $QR=\frac{6}{4}\times 7=10.5$
(ii) ∵ ΔPMN ~ ΔPQR (proved), ∴ $\frac{PM}{PQ}=\frac{MN}{QR}$ (corr. sides, ~Δs),
$\frac{5}{5+MQ}=\frac{7}{10.5}$, 52.5 = 35 + 7MQ, 7MQ = 17.5, ∴ MQ = 2.5

22. (a) In ΔADE and ΔACB, ∠A = ∠A (common angle), ∠DE = ∠ACB = 90° (given), ∠AED = ∠ABC (∠ sum of Δ), ∴ ΔADE ~ ΔACB (AAA)
(b) $\frac{AE}{AB}=\frac{DE}{BC}$ (corr. sides, ~Δs), but DE = EC (given) and EC = AC – AE = 24 – AE
∴ $\frac{AE}{AB}=\frac{24-AE}{BC}$, $\frac{AE}{25}=\frac{24-AE}{7}$, 7AE = 25(24 – AE),
7AE = 25 × 24 – 25AE, 32AE = 25 × 24, $AE=\frac{25\times 24}{32}=18.75$

23. (a) Yes. In ΔBCA and ΔDCB, ∠A = ∠DBC (given), ∠C = ∠C (common), ∠ABC = ∠BDC (∠sum of Δ), ∴ ΔBCA ~ ΔDCB (AAA)
(b) $\frac{BC}{DC}=\frac{AC}{BC}$ (corr. sides, ~Δs), $\frac{15}{12}=\frac{12+AD}{15}$, 225 = 144 + 12AD, ∴ AD = 6.75

24. In ΔABC and ΔCDB, $\frac{AB}{CD}=\frac{14}{21}=\frac{2}{3}$, $\frac{BC}{DB}=\frac{12}{18}=\frac{2}{3}$, $\frac{AC}{CB}=\frac{8}{12}=\frac{2}{3}$,
∴ ΔABC ~ ΔCDB (3 sides prop.), ∴ ∠ACB = ∠CBD (corr. ∠s, ~Δs),
∴ AC // BD (alt. ∠s, eq.)

25. (a) In ΔADB and ΔABC , $\frac{AD}{AB}=\frac{12}{18}=\frac{2}{3}$, $\frac{AB}{AC}=\frac{18}{15+12}=\frac{2}{3}$,
∠DAB = ∠BAC (common), ∴ ΔABC ~ ΔADB (ratio of 2 sides, inc. ∠),
(b) ∴ $\frac{BD}{CB}=\frac{2}{3}$ (corr. sides, ~Δs), $\frac{10}{x}=\frac{2}{3}$, ∴ $x=10\times\frac{3}{2}=15$

26. (a) In ΔPQR and ΔPRS, $\frac{PR}{PS}=\frac{PQ}{PR}$ (given), ∠QPR=∠RPS (common),
∴ ΔPQR ~ ΔPRS (ratio of 2 sides, inc. ∠)
(b) ∠PSR = 180° – 116° = 64° (adj.∠s on st. line),
∠PRQ = ∠PSR = 64° (corr.∠s, ~Δs), ∴ ∠PRS =64° – θ
In ΔPSR, 84° + 64° + (64° – θ) = 180° (∠sum of Δ), ∴ θ = 32°

27. (a) ΔFDA ~ ΔFEB ~ ΔCEA ~ ΔCDB
(b) In ΔCDB and ΔCEA, ∠C = ∠C (common angle), ∠BDC = ∠AEC = 90° (given),

$\angle DBC = \angle EAC$ ($\angle$ sum of Δ), $\therefore$ $\Delta CDB \sim \Delta CEA$ (AAA),

$\therefore$ $\frac{DC}{EC} = \frac{BC}{AC}$ (corr. sides, ~Δs), $\frac{DC}{18} = \frac{6+18}{36} = \frac{2}{3}$,

$\therefore$ $DC = \frac{2}{3} \times 18 = 12$ cm, $\therefore$ $AD = 36 - 12 = 24$ cm.

28. (a) $\Delta LPN \sim \Delta DEN$ (AAA), $\Delta LCD \sim \Delta LMN$ (AAA).

(b) $\frac{DN}{LN} = \frac{DE}{LP}$ (corr. sides, ~Δs), $\frac{m}{m+6} = \frac{5}{8}$, $8m = 5m + 30$, $\therefore$ $m = 10$.

$\frac{LD}{LN} = \frac{DC}{NM}$ (corr. sides, ~Δs), $\frac{6}{6+m} = \frac{n}{12}$, $\therefore$ $n = \frac{6}{6+10} \times 12 = 4.5$

29. (a) $\because$ $\Delta ABC \sim \Delta AEF$ (AAA), $\therefore$ $\frac{8}{8+12} = \frac{x}{x+4+5}$ (corr. sides, ~Δs)

$\frac{2}{5} = \frac{x}{x+9}$, $2(x + 9) = 5x$, $x = 6$

$\because$ $\Delta ABD \sim \Delta AEG$ (AAA), $\therefore$ $\frac{8}{8+12} = \frac{x+4}{x+4+5+y}$ (corr. sides, ~Δs)

$\frac{2}{5} = \frac{10}{15+y}$, $15 + y = 25$, $y = 10$

(b) $\because$ $\Delta PQR \sim \Delta PST$ (AAA), $\therefore$ $\frac{a}{a+b} = \frac{10}{10+5} = \frac{2}{3}$ (corr. sides, ~Δs) ,

$3a = 2a + 2b$, $a = 2b$ ..... (1)

$\because$ $\Delta PQT \sim \Delta PSU$ (AAA), $\therefore$ $\frac{a+b}{a+b+6} = \frac{10}{10+5} = \frac{2}{3}$ (corr. sides, ~Δs)

$3a + 3b = 2a + 2b + 12$, $a + b = 12$ ..... (2)

Solving (1) and (2), $a = 8$, $b = 4$

30. (a) $\Delta HEF \sim \Delta HCD$ (AAA), $\Delta DEF \sim \Delta DGH$ (AAA), $\Delta CDE \sim \Delta HGE$ (AAA)

(b) $\Delta CDE \sim \Delta HGE$, $\therefore$ $\frac{x}{5} = \frac{8}{4}$, $x = 10$

$\Delta HEF \sim \Delta HCD$, $\therefore$ $\frac{HF}{HD} = \frac{EF}{CD} = \frac{HE}{HC}$ (corr. sides, ~Δs), $\frac{y}{y+3} = \frac{z}{5} = \frac{8}{8+4} = \frac{2}{3}$

$\therefore$ $z = \frac{2}{3} \times 5 = \frac{10}{3}$; $3y = 2y + 6$, $\therefore$ $y = 6$

31. Join BE, and mark the intersection of BE and CD as G. $\Delta ECG \sim \Delta EAB$ (AAA),

$\therefore$ $\frac{CG}{AB} = \frac{EC}{EA} = \frac{EG}{EB}$ (corr. sides, ~Δs), $\therefore$ $\frac{CG}{8} = \frac{3}{3+6} = \frac{1}{3}$, $CG = \frac{1}{3} \times 8 = \frac{8}{3}$

$\because$ EG : EB = 1 : 3, $\therefore$ EG : GB : EB = 1 : (3 – 1) : 3 = 1 : 2 : 3, $\therefore$ $\frac{BG}{BE} = \frac{2}{3}$

$\Delta BDG \sim \Delta BFE$ (AAA), $\therefore$ $\frac{DG}{FE} = \frac{BG}{BE}$ (corr. sides, ~Δs)

$\therefore$ $\frac{DG}{6} = \frac{2}{3}$, $DG = \frac{2}{3} \times 6 = 4$, $\therefore$ $CD = \frac{8}{3} + 4 = 6\frac{2}{3}$

32. (a) $\because$ $\Delta ADE \sim \Delta AFG$ (AAA), $\therefore$ $\frac{DE}{FG} = \frac{5+5}{5+5+5} = \frac{2}{3}$ (corr. sides, ~Δs),

$\because$ $\Delta BDH \sim \Delta BFG$ (AAA), $\frac{DH}{FG} = \frac{5}{5+5} = \frac{1}{2}$ (corr. sides, ~Δs),

$\therefore$ DE : FG = 2 : 3 = 4 : 6, DH : FG = 1 : 2 = 3 : 6, $\therefore$ DH : DE = 3 : 4

(b) $\frac{DH}{DE}=\frac{3}{4}$ (proved), $\frac{DH}{24}=\frac{3}{4}$, $DH=\frac{3}{4}\times 24=18$, $\therefore$ HE = 24 – 18 = 6 cm

33. $\because$ ΔBFG ~ ΔBAC (AAA), $\therefore$ $\frac{BF}{BA}=\frac{BG}{BC}$ (corr. sides, ~Δs)

$\frac{42}{BA}=\frac{4+3}{4+3+1}=\frac{7}{8}$, 336 = 7 BA, BA = 48 (corr. sides, ~Δs)

$\because$ ΔCDE ~ ΔCBA (AAA), $\frac{DE}{BA}=\frac{CD}{CB}$ (corr. sides, ~Δs)

$\therefore$ $\frac{DE}{48}=\frac{3+1}{4+3+1}=\frac{1}{2}$, $\therefore$ $DE=\frac{1}{2}\times 48=24$

34. ΔABE ~ ΔACF ~ ΔADG (AAA), $\therefore$ $\frac{AE}{AG}=\frac{BE}{DG}$ and $\frac{BE}{CF}=\frac{AE}{AF}$ (corr. sides, ~Δs)

$\frac{6}{6+EF+3}=\frac{3}{5}$, 30 = 27 + 3 EF, 3 = 3 EF, EF = 1

Also $\frac{3}{CF}=\frac{6}{6+EF}$, $\frac{3}{CF}=\frac{6}{6+1}=\frac{6}{7}$, 21 = 6CF, $\therefore$ $CF=\frac{21}{6}=3.5$

35. (a) In ΔCDF and ΔGEF, ∠CFD = ∠GFE (vert. opp.∠s),
∠CDF = ∠GEF and ∠DCF = ∠EGF (alt.∠s, AD // EG),
$\therefore$ ΔCDF ~ ΔGEF (AAA)

(b) $\because$ ΔCDF ~ ΔGEF (proved), $\therefore$ $\frac{EG}{CD}=\frac{EF}{FD}$ (corr. sides, ~Δs), $\therefore$ $\frac{EG}{CD}=\frac{3}{4}$
EG : CD = 3: 4 and AC : CD = 3 : 2 = 6 : 4, $\therefore$ EG : AC = 3 : 6 = 1 : 2
$\because$ ΔBEG ~ ΔBAC (AAA), $\therefore$ $\frac{BE}{AB}=\frac{EG}{AC}$ (corr. sides, ~Δs)
$\therefore$ BE : AB = 1 : 2, BE : AB : AE = 1 : 2 : (2 – 1) = 1 : 2 : 1
$\therefore$ AE : EB = 1 : 1

36. In ΔBCE and ΔCHF, ∠BEC = ∠CFH = 90° (square),
∠BCE = ∠CHF (corr. sides, CD // HG), ∠CBE = ∠HCF (∠ sum of Δ),
$\therefore$ ΔBCE ~ ΔCHF (AAA), $\therefore$ $\frac{HF}{CE}=\frac{CF}{BE}$ (corr. sides, ~Δs)
CF = DG = 3cm and BE = AD = 1cm (squares)
$\therefore$ $\frac{HF}{3-1}=\frac{3}{1}=3$, HF = 6. GJ = HG = 6 + 3 = 9 cm

37. $\because$ ΔECD ~ ΔEAB (AAA), $\therefore$ $\frac{r}{p}=\frac{CE}{AE}$ (corr. sides, ~Δs),
$\because$ ΔACD ~ ΔAEF (AAA), $\therefore$ $\frac{r}{q}=\frac{AC}{AE}$ (corr. sides, ~Δs),
$\frac{r}{q}=\frac{AE-CE}{AE}=1-\frac{CE}{AE}=1-\frac{r}{p}$, $\frac{r}{p}+\frac{r}{q}=1$, $\therefore$ $\frac{1}{p}+\frac{1}{q}=\frac{1}{r}$

38. (a) ∠AEC = ∠AFD (corr. ∠s, CE // DF),
∠CAE = ∠DAF (common ∠), ∠ACE = ∠ADF (∠ sum of Δ),
$\therefore$ ΔACE ~ ΔADF (AAA), $\therefore$ $\frac{CE}{DF}=\frac{AE}{AF}$ (corr. sides, ~Δs)
$\frac{AE}{AF}=\frac{AE}{AE+EF}=\frac{1}{1+2}=\frac{1}{2}$, $\therefore$ CE : DF = 1 : 2

(b) ∠BEC = ∠BFG and ∠BCE = ∠BGF (corr. ∠s, CE // DF),

$\angle CBE = \angle GBF$ (common $\angle$), $\therefore$ $\Delta BCE \sim \Delta BGF$ (AAA),

$\therefore$ $\frac{CE}{GF} = \frac{BE}{BF}$ (corr. sides, ~Δs), $\frac{BE}{BF} = \frac{BF + FE}{BF} = \frac{3+1}{3} = \frac{4}{3}$, $\therefore$ CE : GF = 4 : 3

(c) CE : GF = 4 : 3 and CE : DF = 1 : 2 = 4 : 8,

$\therefore$ CE : GF : DF = 4 : 3 : 8, $\therefore$ CE : DG = CE : (DF – GF) = 4 : (8 – 3) = 4 : 5

39. (a) BQ = DQ and AQ = EQ (mid-points),

$\angle AQB = \angle EQD$ (vert. opp. $\angle$s), $\therefore$ $\Delta ABQ \sim \Delta EDQ$ (SAS),

$\therefore$ $\angle BAQ = \angle DEQ$ (corr. $\angle$s, ≅Δs), $\therefore$ AB // DE (alt. $\angle$s equal)

(b) EP = PB (mid-point), $\therefore$ EP : EB = 1 : (1 + 1) = 1 : 2

EQ = QA (mid-point), $\therefore$ EQ : EA = 1 : (1 + 1) = 1 : 2

$\angle PEQ = \angle BEA$ (common $\angle$), $\therefore$ $\Delta QPE \sim \Delta ABE$ (ratios of two sides, inc. $\angle$)

(c) $\because$ $\angle EPQ = \angle EBA$ (corr. $\angle$s, ~Δs), $\therefore$ QP // AB (corr. $\angle$s equal)

But AB // DE (from (a)), $\therefore$ AB and DE are parallel to QP.

40. (a) (i) $\frac{PU}{RU} = \frac{8}{5}$, $\frac{QU}{SU} = \frac{6.4}{4} = \frac{64}{40} = \frac{8}{5}$,

$\angle PUQ = \angle SUR$ (vert. opp. $\angle$s), $\therefore$ $\Delta PQU \sim \Delta RSU$ (ratios of two sides, inc. $\angle$)

(ii) RU ≠ SU, $\therefore$ $\angle URS \neq \angle USR$, $\angle URS = \angle UPQ$ (corr. $\angle$s, ~Δs),

$\therefore$ $\angle UPQ \neq \angle USR$, $\therefore$ PQ and RS are not parallel (alt. $\angle$s not eq.)

Ans. The claim is disagreed.

(b) $\frac{RS}{PQ} = \frac{RU}{PU}$ (corr. sides, ~Δs), $\therefore$ $\frac{RS}{12} = \frac{5}{8}$, RS = 7.5cm

(c) $\because$ $\Delta RSU \sim \Delta PQU$ (proved), $\therefore$ $\angle SRU = \angle QPU = 40°$ (corr. $\angle$s, ~Δs)

$\angle PRQ = 180° - \angle SRU - \angle SRT$ (adj. $\angle$s on st. line)

$= 180° - 40° - 60° = 80°$

41. (a) $\angle ADG = \angle CEG = 90°$ (given), $\angle AGD = \angle CGE$ (vert. opp. $\angle$s),

$\angle DAG = \angle ECG$ ($\angle$ sum of Δ), $\therefore$ $\Delta ADG \sim \Delta CEG$ (AAA)

(b) AD : DF = 1 : 2, $\therefore$ AD : AF = 1 : (1 + 2) = 1 : 3

AG : GE = 1 : 2, $\therefore$ AG : AE = 1 : (1 + 2) = 1 : 3

$\angle DAG = \angle FAE$ (common $\angle$), $\therefore$ $\Delta ADG \sim \Delta AFE$ (ratio of 2 sides, inc. $\angle$)

$\therefore$ $\angle ADG = \angle AFE$ (corr. $\angle$s, ~Δs), $\therefore$ CD // EF (corr. $\angle$s eq.)

(c) (i) $\Delta ADG \sim \Delta AFE$ (proved), $\therefore$ $\frac{FE}{DG} = \frac{AF}{AD}$ (corr. sides, ~Δs)

AD : AF = 1 : 3, $\therefore$ AF = 3AD = 3(8) = 24cm, and FE = 3(DG) = 3(6) = 18cm

Area of ΔAFE = (AF)(FE) ÷ 2 = (24)(18) ÷ 2 =216 cm^2

(ii) GE : GA = 2 : 1, $\therefore$ GE = 2(10) = 20cm, $\Delta ADG \sim \Delta CEG$ (proved),

$\therefore$ $\frac{CE}{AD} = \frac{GE}{GD}$ (corr. sides, ~Δs), $\frac{CE}{8} = \frac{20}{6}$, CE = $\frac{80}{3}$

Area of ΔCEG = (GE)(CE) ÷ 2 = $(20)(\frac{80}{3}) \div 2 = \frac{800}{3}$ cm^2 = $266\frac{2}{3}$ cm^2

(iii) $\Delta ADG \sim \Delta AFE$ (proved) and FE : DG = AF : AD =3 : 1 (corr. sides, ~Δs)

$\therefore$ FE = 3(DG) = 3(6) = 18cm

In ΔABE, $\angle FBE = 180° - \angle AEB - \angle DAG$ ($\angle$sum of Δ)

$= 180 - 90° - \angle DAG = 90° - \angle DAG$

In ΔADG, $\angle DGA = 180° - \angle ADG - \angle DAG$ ($\angle$sum of Δ)

$= 180 - 90° - \angle DAG = 90° - \angle DAG$

$\therefore$ $\angle FBE = \angle DGA$; $\angle BFE = \angle GDA = 90°$,

$\therefore$ $\angle$FEB = $\angle$DGA ($\angle$sum of Δ), $\therefore$ ΔEFB ~ ΔADG

$\therefore$ $\dfrac{FB}{DG} = \dfrac{FE}{AD}$ (corr. sides, ~Δs), $\dfrac{FB}{6} = \dfrac{18}{8}$, FB = 13.5cm

Area of ΔEFB = (FB)(FE) ÷ 2 = (13.5)(18) ÷ 2 = 121.5cm^2

42. (a) $\angle$NTQ = $\angle$PTS (vert. opp. $\angle$s)
$\because$ QN // PS (given), $\therefore$ $\angle$QNT = $\angle$SPT (alt. $\angle$s, QN // PS)
$\angle$NQT = $\angle$PST ($\angle$ sum of Δ), $\therefore$ ΔNQT ~ ΔPST (AAA)

(b) $\angle$MTP = $\angle$QTR (vert. opp. $\angle$s)
$\because$ PM // QR (given), $\therefore$ $\angle$PMT = $\angle$RQT (alt. $\angle$s, PM // QR)
$\angle$MPT = $\angle$QRT ($\angle$ sum of Δ), $\therefore$ ΔRTQ ~ ΔPTM (AAA)

(c) $\because$ ΔNQT ~ ΔPST (proved), $\therefore$ $\dfrac{TN}{TP} = \dfrac{TQ}{TS}$ (corr. sides, ~Δs), TN × TS = TP × TQ.
$\because$ ΔRTQ ~ ΔPTM (proved), $\therefore$ $\dfrac{TR}{TP} = \dfrac{TQ}{TM}$ (corr. sides, ~Δs), TM × TR = TP × TQ.
$\therefore$ TN × TS = TM × TR

(d) TN × TS = TM × TR (proved), $\therefore$ $\dfrac{TM}{TS} = \dfrac{TN}{TR}$, $\angle$MTN = $\angle$STR (common $\angle$),
$\therefore$ ΔMTN ~ ΔSTR (ratio of 2 sides, inc. $\angle$), $\therefore$ $\angle$TMN = $\angle$TSR (corr. $\angle$s, ~Δs),
$\therefore$ MN // SR (corr. $\angle$s equal)

43. (a) (i) Let $\angle A = k$. In ΔAGF, $\angle$AFG = 180° – 90° – k = 90° – k ($\angle$sum of Δ)
$\angle$AFD = 90° (given), $\therefore$ $\angle$GFD = 90° – $\angle$AFG = 90° – (90° – k) = k.
In ΔFGD, $\angle$FDG = 180° – 90° – $\angle$GFD = 90° – k ($\angle$sum of Δ) = $\angle$AFG
The three angles of ΔAGF and ΔFGD are k, 90°, 90° – k,
$\therefore$ ΔAGF ~ ΔFGD (AAA)

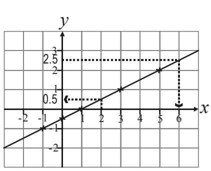
(ii) AG : GC : CD = 1 : 1 : 2, AG = a, $\therefore$ GC = a, CD = 2a
DG = GC + CD = a + 2a = 3a, $\dfrac{AG}{FG} = \dfrac{FG}{DG}$ (corr. sides, ~Δs)
FG^2 = (AG)(DG) = (a)(3a) = 3a^2

(b) (i) In ΔABC, $\angle$B = 180° – 90° – $\angle$A = 90° – k
In ΔEFB, $\angle$BEF = 180° – 90° – $\angle$B =90° – (90° – k) = k
The three angles of ΔFGD and ΔEFB are k, 90°, 90° – k,
$\therefore$ ΔFGD ~ ΔEFB (AAA)

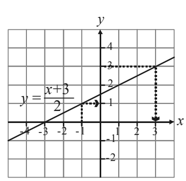
(ii) $\dfrac{EF}{FG} = \dfrac{BF}{DG}$ (corr. sides, ~Δs), $\therefore$ $\dfrac{EF}{BF} = \dfrac{FG}{DG}$, $\therefore$ $\dfrac{EF^2}{BF^2} = \dfrac{FG^2}{DG^2}$.
FG^2 = 3a^2 (from a(ii)), DG = 3a, DG^2 = (3a)^2 = 9a^2, $\therefore$ $\dfrac{EF^2}{BF^2} = \dfrac{3a^2}{9a^2} = \dfrac{1}{3}$

Unit 11 Graphs of linear equations in two unknowns

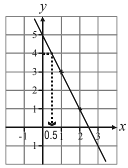
1. (a) –1, –0.5, 0, 1, 2
(b) m = 0.5, n = 6



2. (a) 1
(b) 3

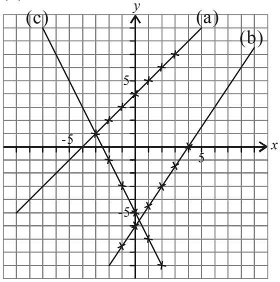


3. (a) 3, 1, –1
   (b) x-axis: (2.5, 0), y-axis: (0, 5)
   (c) From the graph, $k = 0.5$
   (d) $2(k) + 4 = 5$, $2k = 1$, $\therefore\ k = 0.5$

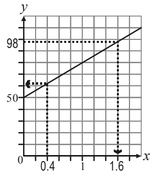


Q.3

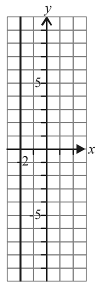
4. (a) 2, 3, 4, 5, 6, 7
   (b) –7.5, –6, –4.5, –3, –1.5, 0
   (c) 1, –1, –3, –5, –7, –9



5. (a) $y = hx + 50$
   (b) $62
   (c) 1.6 kg
   (d) When $x = 1$,
   $y = 80$.
   $80 = h(1) + 50$,
   $\therefore\ h = 30$



6. $x + 2 = 0, x = -2$



7. (a) $2(-7) + k(-6) - 4 = 0$, $-6k = 18$, $\therefore\ k = -3$
   (b) $2h - 3(\frac{5}{2}) - 4 = 0$, $4h - 15 - 8 = 0$, $4h = 23$, $\therefore\ h = 5\frac{3}{4}$
   (c) Let M be $(x, 0)$, $2x - 3(0) - 4 = 0$, $2x = 4$, $\therefore\ x = 2$.
   Let N be $(0, y)$, $2(0) - 3y - 4 = 0$, $-3y = 4$, $\therefore\ y = -1\frac{1}{3}$.
   Ans. The coodinates of M and N are (2, 0) and (0, $-1\frac{1}{3}$*) respectively.*

8. On the x-axis, $y = 0$, $\frac{3}{2}x - \frac{1}{3}(0) = \frac{3}{8}$, $\therefore\ x = \frac{1}{4}$. *Ans. The point is (*$\frac{1}{4}$*, 0).*
   On the y-axis, $x = 0$, $\frac{3}{2}(0) - \frac{1}{3}y = \frac{3}{8}$, $\therefore\ y = -1\frac{1}{8}$. *Ans. The point is (0,* $-1\frac{1}{8}$*).*

9. (a) $a(2) - 3(4) + 2 = 0$, $2a = 10$, $\therefore\ a = 5$
   (b) When $x = 0$, $5(0) - 3y + 2 = 0$, $y = \frac{2}{3}$
   When $y = 0$, $5x - 3(0) + 2 = 0$, $x = -\frac{2}{5}$
   $\therefore$ The shaded area = $\frac{1}{2} \times \frac{2}{3} \times \frac{2}{5} = \frac{2}{15}$ sq. units

10. (a) $4 = 3(0) + k$, $\therefore\ k = 4$
   (b) $0 = 3x + 4$, $x = -\frac{4}{3}$, $\therefore$ Coordinates of Q = $(-\frac{4}{3}, 0)$
   $\because$ OQ = OR, $\therefore$ Coordinates of R = $(0, \frac{4}{3})$ or $(0, -\frac{4}{3})$
   When R = $(0, \frac{4}{3})$, PR = $4 - \frac{4}{3} = \frac{8}{3}$,
   $\therefore$ Area of ΔPQR = $\frac{1}{2} \times \frac{8}{3} \times \frac{4}{3} = \frac{16}{9} = 1\frac{7}{9}$ sq.units

When R = $(0,\ -\frac{4}{3})$, PR = $4 + \frac{4}{3} = \frac{16}{3}$,

$\therefore$ Area of ΔPQR = $\frac{1}{2}\times\frac{16}{3}\times\frac{4}{3} = \frac{32}{9} = 3\frac{5}{9}$ sq. units

11. $3(\frac{1}{2})+4(p)-5=0$, $3+8p-10=0$, $8p=7$, $\therefore\ p=\frac{7}{8}$

$3(\frac{1}{5})+4(q)-5=0$, $3+20q-25=0$, $20q=22$, $\therefore\ q=\frac{11}{10}$

$\therefore\ 2p+q=2(\frac{7}{8})+\frac{11}{10}=\frac{35}{20}+\frac{22}{20}=2\frac{17}{20}$

12. (a) $3(2r)+pr=0$, $pr=-6r$, $\therefore\ p=-6$ ($\because\ r\neq 0$)

(b) Put $(-2s, s)$ into $3x - 6y = 0$,
L.H.S. = $3(-2s)-6s=-12s$, R.H.S. = 0,
If $s=0$, L.H.S. = R.H.S., $\therefore$ D lies on the graph when $s = 0$.
If $s\neq 0$, L.H.S. $\neq$ R.H.S., $\therefore$ D does not lie on the graph when $s\neq 0$.

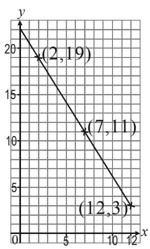
13. $\because$ PR // y-axis, $\therefore$ Coordinates of R are (–10, 0).

Let Q be $(0, q)$, $3(q)+5(0)=4$, $3q=4$, $\therefore\ q=\frac{4}{3}$,

$\therefore$ Area of PROQ = $\frac{(\text{PR}+\text{OQ})\times\text{OR}}{2} = \frac{1}{2}\times(18+\frac{4}{3})\times 10 = 96\frac{2}{3}$ sq. units

14. (a) $2.4x+1.5y=33.3$, $24x+15y=333$,
$\therefore$ $8x + 5y = 111$

(b)

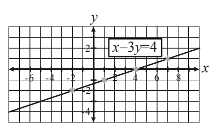


(c) $\because$ x and y must be integers,
$\therefore$ From the graph, the possible answers of (x, y) are (2, 19) or (7, 11) or (12, 3).
Ans. 2 *\$2.4* stamps and 19 *\$1.5* stamps; 7 *\$2.4* stamps and 11 *\$1.5* stamps; or 12 *\$2.4* stamps and 3 *\$1.5* stamps.

Unit 12 Simultaneous linear equations in two unknowns

1

x	–2	1	4	7
y	–2	–1	0	1

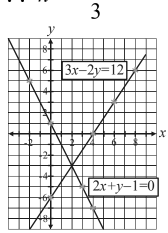


2. (a) When $x = 5$, $y = -7$, $3(5)+k(-7)-1=0$,
$14=7k$, $\therefore\ k=2$

(b) When $x = h$, $y=\frac{5}{2}$, $3(h)+2(\frac{5}{2})-1=0$, $3h+4=0$, $\therefore\ h=-\frac{4}{3}$

3 (a), (b)

(a)	x	–2	0	3	4
	y	5	1	–5	–7
(b)	x	0	4	6	8
	y	–6	0	3	6



(c) From the graph, $x = 2$, $y = -3$

4. (a) $\begin{cases} y = 8x - 19 ......(1) \\ y = 6x - 15 ......(2) \end{cases}$ Put (1) into (2), $8x - 19 = 6x - 15$, $x = 2......(3)$

Put (3) into (1), $y = 8 \times 2 - 19 = -3$. *Ans.* $x = 2, y = -3$.

(b) $\begin{cases} x = 2y + 13 ......(1) \\ 3x + y = 4 ......(2) \end{cases}$ Put (1) into (2), $3(2y + 13) + y = 4$, $6y + 39 + y = 4$,

$y = -5......(3)$. Put (3) into (1), $x = 2(-5) + 13 = 3$. *Ans.* $x = 3, y = -5$.

(c) $\begin{cases} 3y - 7x = 1 ......(1) \\ y + 2x = 9 ....(2) \end{cases}$ From (1), $3y = 1 + 7x$, $y = \frac{1+7x}{3} ......(3)$

Put (3) into (2), $\frac{1+7x}{3} + 2x = 9$, $1 + 7x + 6x = 27$, $x = 2......(4)$

Put (4) into (3), $y = \frac{1+7\times 2}{3} = \frac{15}{3} = 5$. *Ans.* $x = 2, y = 5$.

(d) $\begin{cases} 2y = 5x + 5 ......(1) \\ 2y = 6x + 8 ......(2) \end{cases}$. Put (1) into (2), $5x + 5 = 6x + 8$, $x = -3......(3)$

Put (3) into (1), $2y = 5(-3) + 5$, $y = \frac{-10}{2} = -5$. *Ans.* $x = -3, y = -5$.

(e) $\begin{cases} 8x + 8 = 3y ......(1) \\ 2y = 11 - 6x ......(2) \end{cases}$. From (1), $y = \frac{8x+8}{3} ......(3)$. Put (3) into (2),

$2\left(\frac{8x+8}{3}\right) = 11 - 6x$, $16x + 16 = 33 - 18x$, $34x = 17$, $x = \frac{1}{2} ......(4)$

Put (4) into (3), $y = \frac{8\times\frac{1}{2}+8}{3} = \frac{4+8}{3} = \frac{12}{3} = 4$ *Ans.* $x = \frac{1}{2}, y = 4$.

(f) $\begin{cases} 5x + 3y = 7 ......(1) \\ 5y - 3x = 6 ......(2) \end{cases}$ From (1), $y = \frac{7-5x}{3} ......(3)$. Put (3) into (2),

$5\left(\frac{7-5x}{3}\right) - 3x = 6$, $35 - 25x - 9x = 18$, $-34x = -17$, $x = \frac{1}{2} ......(4)$

Put (4) into (3), $y = \frac{1}{3}\left(7 - 5\times\frac{1}{2}\right) = \frac{1}{3}\times\frac{9}{2} = \frac{3}{2}$ *Ans.* $x = \frac{1}{2}$, $y = \frac{3}{2}$.

5. (a) $\begin{cases} x + y = 11 ......(1) \\ x - y = 7 ......(2) \end{cases}$. (1) + (2), $2x = 18$, $x = 9......(3)$
Put (3) into (1), $9 + y = 11$, $y = 2$
Ans. $x = 9, y = 2$.

(b) $\begin{cases} x + 2y = 3 ......(1) \\ x - y = 9 ......(2) \end{cases}$ (2) × 2, $2x - 2y = 18......(3)$
(1) + (3), $3x = 21$, $x = 7......(4)$

Put (4) into (2), $7 - y = 9$, $7 - 9 = y$, $y = -2$. *Ans.* $x = 7, y = -2$

(c) $\begin{cases} 4x - y = 18 ......(1) \\ x + 2y = -9 ......(2) \end{cases}$ (1) × 2, $8x - 2y = 36......(3)$
(2) + (3), $9x = 27$, $x = 3......(4)$

Put (4) into (2), $3 + 2y = -9$, $2y = -12$, $y = -6$. *Ans.* $x = 3, y = -6$

(d) $\begin{cases} 5x + 3y = 2 ......(1) \\ 3x + 9y = -6 ......(2) \end{cases}$ (1) × 3, $15x + 9y = 6......(3)$
(3) – (2), $12x = 12$, $x = 1......(4)$

Put (4) into (1), $5 + 3y = 2$, $3y = -3$, $y = -1$. *Ans.* $x = 1, y = -1$

(e) $\begin{cases} 3y + 2x = 2 ......(1) \\ 2y - 3x = -16 ......(2) \end{cases}$ (1) × 3, $9y + 6x = 6......(3)$
(2) × 2, $4y - 6x = -32......(4)$
(3) + (4), $13y = -26, y = -2......(5)$

Put (5) into (1), $-6 + 2x = 2$, $2x = 8$, $x = 4$. *Ans.* $x = 4, y = -2$

(f) $\begin{cases} 5x+5y=1......(1) \\ 3x-2y=5......(2) \end{cases}$ (1) × 2, $10x+10y=2......(3)$
(2) × 5, $15x-10y=25......(4)$
(3) + (4), $25x=27$, $x=\frac{27}{25}......(5)$

Put (5) into (1), $\frac{5\times27}{25}+5y=1$, $5y=1-\frac{27}{5}$, $5y=\frac{-22}{5}$, $y=\frac{-22}{25}$

Ans. $x=\frac{27}{25}$, $y=\frac{-22}{25}$

6. (a) $3x+y-120=3x-2y$, $y-120=-2y$, $3y=120$, $y=40$, $3x-2y=10+2x$,
$3x-2\times40=10+2x$, $x=10+80$, $x=90$. *Ans. (90, 40)*

(b) $9x+3y+5=x-2y+7$, $8x+5y=2......(1)$
$x-2y+7=7x-3y-4$, $6x-y=11......(2)$
(2) × 5, $30x-5y=55......(3)$.
(1) + (3), $38x=57$, $x=\frac{3}{2}$ ...... (4)

Put (4) into (2), $9-y=11$, $y=-2$. *Ans.* $(\frac{3}{2}, -2)$

(c) $\frac{5x}{3}-y=6......(1)$, $\frac{x}{3}+y=6......(2)$.
(1) + (2), $\frac{5x}{3}+\frac{x}{3}=12$, $5x+x=36$,
$6x=36$, $x=6......(3)$.
Put (3) into (2), $\frac{6}{3}+y=6$, $2+y=6$, $y=4$. *Ans. (6, 4)*

(d) $\frac{6x-3y}{4}=6......(1)$, $\frac{12x+5y}{30}=6......(2)$.
From (1), $6x-3y=24$, $12x-6y=48$ ......(3)
From (2), $12x+5y=180$ ......(4)
(4) – (3), $\therefore$ $5y+6y=180-48$, $11y=132$, $y=12......(5)$
Put (5) into (4), $12x+5(12)=180$, $12x=120$, $x=10$. *Ans. (10, 12)*

7. (a) $\begin{cases} 9x-5y-11=0......(1) \\ 3x-5y+13=0......(2) \end{cases}$
(1) – (2), $6x-24=0$, $6x=24$, $x=4......(3)$
Put (3) into (1), $36-5y-11=0$, $-5y=-25$, $y=5$. *Ans. (4, 5)*

(b) $\begin{cases} 0.35x-0.15y=1......(1) \\ 0.2x+0.1y=1.5.....(2) \end{cases}$ (1) × 2, $0.7x-0.3y=2......(3)$
(2) × 3, $0.6x+0.3y=4.5.....(4)$
(3) + (4), $1.3x=6.5$, $x=5......(5)$
Put (5) into (2), $0.2(5)+0.1y=1.5$, $0.1y=0.5$, $y=5$. *Ans. (5, 5)*

(c) $\begin{cases} 10x+y=3(x+y),\ 10x+y=3x+3y,\ 7x-2y=0\ ......(1) \\ 9(x-y)=x-y-32,\ 9x-9y=x-y-32,\ x-y=-4\ .....(2) \end{cases}$
From (2), $x=y-4......(3)$.
Put (3) into (1), $7(y-4)-2y=0$, $5y=28$, $y=\frac{28}{5}......(4)$.
Put (4) into (3), $x=\frac{28}{5}-4=\frac{28-20}{5}=\frac{8}{5}$. *Ans.* $(\frac{8}{5}, \frac{28}{5})$

(d) $\begin{cases} 3(x-y) = -(7+2y), \\ 3(2x-y) = 2+y, \end{cases}$ $3x - 3y = -7 - 2y$, $3x - y = -7$, $y = 3x + 7$......(1)
$6x - 3y = 2 + y$, $6x - 4y = 2$......(2)

Put (1) into (2), $6x - 4(3x + 7) = 2$, $6x - 12x - 28 = 2$, $-6x = 30$, $x = -5$......(3).
Put (3) into (1), $y = 3(-5) + 7 = -8$. *Ans.* *(–5, –8)*

(e) $\begin{cases} 3x+2y = 4(4+x)+1, \\ 5x+3y = 7(4+x)-2, \end{cases}$ $3x + 2y = 16 + 4x + 1$, $x = 2y - 17$......(1)
$5x + 3y = 28 + 7x - 2$, $2x - 3y = -26$......(2)

Put (1) into (2), $2(2y - 17) - 3y = -26$, $4y - 34 - 3y = -26$, $y = 8$......(3)
Put (3) into (1), $x = 2(8) - 17 = -1$. *Ans.* *(–1, 8)*

(f) $\begin{cases} -2(2+4y) = 5x-5y, \\ 3(1-x)+2(y+1) = y-x, \end{cases}$ $-4 - 8y = 5x - 5y$, $5x + 3y = -4$......(1)
$3 - 3x + 2y + 2 = y - x$, $y = 2x - 5$.....(2)

Put (2) into (1), $5x + 3(2x - 5) = -4$, $5x + 6x - 15 = -4$, $11x = 11$, $x = 1$......(3).
Put (3) into (2), $y = 2(1) - 5 = -3$. *Ans.* *(1, –3)*

(g) $\begin{cases} 4(x-y)+2(x+y) = -(19+y), \\ 5(x+y)-(2x+y) = -5, \end{cases}$ $6x - 2y = -19 - y$, $6x + 19 = y$.....(1)
$5x + 5y - 2x - y = -5$, $3x + 4y = -5$......(2)

Put (1) into (2),
$3x + 4(6x + 19) = -5$, $3x + 24x + 76 = -5$, $27x = -81$, $x = -3$.....(3).
Put (3) into (1), $y = 6(-3) + 19 = -1$. *Ans.* *(–3, –1)*

8. (a) $\begin{cases} x = 2y+8 \text{......(1)} \\ \dfrac{x}{3}+4 = y \text{......(2)} \end{cases}$ Put (2) into (1), $x = 2(\frac{x}{3}+4)+8 = \frac{2x}{3}+16$,
$3x = 2x + 48$, $x = 48$......(3)

Put (3) into (1), $48 = 2y + 8$, $2y = 40$, $y = 20$. *Ans.* *(48, 20)*

(b) $\begin{cases} y = \dfrac{3}{2}x-4 \text{......(1)} \\ x = -\dfrac{2}{3}y \text{......(2)} \end{cases}$ Put (1) into (2), $x = -\frac{2}{3}(\frac{3}{2}x-4) = -x+\frac{8}{3}$,
$2x = \frac{8}{3}$, $x = \frac{4}{3}$......(3)

Put (3) into (2), $-\frac{4}{3} = -\frac{2}{3}y$, $y = \frac{4}{3}\times(-\frac{3}{2}) = -2$. *Ans.* $(\frac{4}{3}, -2)$

(c) $\begin{cases} \dfrac{2y+x}{3} = 3 \text{......(1)} \\ y-\dfrac{2}{3}x = 1 \text{......(2)} \end{cases}$ From (1), $2y + x = 9$......(3)
From (2), $3y - 2x = 3$……(4)

(3)×2 + (4), $2(2y + x) + (3y - 2x) = 9\times2 + 3$, $7y = 21$, $y = 3$
∴ $2(3) + x = 9$, $x = 3$. *Ans.* *(3, 3)*

(d) $\begin{cases} \dfrac{x}{5}-\dfrac{3y}{2} = 3 \text{......(1)} \\ \dfrac{3x}{8}-\dfrac{5y}{2} = 5 \text{......(2)} \end{cases}$ (1) × 10, $2x - 15y = 30$......(3)
(2) × 8, $3x - 20y = 40$……(4)

(3)×3 − (4)×2, $3(2x - 15y) - 2(3x - 20y) = 90 - 80$,
$-45y + 40y = 10$, $-5y = 10$, $y = -2$
∴ $2x - 15(-2) = 30$, $2x = 0$, $x = 0$. *Ans.* *(0, –2)*

(e) $\begin{cases} \dfrac{5x}{3}-y = -17 \text{......(1)} \\ \dfrac{x}{3}-\dfrac{y}{2} = -4 \text{......(2)} \end{cases}$ (1) × 3, $5x - 3y = -51$......(3)
(2) × 6, $2x - 3y = -24$......(4)
(3) − (4), $3x = -27$, $x = -9$......(5)

Put (5) into (3), $5(-9) - 3y = -51$, $3y = 6$, $y = 2$ *Ans.* *(–9, 2)*

(f) $\begin{cases} \frac{x}{3} - y = -5 \text{......(1)} \\ \frac{3y}{2} + \frac{2x}{3} = \frac{26}{3} \text{......(2)} \end{cases}$ From (1), $y = \frac{x}{3} + 5$ ......(3)

Put (3) into (2),

$\frac{3}{2}(\frac{x}{3} + 5) + \frac{2x}{3} = \frac{26}{3}$, $\frac{x}{2} + \frac{2x}{3} = \frac{26}{3} - \frac{15}{2}$, $\frac{7x}{6} = \frac{52-45}{6}$, $\frac{7x}{6} = \frac{7}{6}$, $x = 1$

$\therefore\ y = \frac{1}{3} + 5 = \frac{16}{3}$ *Ans.* $(1, \frac{16}{3})$

9. (a) Let $m = \frac{1}{x}$, $n = \frac{1}{y}$ $\therefore \begin{cases} 2m + n = 2 \text{......(1)} \\ 3m - 2n = 10 \text{......(2)} \end{cases}$

(1) × 2 + (2), $\therefore$ $2(2m + n) + (3m - 2n) = 4 + 10$, $7m = 14$, $m = 2$ .....(3)

Put (3) into (1), $n = 2 - 4 = -2$ ......(4)

$x = \frac{1}{m} = \frac{1}{2}$, $y = \frac{1}{n} = -\frac{1}{2}$. *Ans.* $(\frac{1}{2}, -\frac{1}{2})$

(b) $\begin{cases} \frac{2}{x} - \frac{1}{y} = \frac{3}{2} \text{......(1)} \\ \frac{1}{x} + \frac{3}{y} = 2 \text{......(2)} \end{cases}$ (1) × 3, $\frac{6}{x} - \frac{3}{y} = \frac{9}{2}$ ......(3)

(2) + (3), $\frac{6}{x} + \frac{1}{x} = \frac{9}{2} + 2$, $\frac{7}{x} = \frac{13}{2}$, $x = \frac{14}{13}$ ......(4)

Put (4) into (2), $\frac{1}{\frac{14}{13}} + \frac{3}{y} = 2$, $\frac{13}{14} + \frac{3}{y} = 2$, $\frac{3}{y} = \frac{15}{14}$, $y = \frac{14}{5}$. *Ans.* $(\frac{14}{13}, \frac{14}{5})$

10. Let the bigger number be y and the smaller number be x.
$\therefore$ $x + y = 91$ and $y - x = 17$. Solving the equations, we get $x = 37$ and $y = 54$.
Ans. The numbers are 37 and 54.

11. Let the bigger number be y and the smaller number be x.
$\therefore$ $\frac{1}{4}(x + y) = 16$ and $2(y - x) = 36$.
Solving the equations, we get $x = 23$ and $y = 41$. *Ans. The numbers are 23 and 41.*

12. Let the bigger number be y and the smaller number be x.
$\therefore \begin{cases} y - x = \frac{3}{5}(x + y),\ 8x - 2y = 0,\ 4x - y = 0 \\ y = 3x + 14 \end{cases}$
Solving the equations, we get $x = 14$ and $y = 56$. *Ans. The numbers are 14 and 56.*

13. Let the numerator be y and the denominator be x. $\therefore \begin{cases} \frac{y+9}{2x} = \frac{2}{3},\ 3y + 27 = 4x \\ \frac{y-1}{x} = \frac{1}{2},\ 2y - 2 = x \end{cases}$
Solving the equations, we get: $x = 12$ and $y = 7$. *Ans. The fraction is* $\frac{7}{12}$.

14. Let the prices of a table and a chair be x and y respectively.
$\therefore$ $3x + 4y = 2\,800$ and $4x + 9y = 4\,650$.
Solving the equations, we get $x = 600$ and $y = 250$.
Ans. The prices of a table and a chair are $600 and $250 respectively .

15. $2m - 3n + 10 = 4m - 5n + 6 = m + n - 9$.
Solving the equations, we get $m = 9$ and $n = 7$.
$\therefore$ The perimeter = 3(9 + 7 – 9) = 3(7) = 21 units.

16. (a) Opposite sides of a rectangle are equal, $\begin{cases} x + 3 = y + 5, & x - y = 2 \\ x - y + 4 = 3x - 4y + 3, & 2x - 3y = 1 \end{cases}$

Solving the equations, we get $x = 5$ and $y = 3$.

(b) AB = 5 + 3 = 8, BC = 5 – 3 + 4 = 6, $\therefore$ perimeter = 2 (8 + 6) = 28 units.

(c) Area = 8 × 6 = 48 square units.

17. Let the numbers of students and pencils be x and y respectively.

$\therefore$ $y = 6x + 5$ and $y = 7x - 8$. Solving the equations, we get $x = 13$ and $y = 83$.

Ans. The number of students and pencils are 13 and 83 respectively .

18. Let the numbers of \$2 coins and \$5 coins be x and y respectively.

$\therefore$ $x + y = 30$ and $2x + 5y = 90$. Solving the equations, we get $x = 20$ and $y = 10$.

Ans. The numbers of \$2 and \$5 coins are 20 and 10 respectively.

19. Let the tens' digit and the units' digit be x and y respectively.
The original no. = $10x + y$,
the reversed no. = $10y + x$

$\therefore \begin{cases} 10x + y = 8(x + y), & 2x - 7y = 0 \\ 10y + x = \frac{1}{3}(10x + y) + 3, & 7x - 29y + 9 = 0 \end{cases}$

Solving the equations, we get $x = 7$ and $y = 2$. *Ans. The number is 72.*

20. $x^2 + Ax - A \equiv x^2 + 3x - Bx - 3B - 11 \equiv x^2 + x(3 - B) - (3B + 11)$

Comparing the coefficients, we get: $A = 3 - B$ and $A = 3B + 11$.

Solving the equations, we get $A = 5$ and $B = -2$.

21. Let the present ages of David and Bobby be x and y respectively.

$\therefore \begin{cases} x = y + 13 \\ y - 11 = \frac{2}{3}(x - 11), & 2x - 3y = -11 \end{cases}$

Solving the equations, we get: $x = 50$ and $y = 37$.

Ans. The present ages of David and Bobby are 50 and 37 respectively .

22. Let the present ages of A and B be x and y respectively. $\therefore \begin{cases} \frac{x}{y} = \frac{5}{3}, & 3x = 5y \\ \frac{x + 9}{y + 9} = \frac{4}{3}, & 3x - 4y = 9 \end{cases}$

Solving the equations, we get $x = 15$ and $y = 9$.

Ans. The present ages of A and B are 15 and 9 respectively .

23. Let the speeds of the cars be x km/h and y km/h. $\therefore \begin{cases} y = x + 12 \\ 0.5(x + y) = 79, & x + y = 158 \end{cases}$

Solving the equations, we get $x = 73$ and $y = 85$.

Ans. The speeds of the cars are 73 km/h and 85 km/h.

24. Let the speeds of the cars be x km/h and y km/h. $\therefore \begin{cases} 4(x + y) = 280, & x + y = 70 \\ 14(y - x) = 280, & y - x = 20 \end{cases}$

Solving the equations, we get $x = 25$ and $y = 45$.

Ans. The speeds of two cars are 25 km/h and 45 km/h.

25. Let the speeds of the boat and the water current be x km/h and y km/h respectively.

$\therefore \begin{cases} 6(x + y) = 108, & x + y = 18 \\ 9(x - y) = 108, & x - y = 12 \end{cases}$

Solving the equations, we get $x = 15$ and $y = 3$.

Ans. The speeds of the boat and the current are 15 km/h and 3km/h respectively.

26. Let x km/h and y km/h be the speeds of the ship in still water and the current of water respectively.

$22.5(x - y) = 360$, $x - y = 16$........(1)

$20(x + y) = 360$, $x + y = 18$.........(2)

Solving (1) and (2), we get $x = 17$ and $y = 1$.
Ans. The speed of the ship against the current = 17 − 1 = 16 km/h.

27. $\frac{4x+3y}{10}-\frac{2x-y}{5}=\frac{x-y}{2}$, $4x+3y-4x+2y=5x-5y$, $x-2y=0$....(1)

$8y-\frac{5x-2}{3}=2x+y$, $24y-5x+2=6x+3y$, $11x-21y=2$.....(2)

Solving (1) and (2), we have: $x = 4$ and $y = 2$.

28. Let $x=\frac{1}{u}$ and $y=\frac{1}{v}$. $\therefore\ 12x-5y+\frac{1}{10}=0$, and $2x+6y-\frac{34}{10}=0$

Solving the equations, we get $x=\frac{1}{5}$ and $y=\frac{1}{2}$. *Ans.* $u=\frac{1}{x}=5,\ v=\frac{1}{y}=2.$

29. $\begin{cases}\frac{6}{x}-\frac{10x}{y}=-5\text{.........(1)}, & (1)\times 2,\ \frac{12}{x}-\frac{20x}{y}=-10\text{..........(3)}\\ \frac{9}{x}-\frac{4x}{y}=9\text{..........(2)}, & (2)\times 5,\ \frac{45}{x}-\frac{20x}{y}=45\text{.........(4)}\end{cases}$

(4) − (3), $\therefore\ \frac{45}{x}-\frac{12}{x}=45+10$, $\frac{33}{x}=55$, $x=\frac{33}{55}=\frac{3}{5}$.........(5)

Put (5) into (1), $6(\frac{5}{3})-\frac{1}{y}(10)(\frac{3}{5})=-5$, $10-\frac{6}{y}=-5$, $15=\frac{6}{y}$, $y=\frac{6}{15}=\frac{2}{5}$

[Or: Let $\frac{1}{x}=a, \frac{x}{y}=b$, then solve for a and b first.] *Ans.* $(\frac{3}{5},\frac{2}{5})$

30. Let the equations be (1) and (2) respectively. From (1), $x=-2y$......(3)

Sub. (3) into (2), $\therefore\ -2y=\dfrac{1}{1-\dfrac{1}{1-(\frac{-2y}{y})}}=\dfrac{1}{1-\frac{1}{3}}=\dfrac{1}{\frac{2}{3}}=\dfrac{3}{2}$,

$\therefore\ y=-\frac{3}{4}$, $\therefore\ x=-2(-\frac{3}{4})=\frac{3}{2}$. *Ans.* $(\frac{3}{2},-\frac{3}{4})$

31. (a) $x-(7+3x)+1=0$, $-2x=6$, $\therefore\ x=-3$, $\therefore\ y=7+3(-3)=-2$

(b) Let $y=\frac{1}{p+q}, x=\frac{1}{p}$.

From (a), $-3=\frac{1}{p}$, $\therefore\ p=-\frac{1}{3}$

$-2=\frac{1}{p+q}$, $-2p-2q=1$, $-2(-\frac{1}{3})-2q=1$, $-2q=\frac{1}{3}$, $\therefore\ q=-\frac{1}{6}$

32. (a) A(2) + B(1) = 18, 2A + B = 18 ...... (1)
A(−6) + B(15) = 18, −2A + 5B = 6 ...... (2)
Solving (1) and (2), we have A = 7 and B = 4.

(b) $7(5)+4(k)=18$, $4k=-17$, $\therefore\ k=-\frac{17}{4}$

33. (a) Two straight lines will never intersect when they are parallel to each other.
(b) $6x-3y+2=0$, $12x-6y+4=0$..........(1)
$4x+ry+4=0$, $12x+3ry+12=0$..........(2)
They are parallel when $3r=-6$, $\therefore\ r=-2$

34. (a) $5x+y-4=0$..........(1) ; $2x+y-10=0$..........(2)
Solving (1) and (2), we have $x=-2$ and $y=14$.
Ans. *The coordinates of P are (–2 , 14).*
(b) $a(-2)+b(14)+16=0$, $a-7b=8$......(1)
$7a(-2)-2b(14)-14=0$, $a+2b=-1$......(2)
Solving (1) and (2), we get $a=1$ and $b=-1$.
35. $a+b=2m$..........(1) ; $a-b=2n$..........(2)
(1) + (2), $2a=2m+2n$, $a=m+n$...........(3)
Put (3) into (1), $(m+n)+b=2m$, $b=m-n$, $\therefore\ \dfrac{a}{b}=\dfrac{m+n}{m-n}$
36. (a) Let the equations be (1), (2), (3) and (4) respectively.
(1) + (2) + (3), $\therefore\ (x_1+x_2)+(x_2+x_3)+(x_3+x_1)=10+(-3)+9$,
$2(x_1+x_2+x_3)=16$, $x_1+x_2+x_3=8$.....(5).
Put (5) into (4), $y=8$.
(b) Put (2) into (5), $\therefore\ x_1+(-3)=8$, $x_1=11$.
Put (3) into (5), $\therefore\ x_2+9=8$, $x_2=-1$.
Put (1) into (5), $10+x_3=8$, $x_3=-2$
Ans. $x_1=11$, $x_2=-1$, $x_3=-2$.
37. (a) $16\,500=a+k(1\,000)$, $a+1\,000k=16\,500$.........(1)
$34\,500=a+k(5\,000)$, $a+5\,000k=34\,500$..........(2)
Solving (1) and (2), we get $a=12\,000$ and $k=4.5$
(b) When $n=40\,000$, $C=12\,000+4.5(40\,000)=192\,000$
Ans. *The average printing cost of 1 copy* = 19 200 ÷ 40 000 = $4.8.
38. Let $x and $y be the amounts that A and B originally had.
$2(x-240)=y+240$, $2x-y=720$..........(1)
$x+1\,010=3(y-1\,010)$, $x-3y=-4\,040$........(2)
Solving (1) and (2), we get $x=1240$ and $y=1760$.
Ans. *The amount they have altogether = 1 240 + 1 760 = $3 000*
39. Let x and y be the numbers of coins in P and Q respectively.
$2(x-22)=y+22$, $2x-y=66$..........(1)
$2(x-22)+46=5y+2(22)$, $2x-5y=42$..........(2)
Solving (1) and (2), we get $x=36$ and $y=6$.
Ans. *There are 36 coins in Box P and 6 coins in Box Q originally.*
40. Let x and y be the present ages of Adam and his sister respectively.
$x-4=4(y-4)$, $x-4y=-12$..........(1)
$x+8=2(y+8)$, $x-2y=8$..........(2)
Solving (1) and (2), we get $x=28$ and $y=10$.
Ans. *The sum of their present ages = 28 + 10 = 38.*
41. Let x and y be the present ages of the elder daughter and the woman respectively,
$\therefore$ the present age of the younger daughter is $(x-4)$.
$x+(x-4)+y=50$, $2x+y=54$..........(1)
$y-x=63-y$, $x-2y=-63$..........(2)
Solving (1) and (2), we get $x=9$ and $y=36$; $\therefore\ x-4=9-4=5$.
Ans. *The woman is 36 years old and the daughters are 9 and 5 years old.*
42. Let x m/s and y m/s be A's speed and B's speed respectively,
$\therefore$ the speed of C is $(y+1.5)$ m/s.
$x\times12\times60=y\times10\times60$, $6x-5y=0$..........(1)

$x \times 18 \times 60 = (y + 1.5) \times (18 - 6) \times 60$, $3x - 2y = 3$..........(2)

Solving (1) and (2), we get $x = 5$ and $y = 6$; $\therefore$ $y + 1.5 = 6 + 1.5 = 7.5$.

Ans. The speed of A, B and C are 5 m/s, 6 m/s and 7.5 m/s respectively.

43. The speeds of the boat against the current and along the current are $(x - y)$ km/h and $(x + y)$ km/h respectively.

$\therefore$ $\frac{315}{x-y} - \frac{315}{x+y} = 2$ .....(1), and $\frac{28}{x-y} = \frac{36}{x+y}$ .....(2)

Let $u = \frac{1}{x-y}$ and $v = \frac{1}{x+y}$.

From (1), $315u - 315v = 2$..........(3), from (2) $7u - 9v = 0$.........(4)

Solving (3) and (4), we get $u = \frac{1}{35}$ and $v = \frac{1}{45}$.

$\therefore$ $\frac{1}{x-y} = \frac{1}{35}$, $x - y = 35$.......(5), and $\frac{1}{x+y} = \frac{1}{45}$, $x + y = 45$.......(6)

Solving (5) and (6), we get x = 40 and y = 5.

44. Let x h and y h be the time taken by A and B respectively.

$\frac{1}{x} + \frac{1}{y} = \frac{1}{3}$..........(1), $\frac{1}{x} \times 4 + (\frac{1}{x} + \frac{1}{y}) \times 2 = 1$.........(2)

Put (1) into (2), $\frac{4}{x} + \frac{2}{3} = 1$, $\frac{1}{x} = \frac{1}{12}$, $\therefore$ $x = 12$.

$\frac{1}{12} + \frac{1}{y} = \frac{1}{3}$; $\frac{1}{y} = \frac{1}{4}$, $\therefore$ $y = 4$

Ans. A and B take 12 h and 4 h respectively to fill up the pool alone.

45. Let x days and y days be the time taken by A and B respectively.

$x = 2y$.........(1); $\frac{1}{x} + \frac{1}{y} = \frac{1}{14}$.........(2)

Put (1) into (2), $\therefore$ $\frac{1}{2y} + \frac{1}{y} = \frac{1}{14}$, $\frac{3}{2y} = \frac{1}{14}$, $y = \frac{3}{2} \times 14 = 21$; $\therefore$ $x = 2(21) = 42$

Ans. A and B take 42 days and 21 days respectively.

46. Let x and y be the original numbers of workers and days required respectively,

$(x + 8)(y - 10) = xy$, $-10x + 8y = 80$..........(1)

$(x - 8)(y + 20) = xy$, $20x - 8y = 160$.........(2)

Solving (1) and (2), we get $x = 24$ and $y = 40$.

Ans. The numbers of workers and days required are 24 and 40 respectively.

47. (a) Let \$$x$, \$$y$ and \$$z$ be the costs of an apple, an orange and a pear respectively.

$x + 3y + 2z = 22.6$..........(1); $5x + 8y + 3z = 62.6$........(2)

$(1) \times 5 - (2)$, $7y + 7z = 50.4$, $y + z = 7.2$.........(3)

Ans. The total cost of 1 orange and 1 pear is \$7.2.

(b) $5y = 4z$, $5y - 4z = 0$..........(4).

Solving, (3) and (4), we have $y = 3.2$, $z = 4$.

From (1), $x + 3(3.2) + 2(4) = 22.6$, $x = 5$

Ans. The costs of an apple, an orange and a pear are \$5, \$3.2 and \$4 respectively.

48. Let x and y be the numbers of males and females in project A respectively,

$\therefore$ in project B, the no. of males = $181 - x$, and the no. of females = $154 - y$.

$x = 2y + 3$, $x - 2y = 3$.....(1);

$3(181 - x) = 154 - y$, $3x - y = 389$......(2)

Solving (1) and (2), we get $x = 155$ and $y = 76$.

Ans. There are 155 males and 76 females in project A, while there are 26 males and 78 females in project B.

49. (a) $\begin{cases} 5x - y = 2 & \cdots\cdots(1) \\ 10x + 2y = 12 & \cdots\cdots(2) \end{cases}$, (2) – (1) × 2, $4y = 8$, $y = 2$.

Sub. $y = 2$ into (1), $x = \dfrac{2+2}{5} = \dfrac{4}{5}$. $\therefore\ x = \dfrac{4}{5}$, $y = 2$

(b) $\begin{cases} \dfrac{5}{2m+5n} - \dfrac{m}{2} + n = 2 \\ \dfrac{10}{2m+5n} + m - 2n = 12 \end{cases}$, $\begin{cases} 5\left(\dfrac{1}{2m+5n}\right) - \dfrac{m-2n}{2} = 2 \\ 10\left(\dfrac{1}{2m+5n}\right) + 2\left(\dfrac{m-2n}{2}\right) = 12 \end{cases}$.

From (a), $\begin{cases} \dfrac{1}{2m+5n} = \dfrac{4}{5} \\ \dfrac{m-2n}{2} = 2 \end{cases}$, $\therefore$ $\begin{cases} 2m + 5n = \dfrac{5}{4} & \cdots\cdots(3) \\ m - 2n = 4 & \cdots\cdots(4) \end{cases}$,

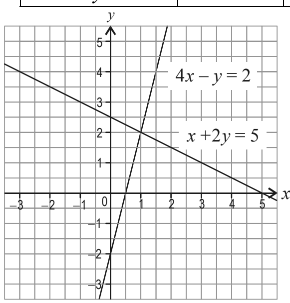
(3) – (4) × 2, $9n = -\dfrac{27}{4}$, $n = -\dfrac{3}{4}$.

Sub. $n = -\dfrac{3}{4}$ into (4), $m = 4 + 2\left(-\dfrac{3}{4}\right) = \dfrac{5}{2}$. $\therefore\ m = \dfrac{5}{2}$, $n = -\dfrac{3}{4}$

50. (a)

x	–3	0	4
y	4	2.5	0.5

(b)



(c) (i) From the graph, the solutions are $(x, y) = (1, 2)$.

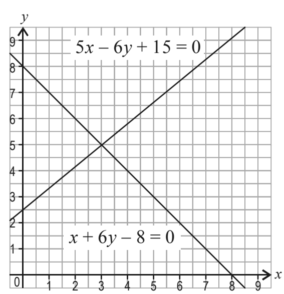
(ii) $\begin{cases} \dfrac{1}{2a} + \dfrac{2}{b} = 5 \\ \dfrac{2}{a} - \dfrac{1}{b} = 2 \end{cases}$, $\begin{cases} \dfrac{1}{2a} + 2\left(\dfrac{1}{b}\right) = 5 \\ 4\left(\dfrac{1}{2a}\right) - \dfrac{1}{b} = 2 \end{cases}$.

From (c)(i), $\dfrac{1}{2a} = 1$, $\dfrac{1}{b} = 2$,

$\therefore\ a = \dfrac{1}{2}$, $b = \dfrac{1}{2}$

51. (a)

x	0	6	9
y	2.5	7.5	10



(b) (i) $\begin{cases} \dfrac{x+3}{y} = \dfrac{6}{5} \\ \dfrac{x}{y-8} = -1 \end{cases}$ $\therefore$ $\begin{cases} 5x - 6y + 15 = 0 \\ x + y - 8 = 0 \end{cases}$.

(ii) From (a), the solutions of $\begin{cases} 5x - 6y + 15 = 0 \\ x + y - 8 = 0 \end{cases}$ are $(x, y) = (3, 5)$.

$\therefore$ the fraction is $\dfrac{3}{5}$.

52. (a) $\begin{cases} a - 2b = -0.4 & \cdots\cdots(1) \\ a + b = 3.2 & \cdots\cdots(2) \end{cases}$.

(2) – (1), $3b = 3.6$, $b = 1.2$
Sub. $b = 1.2$ into (2), $a = 3.2 - 1.2 = 2$,
$\therefore\ a = 2$, $b = 1.2$.

(b) (i) Let w hours be the driving time from city A to city B.

$\begin{cases} x = 2w - 0.4 \\ x + w = 3.2 \end{cases}$, $\begin{cases} x - 2w = -0.4 \\ x + w = 3.2 \end{cases}$. From (a), $x = 2$, $w = 1.2$.

$y = \dfrac{96}{w} = \dfrac{96}{1.2} = 80$. *Ans.* $x = 2$, $y = 80$.

(ii) Speed from home to City A = $\dfrac{96}{2} =$ 48 km/h

Speed from home to City B = $\dfrac{96}{1.2} =$ 80 km/h

The required difference = 80 – 48 = 32 km/h

(iii) Time for his whole journey = 2 + 2 = 4 hours.

$\therefore$ The claim is disagreed.

Unit 13 Rational & irrational numbers

1. (a) 5.20 (b) 3.48 (c) 63.2 (d) $= \sqrt{65} = 8.06$
   (e) $= \sqrt{99} = 9.95$ (f) $= \sqrt{1600} = 40$
2. (a) $= \sqrt{12^2} = 12$ (b) $= \sqrt{13^2} = 13$ (c) $= -\sqrt{4 \times 144} = -2 \times 12 = -24$
   (d) $= \dfrac{5}{9}$ (e) $= -\sqrt{\dfrac{64}{49}} = -\dfrac{8}{7}$ (f) $= \sqrt{\dfrac{225}{10\,000}} = \sqrt{\dfrac{9}{400}} = \dfrac{3}{20}$
3. (a) $\because$ $25 < 35 < 36$, $\sqrt{25} < \sqrt{35} < \sqrt{36}$, $5 < \sqrt{35} < 6$, $\therefore$ the integral part of $\sqrt{35}$ is 5.
   (b) $\because$ $81 < 92 < 100$, $\sqrt{81} < \sqrt{92} < \sqrt{100}$, $9 < \sqrt{92} < 10$,
   $\therefore$ the integral part of $\sqrt{92}$ is 9.
   (c) $\because$ $100 < 106 < 121$, $\sqrt{100} < \sqrt{106} < \sqrt{121}$, $10 < \sqrt{106} < 11$,
   $\therefore$ the integral part of $\sqrt{106}$ is 10.
4. (a) 4.0 (b) –4.6 (c) –2
5. (a) $= 1 - 8 = -7$, $\therefore$ rational (b) $= \sqrt{3 \times 81} = \sqrt{3} \times 9$, $\therefore$ irrational
   (c) recurring decimal, $\therefore$ rational
6. (a) $= 2\sqrt{10}$, $\therefore$ yes (b) $= \dfrac{16}{17}$, $\therefore$ no
   (c) $= 30\sqrt{10}$, $\therefore$ yes (d) $= \sqrt{\dfrac{36}{3}} = \sqrt{12} = 2\sqrt{3}$, $\therefore$ yes
   (e) $= \sqrt{45} = 3\sqrt{5}$, $\therefore$ yes (f) $= 3$, $\therefore$ no
7. (a) $= \sqrt{4 \times 2} = 2\sqrt{2}$ (b) $= \sqrt{9 \times 2} = 3\sqrt{2}$ (c) $= \sqrt{9 \times 3} = 3\sqrt{3}$
   (d) $= \sqrt{36 \times 2} = 6\sqrt{2}$ (e) $= \sqrt{9 \times 5} = 3\sqrt{5}$ (f) $= \sqrt{16 \times 3} = 4\sqrt{3}$
   (g) $= \sqrt{49 \times 2} = 7\sqrt{2}$ (h) $= \sqrt{25 \times 3} = 5\sqrt{3}$ (i) $= \sqrt{121 \times 3} = 11\sqrt{3}$
   (j) $= \sqrt{81 \times 3} = 9\sqrt{3}$ (k) $= \sqrt{81 \times 5} = 9\sqrt{5}$ (l) $= \sqrt{25 \times 5} = 5\sqrt{5}$
   (m) $= \sqrt{2} \times \sqrt{5^2} \times \sqrt{m^2} \times \sqrt{n^4} = 5mn^2\sqrt{2}$ (n) $= \sqrt{2 \times 3^2 \times ab \times b^4} = 3b^2\sqrt{2ab}$
8. (a) $5\sqrt{3} - 8\sqrt{5}$ (b) $7\sqrt{7} - 2\sqrt{11}$
   (c) $3\sqrt{w}$ (d) $5\sqrt{y} - 2\sqrt{x}$
9. (a) $= 2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$ (b) $= 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$
   (c) $= 3\sqrt{2} - 4\sqrt{2} + 5\sqrt{2} = 4\sqrt{2}$ (d) $= 2\sqrt{3} + 5\sqrt{3} - 3\sqrt{2} = 7\sqrt{3} - 3\sqrt{2}$
   (e) $= 2\sqrt{3} + 6\sqrt{3} - 5\sqrt{3} = 3\sqrt{3}$ (f) $= 2\sqrt{7} - 5\sqrt{7} - 3\sqrt{7} = -6\sqrt{7}$

(g) $= 8\sqrt{x} - 7\sqrt{x} = \sqrt{x}$ (h) $= 2m\sqrt{2mn} + 3m\sqrt{2mn} = 5m\sqrt{2mn}$

10. (a) $= 24 \times 2 = 48$ (b) $= 10 \times 3 \times \sqrt{3} = 30\sqrt{3}$
(c) $= 36 \times 2 = 72$ (d) $= 9 \times 5 = 45$
(e) $= (2 \times \sqrt{2}) \times (\sqrt{2} \times \sqrt{13}) = 4\sqrt{13}$ (f) $= (\sqrt{6} \times \sqrt{10}) \times \sqrt{10} = 10\sqrt{6}$
(g) $= (6 \times \sqrt{3}) \times (5 \times \sqrt{3} \times \sqrt{2}) = 90\sqrt{2}$ (h) $= 15\sqrt{6}$
(i) $= 2\sqrt{33} \times \sqrt{33} = 66$ (j) $= 3\sqrt{17} \times 2\sqrt{17} = 6 \times 17 = 102$
(k) $= 3\sqrt{7ab} \times (a\sqrt{7ab} \times \sqrt{2}) = 3a \times 7ab \times \sqrt{2} = 21a^2b\sqrt{2}$
(l) $= (\sqrt{3n} \times \sqrt{7mn}) \times \sqrt{7mn} = 7mn\sqrt{3n}$

11. (a) 6 (b) $= \sqrt{10} \times 2 = 2\sqrt{10}$
(c) $= \dfrac{\sqrt{2} \times 3\sqrt{7}}{3 \times 3\sqrt{7}} = \dfrac{\sqrt{2}}{3}$ (d) $= \dfrac{5 \times \sqrt{3}}{2 \times \sqrt{3}} = \dfrac{5}{2}$
(e) $= \dfrac{2\sqrt{2} \times \sqrt{14}}{3 \times \sqrt{14}} = \dfrac{2\sqrt{2}}{3}$ (f) $= \dfrac{(4 \times \sqrt{3} \times \sqrt{5}) \times (2 \times 3 \times \sqrt{3})}{4 \times 3 \times \sqrt{5}} = 2 \times 3 = 6$

12. (a) $= 2 - 4\sqrt{2} - 5 = -3 - 4\sqrt{2}$
(b) $= 30 - 17\sqrt{5} + 12 = 42 - 17\sqrt{5}$
(c) $= (7 - 3\sqrt{3})(\sqrt{3} - 2) = 7\sqrt{3} - 14 - 9 + 6\sqrt{3} = 13\sqrt{3} - 23$
(d) $= \sqrt{14} \cdot \sqrt{7} - \sqrt{14} \cdot \sqrt{2} + 5\sqrt{7} - 5\sqrt{2} = 7\sqrt{2} - 2\sqrt{7} + 5\sqrt{7} - 5\sqrt{2} = 2\sqrt{2} + 3\sqrt{7}$
(e) $= 5y - 6 + 7\sqrt{y}$
(f) $= 3x - 4\sqrt{xy} + 6\sqrt{xy} - 8y = 3x - 8y + 2\sqrt{xy}$

13. (a) $= 7 - 2\sqrt{35} + 5 = 12 - 2\sqrt{35}$ (b) $= 18 + 12\sqrt{2} + 4 = 22 + 12\sqrt{2}$
(c) $= 48 + 8\sqrt{15} + 5 = 53 + 8\sqrt{15}$ (d) $= 24 - 2\sqrt{24}\sqrt{12} + 12 = 36 - 24\sqrt{2}$
(e) $= 25x + 4 - 20\sqrt{x}$ (f) $= 9x + 49y + 42\sqrt{xy}$

14. (a) $= 5 - 4 = 1$ (b) $= 9 - 20 = -11$ (c) $x - 4y^2$
(d) $= (4\sqrt{a} - 3\sqrt{b})(4\sqrt{a} + 3\sqrt{b}) = 16a - 9b$

15. (a) $= \dfrac{2\sqrt{7} \times \sqrt{7}}{\sqrt{7}} = 2\sqrt{7}$ (b) $= \dfrac{8 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \dfrac{8\sqrt{5}}{5}$ (c) $= \dfrac{4\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = 4\sqrt{3}$
(d) $= \dfrac{4}{2\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$ (e) $= \dfrac{24}{4\sqrt{3}} = \dfrac{2 \times 3}{\sqrt{3}} = 2\sqrt{3}$ (f) $= \dfrac{6\sqrt{7} \times \sqrt{7}}{2\sqrt{7}} = 3\sqrt{7}$
(g) $= \dfrac{5 \times 2}{\sqrt{5} \times \sqrt{3}} = \dfrac{\sqrt{5} \times 2}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{15}}{3}$
(h) $= \dfrac{\sqrt{4x} \times \sqrt{a}}{\sqrt{a} \times \sqrt{a}} = \dfrac{2\sqrt{ax}}{a}$ (i) $= \dfrac{2m \times \sqrt{n}}{3\sqrt{n} \times \sqrt{n}} = \dfrac{2m\sqrt{n}}{3n}$

16. $= \dfrac{1}{4}(10)\sqrt{3} + \dfrac{\sqrt{3}}{2} - 2\sqrt{3} + \dfrac{\sqrt{3}}{3} = \sqrt{3}(\dfrac{5}{2} + \dfrac{1}{2} - 2 + \dfrac{1}{3}) = \dfrac{4\sqrt{3}}{3}$

17. (a) $12\,100 = 121 \times 100 = (11 \times 10)^2 = 110^2$. Also, note that $(-110)^2 = 12\,100$,
$\therefore$ the square roots of 12100 are 110 and –110.
(b) $-8\,000 = -(8 \times 1\,000) = (-1)^3(2 \times 10)^3 = (-20)^3$, $\therefore$ the cube root of –8 000 is –20.
(c) $0.0625 = \dfrac{625}{10\,000} = \dfrac{5^4}{10^4} = \left(\dfrac{1}{2}\right)^4 = 0.5^4$. Also, note that $(-0.5)^4 = 0.0625$,
$\therefore$ the fourth roots of 0.0625 are 0.5 and –0.5.

18. $x=\sqrt{3}, y=\sqrt{27}=\sqrt{3^3}=(\sqrt{3})^3$, $\therefore$ $y=x^3$; but $y=\sqrt{27}=\sqrt{3^3}=3\sqrt{3}$. $\therefore$ $y=3x$.
Ans. Both of them are correct.

19. (a) $\sqrt{12}=2\sqrt{3}\approx 2\times 1.732=3.464$ (b) $\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}\approx\frac{1.732}{3}=0.577$ (3 sig. fig.)

20. (a) $\sqrt{1.8}=\sqrt{\frac{18}{10}}=\sqrt{\frac{3^2}{5}}=\frac{3}{\sqrt{5}}=\frac{3\sqrt{5}}{5}=\frac{3\times 2.236}{5}=1.3416$
(b) $\sqrt{12}-\sqrt{8}=2\sqrt{3}-2\sqrt{2}=2(\sqrt{3}-\sqrt{2})=2\ (1.732-1.414)=0.636$

21. (a) $=(10)-1=9$
(b) $9=(\sqrt{10}-1)(\sqrt{10}+1)$, $\therefore$ $3=\frac{(\sqrt{10}-1)(\sqrt{10}+1)}{3}$, $\frac{3}{\sqrt{10}+1}=\frac{\sqrt{10}-1}{3}$
(c) L.H.S. $=\frac{3}{\sqrt{10}+1}\cdot\frac{\sqrt{10}-1}{\sqrt{10}-1}=\frac{3(\sqrt{10}-1)}{10-1}=\frac{3(\sqrt{10}-1)}{9}=\frac{\sqrt{10}-1}{3}=$ R.H.S.

22. $=\frac{(\sqrt{a})^2}{\sqrt{a}\sqrt{a+1}}-\frac{(\sqrt{a+1})^2}{\sqrt{a}\sqrt{a+1}}-\frac{1}{\sqrt{a}\sqrt{a+1}}=\frac{a-a-1-1}{\sqrt{a}\sqrt{a+1}}=\frac{-2}{\sqrt{a(a+1)}}$

23. $(\sqrt{3}+1)\sqrt{x}=2, \sqrt{x}=\frac{2}{\sqrt{3}+1}=\frac{2(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{2(\sqrt{3}-1)}{3-1}=\sqrt{3}-1$
$\therefore$ $x=(\sqrt{3}-1)^2=3-2\sqrt{3}+1=4-2\sqrt{3}$. $\therefore$ $a=4, b=-2$.

24. $(x-\frac{1}{x})^2=(3\sqrt{5}-2)^2, x^2-2+\frac{1}{x^2}=45-12\sqrt{5}+4.$ $\therefore$ $x^2+\frac{1}{x^2}=51-12\sqrt{5}$

25. $\text{AB}=\sqrt{27}cm=3\sqrt{3}cm$, $\text{BC}=\sqrt{75}cm=5\sqrt{3}cm$, $\text{AC}=12cm=2\sqrt{3}cm$
$\text{AB}+AC=3\sqrt{3}+2\sqrt{3}=5\sqrt{3}=\text{BC}$. $\therefore$ A, B, C form a straight line (line segment).

26. $\sqrt{32}+\sqrt{18}=4\sqrt{2}+3\sqrt{2}=7\sqrt{2}$, but $\sqrt{147}=\sqrt{7\times 7\times 3}=7\sqrt{3}$, $\therefore$ $\sqrt{32}+\sqrt{18}<\sqrt{147}$
Ans. Since the sum of 2 sides is smaller than the third side, they cannot form a triangle.

27. $(2\sqrt[3]{3})^3=(\sqrt{x})^3,\ 2^3\cdot 3=\sqrt{x^3}$, $\therefore$ $2^3\cdot 3=\sqrt{y},\ \sqrt{y}=24, y=24^2=576$

28. (a) $\frac{m}{n}=(\sqrt{5}-2)\div\frac{1}{\sqrt{5}+2}=(\sqrt{5}-2)(\sqrt{5}+2)=5-4=1$
(b) $\frac{m^{2005}}{n^{2003}}=(\frac{m}{n})^{2003}\times m^2=(1)^{2003}\times(\sqrt{5}-2)^2=5-4\sqrt{5}+4=9-4\sqrt{5}$

29. $=\left[\left(\sqrt{3}-\sqrt{2}+1\right)\left(\sqrt{3}+\sqrt{2}-1\right)\right]^5$
$=\left\{\left[\sqrt{3}-\left(\sqrt{2}-1\right)\right]\left[\sqrt{3}+\left(\sqrt{2}-1\right)\right]\right\}^5$
$=\left[(\sqrt{3})^2-\left(\sqrt{2}-1\right)^2\right]^5=\left[3-\left(2-2\sqrt{2}+1\right)\right]^5=\left(2\sqrt{2}\right)^5$
$=2^5\times\left(\sqrt{2}\right)^5=32\times\left(\sqrt{2}\right)^4\sqrt{2}=32\times 2^2\times\sqrt{2}=128\sqrt{2}$

30. (a) L.H.S. $=\frac{a-b}{\sqrt{a}-\sqrt{b}}=\frac{(\sqrt{a})^2-(\sqrt{b})^2}{\sqrt{a}-\sqrt{b}}=\frac{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}{\sqrt{a}-\sqrt{b}}=\sqrt{a}+\sqrt{b}$
R.H.S. $=\sqrt{a}+\sqrt{b}=$ L.H.S. $\therefore$ It is an identity.

(b) From (a), $\dfrac{a-b}{\sqrt{a}-\sqrt{b}} \equiv \sqrt{a}+\sqrt{b}$, $\therefore \dfrac{1}{\sqrt{a}+\sqrt{b}} = \dfrac{\sqrt{a}-\sqrt{b}}{a-b}$ ...... (*)

$$\frac{1}{\sqrt{2}+\sqrt{1}}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+...+\frac{1}{\sqrt{100}+\sqrt{99}}$$

$$=(\frac{\sqrt{2}-\sqrt{1}}{2-1})+(\frac{\sqrt{3}-\sqrt{2}}{3-2})+(\frac{\sqrt{4}-\sqrt{3}}{4-3})+\ldots+\frac{\sqrt{100}-\sqrt{99}}{100-99} \quad \text{[From (*)]}$$

$$=(\sqrt{2}-\sqrt{1})+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+.....+(\sqrt{100}-\sqrt{99})$$

$$=-\sqrt{1}+\sqrt{100}=-1+10=9$$

Unit 14 Pythagoras' theorem

1. (a) $x^2 = 10^2 + 24^2$, $x = \sqrt{100+576} = \sqrt{676} = 26$

   (b) $17^2 = y^2 + 8^2$, $y^2 = 17^2 - 8^2$, $y = \sqrt{289-64} = \sqrt{225} = 25$

   (c) $x^2 = 15^2 - 10^2 = 225 - 100 = 125$, $x = \sqrt{125} = 11.2$

   $y^2 = x^2 - 8^2 = 125 - 64 = 61$, $y = \sqrt{61} = 7.8$

   (d) $20^2 = h^2 + (\frac{24}{2})^2$, $h = \sqrt{400-144} = \sqrt{256} = 16$ cm

2. (a) PS = $\sqrt{10^2 - 6^2} = \sqrt{64} = 8$; SR = $\sqrt{8^2 - 6^2} = \sqrt{28}$, a = PS + SR = 8 + $\sqrt{28}$ = 13.3

   (b) AC = 3 + 6 = 9, $BC^2 = 10^2 - 9^2 = 19$,

   $m^2 = BC^2 + 6^2 = 19 + 36 = 55$, $m = \sqrt{55} = 7.42$

   (c) GF = $\sqrt{13^2 - 12^2} = \sqrt{169-144} = \sqrt{25} = 5$, EF = $\sqrt{15^2 - 12^2} = \sqrt{225-144} = \sqrt{81} = 9$

   x = EF – GF = 9 – 5 = 4

   (d) $x = \sqrt{39^2 - 36^2} = \sqrt{225} = 15$, ML = $\sqrt{45^2 - 36^2} = \sqrt{729} = 27$

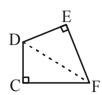
   y = ML – x = 27 – 15 = 12

   (e) In ΔPQR, $PQ^2 + RQ^2 = PR^2$,

   $(n+8)^2 + 10^2 = 26^2$, $(n+8)^2 = 26^2 - 10^2$,

   (f) Join DF. $DF^2 = 6^2 + 10^2$, and $DF^2 = 7^2 + y^2$,

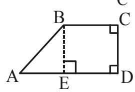
   $\therefore$ $y^2 + 49 = 36 + 100$, $y^2 = 87$, $\therefore$ $y = \sqrt{87} = 9.33$



3. (a) Draw BE $\perp$ AD. BE = CD = 8, ED = BC = 7,

   AE = AD – ED = 13 – 7 = 6

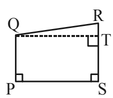
   $y = \sqrt{BE^2 + AE^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$



   (b) Draw QT $\perp$ RS. QT = PS = 40, TS = QP = 15

   RT = $\sqrt{QR^2 - QT^2} = \sqrt{41^2 + 40^2} = \sqrt{81} = 9$

   x = RT + TS = 9 + 15 = 24



4. $(x+2)^2 = x^2 + 12^2$, $x^2 + 4x + 4 = x^2 + 144$, $4x = 140$, $\therefore$ $x = 35$

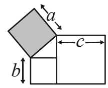
5. Let BC = x cm. $\therefore$ AB = $2x$ cm.

   $AB^2 = 4^2 + BC^2$, $(2x)^2 = 16 + x^2$, $3x^2 = 16$, $x = \sqrt{\frac{16}{3}}$; AB = $2x$ = $2(\sqrt{\frac{16}{3}})$ = 4.62 cm

6. BD = 14 + 10 = 24; AB = $\sqrt{25^2 - BD^2} = \sqrt{25^2 - 24^2} = \sqrt{49} = 7$

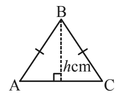
   Area of ΔACD = $\frac{1}{2}$(AB)(CD) = $\frac{1}{2}$(7)(10) = 35 sq. units

7. $b=\sqrt{49}=7$; $c=\sqrt{169}=13$;
$a^2=b^2+(c-b)^2=7^2+(13-7)^2=49+36=85$
The shaded area $=a^2=85\text{ cm}^2$



8. Let the altitude from B to AC be h cm. $\frac{8h}{2}=24, h=6$

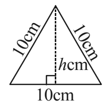
$AB^2=h^2+(\frac{8}{2})^2=6^2+4^2=52$, $AB=\sqrt{52}=7.21\text{ cm}$



9. Let the height of the equilateral triangle be h cm.

$10^2=h^2+(\frac{10}{2})^2$, $h^2=100-25=75$, $h=\sqrt{75}$ cm

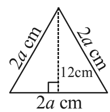
$\therefore$ Area $=\frac{1}{2}(10)(\sqrt{75})=43.3\text{ cm}^2$



10. Let the side of the equilateral triangle be $2a$ cm.

$(2a)^2=12^2+a^2$, $4a^2=144+a^2$, $3a^2=144$, $a=\sqrt{\frac{144}{3}}=\sqrt{48}$

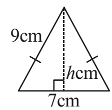
$\therefore$ Area $=\frac{1}{2}(2\sqrt{48})(12)=8.31\text{ cm}^2$



11. Let h cm be the altitude.

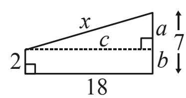
$9^2=h^2+(\frac{7}{2})^2$, $h^2=81-\frac{49}{4}=\frac{275}{4}$, $h=\sqrt{\frac{275}{4}}$

$\therefore$ Area $=\frac{1}{2}(7)(\sqrt{\frac{275}{4}})=29.0\text{ cm}^2$



12. Let its sides be x m. $x^2+x^2=30$, $2x^2=30$, $x^2=15$, $x=\sqrt{15}$
$\therefore$ Area $=x^2=15\text{ m}^2$; perimeter $=4x=4\sqrt{15}=15.5$ m

13. Let x m be the distance between the tops of the two posts.
$c=18$, $b=2$, $a=7-b=7-2=5$
$x=\sqrt{a^2+c^2}=\sqrt{5^2+18^2}=18.7$
Ans. The distance between the tops is 18.7 m.



14. (a) $AC=\sqrt{15^2+20^2}=\sqrt{625}=25$ cm
(b) $AS=\sqrt{60^2+AC^2}=\sqrt{3\,600+625}=\sqrt{4225}=65$ cm

15. The diagonal of the rectangle with dimensions 8cm × 10cm $=b\sqrt{8^2+10^2}=\sqrt{164}$ cm.
$\therefore$ The longest pencil $=\sqrt{4^2+164}=\sqrt{180}=13.4$ cm

16. $BC=\sqrt{50^2-48^2}=\sqrt{196}=14$ cm. Area $=(48)(14)=672\text{ cm}^2$

17. Let his speed be x km/h. The distance be traveled in 40 mins $=x\times\frac{40}{60}=\frac{2}{3}x$ km

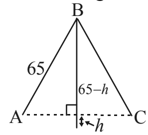
$(\frac{2}{3}x)^2+35^2=37^2, \frac{4}{9}x^2=37^2-35^2=144$, $x=\sqrt{\frac{144\times 9}{4}}=18$

Ans. His speed was 18 km/h.

18. Let h cm be the distance between the highest and the lowest point of the swing.

AC = 25 cm, $\therefore$ $(65-h)^2+(\frac{25}{2})^2=65^2$,

$(65-h)^2=\sqrt{65^2-12.5^2}$, $65-h=63.79$, $h=1.21$
Ans. The distance is 1.21 cm.



19. (a) $AB^2=52^2=2704$, $AC^2+BC^2=48^2+20^2=2704$
$\because$ $AB^2=AC^2+BC^2$, $\therefore$ ΔABC is right-angled.

(b) $PR^2 = 41^2 = 1681$, $PQ^2 + QR^2 = 38^2 + 15^2 = 1669$

$\because$ $PR^2 \neq PQ^2 + QR^2$, $\therefore$ ΔPQR is not right-angled.

(c) $CE^2 = 137^2 = 18769$, $CD^2 + DE^2 = 88^2 + 105^2 = 18769$

$\therefore$ $CE^2 = CD^2 + DE^2$, $\therefore$ ΔCDE is right-angled.

20. (a) $PQ^2 = 93$, $PR^2 + QR^2 = (3\sqrt{5})^2 + (4\sqrt{3})^2 = 45 + 48 = 93$

$\because$ $PR^2 + QR^2 = PQ^2$, $\therefore$ ΔPQR is a right-angled triangle.

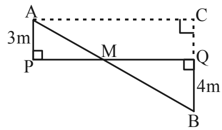
(b) $\because$ $PR \perp QR$, $\therefore$ area $= \frac{1}{2}(3\sqrt{5})(4\sqrt{3}) = 6\sqrt{15}$ cm^2

21. No, the longest side is 175. He should compare: "$49^2 + 168^2$" with 175^2.

$\because$ $49^2 + 168^2 = 2\,401 + 28\,224 = 30\,625 = 175^2$

$\therefore$ It is a right-angled triangle (converse of Pyth. Thm.)

$\therefore$ I disagree with Simon.



22. $AC = PQ = 24$, $CB = CQ + QB = AP + QB = 3 + 4 = 7$

$\therefore$ $AB = \sqrt{24^2 + 7^2} = \sqrt{625} = 25$ m

23. $OQ = OC + CQ = 9 + 6 = 15$ (cm),

$OB = OQ = 15$ (radii), $OC^2 + BC^2 = OB^2$,

$9^2 + BC^2 = 15^2$, $BC^2 = 15^2 - 9^2$, $BC = \sqrt{225 - 81} = 12$ cm

$\therefore$ The area of the rectangle $= OC \times BC = 9 \times 12 = 108$ cm^2

24. The nearer vertical distance $= 12 - 4 = 8$ cm

$\therefore$ The distance the foot of the ladder will slide

$= \sqrt{20^2 - 8^2} - \sqrt{20^2 - 12^2} = \sqrt{336} - 16 = 2.33$ cm

25. The horizontal distance between P and Q $= 112 + 14 + 14 = 140$

The vertical distance between P and Q $= 16 + 16 + 16 = 48$

The shortest distance between P and Q $= \sqrt{140^2 + 48^2} = \sqrt{21904} = 148$ (Pyth. Thm.)

26. $AC = \sqrt{AB^2 + BC^2} = \sqrt{7^2 + 24^2} = 25$

Area of $\Delta ABC = \frac{1}{2}(AC)(BM) = \frac{1}{2}(AB)(BC)$, $25BM = (7)(24)$, $BM = 6.72$ cm

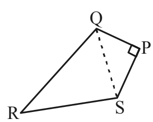
27. Let $BC = x$ cm, $\therefore$ $CD = (24 - x)$ cm.

$AC^2 = 16^2 + x^2$, and $CE^2 = 12^2 + (24 - x)^2$

$\because$ $AC = CE$, $\therefore$ $16^2 + x^2 = 12^2 + (24 - x)^2 = 12^2 + 24^2 - 48x + x^2$

$48x = 12^2 + 24^2 - 16^2$, $x = \frac{464}{48} = 9\frac{2}{3}$, $\therefore$ $BC = 9\frac{2}{3}$ cm

28. Join QS.



$QS = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm (Pyth. Thm.)

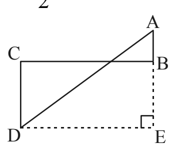
In $\triangle QSR$, $QS^2 + SR^2 = 10^2 + 24^2 = 100 + 576 = 676$

$QR^2 = 26^2 = 676$, $\because$ $QS^2 + SR^2 = QR^2$,

$\therefore$ $\angle QSR = 90°$ (converse of Pyth. Thm.)

$\therefore$ Area of PQRS $= \frac{1}{2}(PQ)(PS) + \frac{1}{2}(QS)(SR) = \frac{1}{2}(6)(8) + \frac{1}{2}(10)(24) = 144$ cm^2

29. $AB = 72 \times \frac{6}{60} = 7.2$ km, $DE = BC = 72 \times \frac{10}{60} = 12$ km,



$BE = CD = 8$ km. $\therefore$ $AE = 7.2 + 8 = 15.2$ km

$AD = \sqrt{DE^2 + AE^2} = \sqrt{12^2 + 15.2^2} = 19.4$ km

Ans. He is 19.4km from the starting point.

30. $DQ = \sqrt{24^2 + 18^2} = \sqrt{900} = 30$ cm.

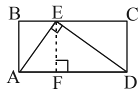
Let h cm be the distance between PB and DQ.

Area of ABCD = $2 \times \frac{1}{2}(24)(18) + (DQ)(h) = 24^2$

$432 + 30h = 576, \quad h = \frac{144}{30} = 4.8$. *Ans. The distance between PB and DQ is 4.8 cm.*

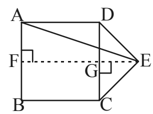
31. In ΔABC, $AB^2 + BC^2 = 24^2 + 10^2 = 676$, $AC^2 = 26^2 = 676$
$\because AC^2 + BC^2 = AB^2$, $\therefore \angle B = 90°$ (Converse of Pyth. Thm.)
$\angle B = \angle Q = 90°$ (proved), AB = PQ and BC = QR (given), $\therefore \Delta ABC \cong \Delta PQR$ (S.A.S.)

32. $ED^2 = \sqrt{25^2 - 7^2} = \sqrt{576} = 24$.
Let EF be the height from E to AD.



Area of $\Delta AED = \frac{1}{2}(AE)(ED) = \frac{1}{2}(AD)(EF)$
$\therefore$ 25EF = (7)(24), EF = 6.72; AB = EF = 6.72
$\therefore$ Area of ABCD = (25)(6.72) = 168 sq. units

33. From E, draw $EF \perp AB$. $DG = AF = \frac{1}{2}(12) = 6$ cm.



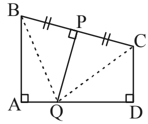
$GE = \sqrt{DE^2 - DG^2} = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$ cm
FG = BC = 12 cm; FE = FG + GE = 12 + 8 = 20 cm
In ΔAFE, $AE = \sqrt{AF^2 + FE^2} = \sqrt{6^2 + 20^2} = \sqrt{436} = 20.9$ cm

34. (a) DQ = 18 – 16 = 2 cm; $BQ = \sqrt{16^2 + 12^2} = 20$ cm (Pyth. Thm.)
(b) In ΔDPQ, $PQ^2 = PD^2 + DQ^2 = x^2 + 2^2 = x^2 + 4$
In ΔPQB, $y^2 = PQ^2 + BQ^2 = (x^2 + 4) + 20^2 = x^2 + 404$
(c) In ΔAPB, $AP = (12 - x)$cm, $\therefore y^2 = AP^2 + 18^2 = (12 - x)^2 + 18^2$
$\therefore (12 - x)^2 + 18^2 = x^2 + 404$,
$144 - 24x + x^2 + 324 = x^2 + 404$, $64 = 24x$, $\therefore x = \frac{8}{3}$
From (b), $y^2 = (\frac{8}{3})^2 + 404 = \frac{3700}{9}$, $y = \sqrt{\frac{3700}{9}} = \frac{10\sqrt{37}}{3}$

35. $AC = EG = \sqrt{1^2 + 1^2} = \sqrt{2}$ (Pyth. Thm.)
In ΔDEG and ΔACG, $\angle DEG = \angle ACG = 90° + 45° = 135°$ (square),
$\frac{DE}{AC} = \frac{1}{\sqrt{2}}$, $\frac{EG}{CG} = \frac{\sqrt{2}}{1+1} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$, $\frac{DE}{AC} = \frac{EG}{CG}$,
$\therefore \Delta DEG \sim \Delta ACG$ (ratio of 2 sides, inc. $\angle$)

36. Let QD = x cm. QC = AQ = AD – QD = $(16 - x)$ cm; CD = AB = 10 cm
In ΔQCD, $QC^2 = QD^2 + CD^2$,
$(16 - x)^2 = x^2 + 10^2$, $256 - 32x + x^2 = x^2 + 100$, $156 = 32x$, $x = 4.875$, $\therefore$ QD = 4.875 cm

37. Join BQ and CQ.



$BQ^2 = QP^2 + BP^2$, and $BQ^2 = AB^2 + 3^2$
$\therefore AB^2 + 9^2 = QP^2 + BP^2$ …… (i)
$CQ^2 = QP^2 + PC^2 = QP^2 + BP^2$, and $CQ^2 = 5^2 + 4^2 = 41$
$\therefore QP^2 + BP^2 = 41$ …… (ii)
From (i) and (ii), $AB^2 + 9 = 41$, $AB^2 = 32$, AB = 5.66

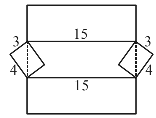
38. $DE^2 = EA^2 - AD^2$, and $DE^2 = EC^2 - DC^2$
$\therefore EA^2 - AD^2 = EC^2 - DC^2$, $EA^2 - EC^2 = AD^2 - DC^2$ ……(i)
$BD^2 = AB^2 - AD^2$, and $BD^2 = BC^2 - DC^2$
$\therefore AB^2 - AD^2 = BC^2 - DC^2$, $AB^2 - BC^2 = AD^2 - DC^2$ …… (ii)
From (i) and (ii), $EA^2 - EC^2 = AB^2 - BC^2$

39. (a) $PR^2 + QR^2 = (m^2 - n^2)^2 + (2mn)^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 + 2m^2n^2 + n^4$.
$PR^2 = (m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4$
$\because$ $PR^2 + QR^2 = PQ^2$, $\therefore$ PQR is a right-angled triangle.
(b) Let $24 = 2mn$.
The table below shows the possible values of m, n, $m^2 - n^2$ and $m^2 + n^2$.

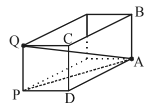
$m = 4$, $n = 3$	$m^2 - n^2 = 4^2 - 3^2 = 7$	$m^2 + n^2 = 4^2 + 3^2 = 25$
$m = 6$, $n = 2$	$m^2 - n^2 = 6^2 - 2^2 = 32$	$m^2 + n^2 = 6^2 + 2^2 = 40$
$m = 12$, $n = 1$	$m^2 - n^2 = 12^2 - 1^2 = 143$	$m^2 + n^2 = 12^2 + 1^2 = 145$

Ans. Three possible sets of values of the side:
24 cm, 7 cm, 25 cm; 24 cm, 32 cm, 40 cm; 24 cm, 143 cm, 145 cm.

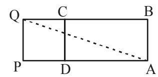
40. The diagonal of the small rectangular board = $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$ cm
The new enclosed area = $15 \times 5 - 2 \times (\frac{1}{2} \times 3 \times 4) = 75 - 12 = 63$ cm^2



41. (a) The shortest distance = AQ. $AP^2 = 10^2 + 20^2 = 500$
$AQ = \sqrt{QP^2 + AP^2} = \sqrt{10^2 + 500} = \sqrt{600} = 24.5$ cm
Ans. The shortest distance to fly from Q to A is 24.5 cm.



(b) Imagine cutting the box along QP and BA so that QPDABCQ becomes a rectangle. The shortest distance from A to Q is the diagonal AQ in the rectangle.
AP = AD + DP = 20 + 10 = 30 cm,
$AQ = \sqrt{10^2 + 30^2} = \sqrt{1\,000} = 31.6$ cm
Ans. The shortest distance to crawl from A to Q is 31.6 cm.



42. (a) $\frac{AB}{BE} = \frac{20}{15} = \frac{4}{3}$, $\frac{BC}{BD} = \frac{4}{3}$, $\angle ABC = \angle EBD$ (common $\angle$),
$\therefore$ $\Delta ABC \sim \Delta EBD$ (ratios of two sides, inc. $\angle$s)
(b) (i) $\angle BDE = \angle BCA = 90°$ (corr. $\angle$s, $\sim\Delta$s)
(ii) $BD^2 = BE^2 - DE^2$ (Pyth. Thm.),
$BD = \sqrt{15^2 - 9^2} = 12$, AD = AB – BD = 20 – 12 = 8

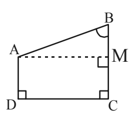
43. (a) $\angle PQT = \angle RST = 90°$ and PQ = RS (rectangle)
$\angle PTQ = \angle RTS$ (vert. opp. $\angle$s), $\therefore$ $\Delta PQT \cong \Delta RST$ (AAS)
(b) Let ST = x cm. QT = ST = x cm (corr. sides, $\cong \Delta$s)
RT = QR – QT = $(24 - x)$ cm; RS = PQ = 12 cm
In ΔRST, $RT^2 = RS^2 + ST^2$ (Pyth. Thm.),
$(24 - x)^2 = 12^2 + x^2$, $24^2 - 48x + x^2 = 12^2 + x^2$, $48x = 432$, $x = 9$, $\therefore$ ST = 9 cm

44. (a) $PR^2 = PQ^2 + QR^2$ (Pyth. Thm.), $PR = \sqrt{16^2 + 30^2} = 34$ cm
(b) (i) PQ + QR + RS + PS = $\left(78 + 2\sqrt{33}\right)$ cm,
$\therefore$ RS = $\left(78 + 2\sqrt{33}\right) - 16 - 30 - 32 = 2\sqrt{33}$ cm
$\because$ $PS^2 + RS^2 = [32^2 + \left(2\sqrt{33}\right)^2]$ cm^2 = 1156 cm^2 = 34^2 cm^2 = PR^2,
$\therefore$ ΔPRS is a right-angled triangle. (Converse of Pyth. Thm.)
(ii) Let N be a point on PR such that SN $\perp$ PR.
When T moves to N, area of $\Delta PRS = \frac{(PS)(RS)}{2} = \frac{(PR)(SN)}{2}$
$\therefore$ $SN = \frac{(PS)(RS)}{PR} = \frac{(32)(2\sqrt{33})}{34}$ cm = 10.8 cm (corr. to 3 sig fig.)
$\because$ SN < 11 cm, $\therefore$ the claim is disagreed.

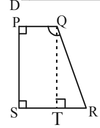
45. (a) CD = AB. CQ = CD – DQ = (15 – t) cm
(b) BQ = DQ = t cm, BC = AD = 6 cm
In ΔBCQ , $BQ^2 = BC^2 + CQ^2$,
$t^2 = 6^2 + (15 - t)^2$, $t^2 = 36 + 15^2 - 30t + t^2$, 30t = 261, t = 8.7
(c) (i) In figure (b):
$\angle BAP = 90° = \angle BCQ$, and BA = BC = 6 cm (rectangle)
$\angle ABQ = \angle PBC = 90°$ (rectangle),
i.e. $\angle ABP + \angle PBQ = \angle PBQ + \angle CBQ$,
$\therefore \angle ABP = \angle CBQ$, $\therefore \Delta BAP \cong \Delta BCQ$ (ASA)
(ii) CQ = 15 – t = 15 – 8.7 = 6.3 cm.
AP = CQ (corr. sides, $\cong \Delta$s), $\therefore$AP = = 6.3 cm

Unit 15 Introduction to trigonometric ratios

1. (a) $\theta = 70.3°$ (b) $\cos\theta = \frac{2}{3}$, $\theta = 48.2°$ (c) $\sin\theta = \frac{4}{5}$, $\theta = 53.1°$
2. (a) $\cos\theta = \frac{10}{12}$, $\theta = 33.6°$ (b) $\sin x = \frac{21}{25}$, $x = 57.1°$ (c) $\tan y = \frac{6}{17}$, $y = 19.4°$
3. (a) $a = 13\tan 36° = 9.45$ (b) $y = \frac{24}{\cos 55°} = 41.8$
(c) $k = \frac{37}{\tan 61°} = 20.5$ (d) $x = 8 \sin 26° = 3.51$
4. (a) $\tan\theta = \frac{14}{20}, \theta = 35.0°$; $x = \sqrt{20^2 + 14^2} = \sqrt{596} = 24.4$
(b) $y = 12\cos 56° = 6.71$ cm; $\sin\theta = \frac{5}{y} = \frac{5}{6.71}$, $\theta = 48.2°$
(c) In ΔABD, $x = \frac{11}{\sin 27°} = 24.2$.
In ΔABC, $\frac{y}{x} = \tan 27°$, $y = x\tan 27° = 24.2\tan 27° = 12.3$
(d) In ΔABD, BD = $16\sin(30° + 20°) = 16\sin 50°$; AD = $16\cos 50°$
In ΔACD, CD = AD $\tan 20° = 16\cos 50° \tan 20°$
$x = \text{BD} - \text{CD} = 16\sin 50° - 16\cos 50° \tan 20° = 8.51$
(e) $x = 8\tan 17°$; $\tan(17° + \theta) = \frac{2x}{8} = \frac{2 \times 8\tan 17°}{8} = 0.61146$
$\therefore$ $17° + \theta = 31.4°$, $\theta = 14.4°$
(f) $a = 60\sin 33° = 32.7$ cm;
$c = \frac{a}{\cos 44°} = \frac{60\sin 33°}{\cos 44°} = 45.4$ cm
$b = a\tan 44° = 60\sin 33° \tan 44° \approx 31.557 \approx 31.6$ cm
PR $= 60\cos 33°$; $d = \text{PR} - b = 60\cos 33° - 31.557 = 18.8$ cm
5. (a) Draw AM $\perp$ BC. In ΔABM, $a = \frac{\text{BM}}{\cos 50°} = \frac{7-4}{\cos 50°} = 4.67$



$b = \text{AM} = \text{BM}\tan 50° = 3\tan 50° = 3.58$
(b) Draw QT $\perp$ SR. $\cos\angle RQT = \frac{QT}{18} = \frac{15}{18}$, $\angle RQT = 33.6°$
$x = 90° + \angle RQT = 90° + 33.6° = 123.6°$



6. (a) $\cos 32° = \dfrac{0.5y}{17}$, $y = \dfrac{17\cos 32°}{0.5} = 28.8$

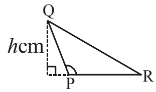
(b) $\sin(\dfrac{50°}{2}) = \dfrac{0.5x}{8}$, $x = \dfrac{8\sin 25°}{0.5} = 6.76$

(c) $\sin\dfrac{\theta}{2} = \dfrac{9 \div 2}{15} = \dfrac{3}{10}$, $\dfrac{\theta}{2} = 17.4576°$, $\theta = 34.9°$

7. (a) $\text{DE} = 8\sin 56°$, $\text{CE} = 8\cos 56°$,

area = $\dfrac{1}{2}(\text{DE})(\text{CE}) = \dfrac{1}{2}(8\sin 56°)(8\cos 56°) = 14.8\ \text{cm}^2$

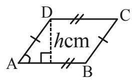
(b) Let h cm be the height from Q to PR.



$h = 5\sin(180° - 110°) = 5\sin 70°$

Area = $\dfrac{1}{2}(\text{PR})(\text{h}) = \dfrac{1}{2}(7)(5\sin 70°) = 16.4\ \text{cm}^2$

(c) Let h cm be the height from D to AB.

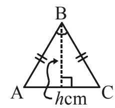


$h = 10\sin 60°$; area = $12 \times (10\sin 60°) = 103.9\ \text{cm}^2$

(d) LM = LN = 7 cm; $\angle$L = 60°; the height = $7\sin 67°$

Area = $\dfrac{1}{2}(7)(7\sin 60°) = 21.2\ \text{cm}^2$

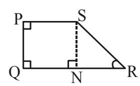
(e) Let h cm be the height from B to AC.



$h = 10\cos\dfrac{46°}{2} = 10\cos 23°$; $\dfrac{0.5\text{AC}}{10} = \sin\dfrac{46°}{2}$, $\text{AC} = 20\sin 23°$

Area = $\dfrac{1}{2}(\text{AC})(\text{h}) = \dfrac{1}{2}(20\sin 23°)(10\cos 23°) = 36.0\ \text{cm}^2$

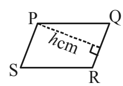
(f) Draw SN $\perp$ QR. SN = PQ = 3 cm; $\tan 58° = \dfrac{\text{SN}}{\text{NR}}$, $\text{NR} = \dfrac{3}{\tan 58°}$



$\therefore$ $\text{QR} = 4 + \dfrac{3}{\tan 58°} = 5.8746$; area = $\dfrac{1}{2}(4 + 5.8746)(3) = 14.8\ \text{cm}^2$

8. $h = 3.5\tan 30°$, area = $\dfrac{1}{2}(7)(3.5\tan 30°) = 7.07\ \text{cm}^2$

9. Let h cm be the height from P to QR. $h = 26\sin\angle\text{Q}$,



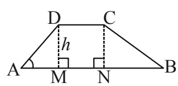
$\therefore$ $14 \times 26\sin\angle\text{Q} = 160$, $\sin\angle\text{Q} = \dfrac{160}{14 \times 26} = 0.43956$, $\angle\text{Q} = 26.1°$

10. (a) $\text{BD} = 18\tan 43°$, area of ΔABC = $\dfrac{1}{2}(18 + 40)(18\tan 43°) = 486.8$ sq. units

(b) $a = \dfrac{18}{\cos 43^o} = 24.6$; $b = \sqrt{\text{BD}^2 + 40^2} = \sqrt{(18\tan 43°)^2 + 40^2} = 43.4$

$\tan\theta = \dfrac{40}{\text{BD}} = \dfrac{40}{18\tan 43°} = 2.383$, $\therefore$ $\theta = 67.2°$

11. (a) Let h = DM = CN. $h = 8\sin 54° = 13\sin\angle\text{B}$,



$\therefore$ $\sin\angle\text{B} = \dfrac{8\sin 54°}{13} = 0.49786$, $\angle\text{B} = 29.86°$

(b) $\text{AM} = 8\cos 54°$, $\text{NB} = 13\cos 29.86°$,

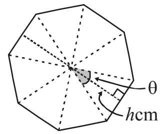
$\therefore$ $\text{AB} = 8\cos 54° + 5 + 13\cos 29.86° = 21.0$ cm

(c) Area of ABCD = $\dfrac{1}{2}(5 + 20.976)(8\sin 54°) = 84.1\ \text{cm}^2$

12. Let h cm be the height from the centre to a circle.

$\theta = \dfrac{360°}{8} = 45°$, $\dfrac{6 \div 2}{h} = \tan\dfrac{45°}{2}$, $h = \dfrac{3}{\tan 22.5°}$

Area of the octagon = $8 \times \dfrac{1}{2}(6)(\dfrac{3}{\tan 22.5°})$ $= 173.8 \text{ cm}^2$



13. Let the length of the rope be x cm. $\cos 72° = \dfrac{4 - 2.5}{x}$, $x = \dfrac{1.5}{\cos 72°} = 4.85$

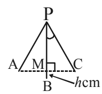
Ans. The rope is 4.85 m long.

14. $\sin\dfrac{\theta}{2} = \dfrac{8 \div 2}{10} = \dfrac{4}{10}$, $\therefore$ $\dfrac{\theta}{2} = 23.578$, $\theta = 47.2°$

15. Let $\angle POQ = \theta$. $\cos\dfrac{\theta}{2} = \dfrac{2.6}{3}$, $\therefore$ $\dfrac{\theta}{2} \approx 29.9264$, $\theta = 60°$ (to the nearest degree)

16. Let h cm be the greatest height.

$\cos 28° = \dfrac{PM}{PC} = \dfrac{24 - h}{24}$, $h = 24 - 24\cos 28°$ $= 2.81$ cm



17. The vertical distance he has risen = $450\sin 11° + 200\sin 34°$ $= 198$ m

18. The distance between the two ends of the ropes = $\dfrac{19}{\tan 38°} + \dfrac{19}{\tan 48°}$ $= 41.4$ m

19. Let the distance the plane traveled be x km.

$\sin 32° = \dfrac{1.5 \times 1000}{x}$, $x = \dfrac{1500}{\sin 32°}$

The time taken $= x \div 900 = \dfrac{1500}{\sin 32°} \div 900 =$ 3.15 minutes (2 d.p.)

20. Let x m be the length of the post. $\dfrac{x}{3.8} = \tan 40°$, $x = 3.8\tan 40°$

Let y m be the length of the new shadow.

$\dfrac{x}{y} = \tan 28°$, $y = \dfrac{x}{\tan 28°} = \dfrac{3.8\tan 40°}{\tan 28°}$ $= 6.00$ m

Ans. The new length of the shadow is 6.00 m.

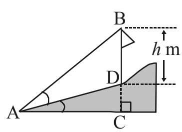
21. Let the length of the pole be h m.

DC = 4 sin 15°, AC = 4 cos 15°.

In ΔABC, BC = AC tan (27° + 15°),

h + 4 sin 15° = 4 cos 15° tan 42°, h = 2.44

Ans. The vertical pole is 2.44 m long.



22. (a) DE = $\dfrac{8}{\tan 30°}$, $\therefore$ area of ΔCDE = $\dfrac{1}{2}(8)(\dfrac{8}{\tan 30°}) = 55.4$ cm^2

(b) DC = $\dfrac{8}{\sin 30°} = 16$, $\therefore$ area of ABCD = $16^2 = 256$ cm^2

(c) ∠ADF = 180° – 30° – 90° = 60°, AD = DC = 16 cm,

The vertical distance from A to DE = AD sin ∠ADF = 16 sin 60° = 13.9 cm

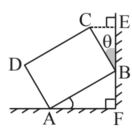
23. Draw CE ⊥ EF. In ΔABF, BF = AB sin 30° = 30 sin 30°.

∠ABE = 30° + 90° (ext. ∠ of Δ), $\therefore$ θ + 90° = 30° + 90°, θ = 30°

In ΔCBE, BE = BC cos θ = 20 cos 30°

The vertical distance from C to the horizontal

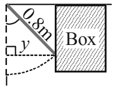
= BF + BE = 30 sin 30° + 20 cos 30° = 3.23 cm



24. In ΔBEC, $\dfrac{6}{EC} = \tan\dfrac{50°}{2}$, EC = $\dfrac{6}{\tan 25°}$

The distance from the vertex to the bottom of the cylinder = $15 - \dfrac{6}{\tan 25°}$ $= 2.13$ cm

25. Let y m be the original distance between the box and the closed door.



$\frac{y}{0.8} = \sin 50°$, $y = 0.8\sin 50° = 0.613$

$x = 0.8 - y = 0.8 - 0.613 = 0.187$

26. (a) PR = 3 + 4 = 7, PQ = $\sqrt{12^2 - 7^2} = \sqrt{95}$. ∴ cos ∠PQR= $\frac{\sqrt{95}}{12}$, ∠PQR = 35.7°

tan ∠PQS = $\frac{3}{\sqrt{95}}$, ∠PQS = 17.1°, ∴ $n = 35.7° - 17.1° = 18.6°$

(b) QS = $\sqrt{8^2 + 6^2} = 10$, RQ = QS = 10, ∴ PR = 10 + 8 = 18

tan ∠R = $\frac{6}{18}$, ∠R = 18.4°, θ = ∠R = 18.4° (base ∠s, isos.Δ)

(c) EF = $\frac{5}{\tan 35°}$, CE = $\sqrt{13^2 - 5^2} = 12$, ∴ $m = 12 - \frac{5}{\tan 35°} = 4.86$

sin ∠DCE = $\frac{5}{13}$, ∠DCE = 22.6°; ∴ θ = 35° – 22.6° = 12.4° (ext.∠of Δ)

(d) $\sin\theta = \frac{12}{13}$, θ = 67.4°, θ + ∠F + 90° = 180°, ∴ ∠D = θ = 67.4°

cos ∠D = $\frac{7}{y+12}$, cos 67.4° (y + 12) = 7, $y = \frac{7}{\cos 67.4°} - 12 = 6.22$

27. (a) 3θ = 64.158°, θ = 21.4°

(b) tan(θ + 18°) = 1.37374, θ + 18° = 53.9°, θ = 53.9° – 18° = 35.9°

(c) $\cos(2\theta - 15°) = \frac{1}{5}$, 2θ – 15° = 78.462°, $\theta = \frac{78.463° + 15°}{2} \approx 46.7°$

28. No. Because we don't know whether the triangle is right-angled.

29. (a) From the graph, $\sin 20° = 0.35$; $\sin 38° = 0.60$;
$\sin 64° = 0.90$ (correct to the nearest 0.05)

(b) From the graph, $\cos 20° = 0.95$; $\cos 38° = 0.80$;
$\cos 64° = 0.45$ (correct to the nearest 0.05)

(c) When θ = 45°; cos θ = sin θ.
When θ increases, cos θ decreases but sin θ increases.
∴ If cos θ > sin θ, θ < 45°. (OR: 0° < θ < 45°)

30. ∠B = 180° – 45° – 90° = 45°. ∴ ΔABC is isosceles, and CA = CB.

Let CA = CB = 1 unit, ∴ AB = $\sqrt{1^2 + 1^2} = \sqrt{2}$ units, $\frac{AC}{AB} = \frac{1}{\sqrt{2}}$, ∴ $\cos 45° = \frac{1}{\sqrt{2}}$

31. PQ = RQ (equil. Δ), ∠QLP = ∠QLR = 90° (given), QL = QL (common),
∴ ΔQLP ≅ ΔQLR (R.H.S.), ∴ PL = LR (corr. sides, ≅Δs)

Let PQ = 2 units, PL = $\frac{1}{2}$ PR = $\frac{1}{2}$(2) =1 unit, ∴ QL = $\sqrt{2^2 - 1^2} = \sqrt{3}$,

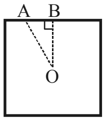
∠P = 60° (equil.Δ), ∠PQL = 180° – 90° – 60° = 30°,

$\tan 60° = \frac{QL}{PL} = \frac{\sqrt{3}}{1} = \sqrt{3}$, $\sin 30° = \frac{PL}{PQ} = \frac{1}{2}$

32. In ΔABC, sin ∠CBD = $\frac{12}{20}$, ∠CBD =36.9°;

∠DCB = ∠CBD (base ∠s, isos. Δ), ∴ ∠DCB = 36.9°

33. $\angle AOB = \frac{360°}{12} = 30°$, $OB = \frac{15}{2} = 7.5$ cm



The distance between the two marks

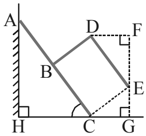
= AB = OB tan 30° = 7.5 tan 30° = 4.33 cm

34. (a) BCED is a rectangle.

$\angle ECG = 180° - 90° - 57° = 33°$; CE = BD = 28 cm.

The vertical distance from E to the ground = EG

= 28 sin 33° = 15.2 cm



(b) $\angle ECG + \angle CGE = \angle CEF = \angle CED + \angle DEF$ (ext.∠ of Δ)

∴ $33° + 90° = 90° + \angle DEF$, $\angle DEF = 33°$; EF = 40 cos 33°

Vertical distance from D to the ground = FE + EG

= 40 cos 33° + 28 sin 33° = 48.8 cm

(c) AC = 50 + 40 = 90, ∴ HC = 90 cos 57°

DF = 40 sin 33°; CG = 28 cos 33°

Horizontal distance from D to the wall = HC + CG − DF

= 90 cos 57° + 28 cos 33° − 40 sin 33° = 50.7 cm

35. (a) PS = 16 cm, ∴ $PB = \frac{16}{\sin 30°}$.

QP = 20 cm, $\angle APQ = 30°$, ∴ $AP = \frac{20}{\cos 30°}$

The length of the rod = AP + PB = $\frac{20}{\cos 30°} + \frac{16}{\sin 30°} = 55.1$ cm

(b) $SB = \frac{PS}{\tan 30°} = \frac{16}{\tan 30°}$.

Let x cm be the new horizontal distance.

∵ the length of the rod = 55.1 cm, ∴ $x = 55.1 \cos 20°$

The distance top B slides = x − (PS + SB) = $55.1\cos 20° - (20 + \frac{16}{\tan 30°}) = 4.06$ cm

36. Let r cm be the radius of the sphere.

In ΔDAE, DE = r, $\angle DAE = \frac{52°}{2} = 26°$, ∴ $DA = \frac{r}{\sin 26°}$

BD + DA = 8, ∴ $r + \frac{r}{\sin 26°} = 8$, $r = 8 \div (1 + \frac{1}{\sin 26°}) = 2.44$

Ans. The radius of the sphere is 2.44 cm.

37. (a) In ΔPQS, PS = QS tan 24° = (3 + 5) tan 24° = 8 tan 24°

Area of ΔPQT = $\frac{1}{2}$(QT)(PS) = $\frac{1}{2}$(3)(8 tan 24°) = 5.34 cm^2

(b) In ΔTSR, SR = $\sqrt{13^2 - 5^2} = \sqrt{144} = 12$

$\tan \angle QRS = \frac{3+5}{12} = \frac{2}{3}$, $\angle QRS = 33.69°$

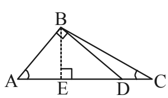
$\sin \angle TRS = \frac{5}{13}$, $\angle TRS = 22.62°$; $\angle QRT = \angle QRS - \angle TRS = 33.69° - 22.62° \approx 11.1°$

38. (a) In ΔABD, BD = 12 sin 48° = 8.92

(b) Draw BE ⊥AC. $\angle ADB = 180° - 90° - 48° = 42°$

In ΔBED, BE = BD sin 42° = (12 sin 48°)(sin 42°) = 5.967

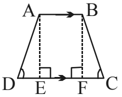
∴ EC = 12 sin 48°cos 42° + 4 = 10.627



$\tan\theta = \frac{BE}{EC} = \frac{5.967}{10.627} = 0.561494$, $\theta = 29.3°$

39. In ΔBAC, AC = $x\cos\theta$. ∵ ∠BCD + ∠ADC = 90° + 90° = 180°,
∴ BC // AD (int. ∠s supp.). ∠CAD = ∠ACB = θ (alt. ∠s, BC // AD)
In ∠ACD, $\sin\angle CAD = \frac{CD}{AC}$, ∴ $\sin\theta = \frac{CD}{x\cos\theta}$, CD = $x\cos\theta\sin\theta$

40. $p^2 = q^2 + 12$, ∴ $p^2 - q^2 = 12$.
Draw AE and BF ⊥ DC. Let h = AE = BF.
DE = FC = $\frac{p-q}{2}$, AE = DE tan θ = $(\frac{p-q}{2})\tan\theta$

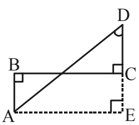


Area of the trapezium
$= \frac{1}{2}(p+q)(AE) = \frac{1}{2}(p+q)(\frac{p-q}{2})\tan\theta$
$= \frac{1}{4}(p^2-q^2)\tan\theta = \frac{1}{4}(12)\tan\theta = 3\tan\theta$.

41. (a) $\angle F = \frac{180°\times 4}{6} = 120°$, AF = EF = 6 cm, $\frac{0.5AE}{AF} = \sin\frac{120°}{2}$,
∴ AE = 2(6) sin60° = 10.4 cm
(b) ∠BAF = ∠F = 120^o, $\angle EAF = \frac{180°-120°}{2} = 30°$ (base ∠s, isos. Δ)
∴ ∠BAE = ∠BAF – ∠EAF = 120° – 30° = 90°
∴ ABE is a right-angled triangle.
(c) Area of ΔABE = $\frac{1}{2}$(AB)(AE) = $\frac{1}{2}$(6)(10.4) = 31.2 cm^2

42. (a) In ΔABD, ∠ABD = 180° – 90° – θ = 90° – θ (∠sum of Δ)
In ΔABC, ∠BCD = 180°– 90° – θ = 90° – θ (∠sum of Δ)
∴ ∠ABD = ∠BCD;
∠DBC = 90° – ∠ABD = 90° – (90° – θ) = θ, ∴ ∠A = ∠DBC;
∠ADB = ∠BDC = 90°, ∴ ΔABD ~ ΔBCD (AAA)
(b) $\frac{BD}{CD} = \frac{AD}{BD}$ (corr. sides, ~Δs), ∴ $\frac{BD}{27} = \frac{48}{BD}$, BD = $\sqrt{48\times 27} = 36$
In ΔABD, $\tan\theta = \frac{36}{48}$, θ = 36.9°

43. Produce DC to E, so that AE ⊥ DE.
AE = BC = 56, CE = AB = 15.
DE = DC + CE = 18 + 15 = 33.
In ΔADE, $\tan\theta = \frac{AE}{DE} = \frac{56}{33}$, θ = 59.5°



44. Let y = AB = CD. ∴ DE = y sin 70°, BE = y sin 40°, DE – BE = 4,
∴ $y\sin 70° - y\sin 40° = 4$, $y = \frac{4}{\sin 70° - \sin 40°} = 13.47$.
AE = y cos 40°, CE = y = cos 70°
x = AE – CE = $y(\cos 40° - \cos 70°) = (13.47)(\cos 40° - \cos 70°) = 5.71$

45. Let PB = x. ∴ AP = $2x$; AB = $x + 2x = 3x$
In ΔABC, AC = AB tan 45° = $3x$ (1) = $3x$.
tan ∠APC = $\frac{AC}{AP} = \frac{3x}{2x} = \frac{3}{2}$, ∴ ∠APC = 56.3°
∠CPQ + 56.3° = 45° + 90° (ext.∠ of Δ), ∴ ∠CPQ = 78.7°

46. (a) Let x = PC = CB. $\tan 56° = \dfrac{2x}{AB}$, $\therefore\ \dfrac{x}{AB} = \dfrac{\tan 56°}{2}$, but $\tan\theta = \dfrac{x}{AB}$,

$\therefore\ \tan\theta = \dfrac{\tan 56°}{2} = 0.74128$, $\theta \approx 36.549° \approx 36.5°$

(b) PC = 10 sin θ = 10 sin 36.549° = 5.955, $\dfrac{2PC}{PA} = \sin 56°$, $PA = \dfrac{2(5.955)}{\sin 56°} = 14.4$ cm

47. (a) Let a = AE = ED = DC. $\tan 30° = \dfrac{BC}{3a}$, $\tan\angle BDC = \dfrac{BC}{a}$

$\therefore\ \tan 30° = \dfrac{1}{3}(\tan\angle BDC)$, $\tan\angle BDC = 3\tan 30° = 1.7321$, $\angle BDC = 60°$

(b) $\angle DBA = 60° - 30°$ (ext.∠ of Δ)
$\because\ \angle BAD = \angle DBA = 30°$, $\therefore$ DA = DB (sides opp. equal ∠ s)

48. (a) In ΔABD, let ∠B = x. ∠BAD = ∠B = x (base ∠s, isos. Δ)
In ΔADC, let ∠C = y. ∠CAD = ∠C = y (base ∠s, isos. Δ)
In ΔABC, let ∠B + ∠BAD + ∠CAD + ∠C = 180° (∠ sum of Δ)
$x + x + y + y = 180°$, $x + y = 90°$, $\angle BAC = x + y$, $\therefore\ \angle BAC = 90°$

(b) $\sin\angle C = \dfrac{12}{10+10} = \dfrac{12}{20}$, $\therefore\ \angle C = 36.9°$

49. (a) $PQ^2 + QR^2 = 10.5^2 + 10^2 = 210.25$, $PR^2 = 14.5^2 = 210.25$
$\because\ PQ^2 + QR^2 = PR^2$, $\therefore\ \angle Q = 90°$ (converse of Pyth. Thm.)
i.e. PQR is a right-angled triangle.

(b) ∠Q = 90° (proved); $\cos\angle P = \dfrac{10.5}{14.5}$, ∠P = 43.6°, ∠R = 180° – 90° – 43.6° = 46.4°

50. (a) $AB^2 + BC^2 = 13^2 + 15^2 = 394$, $AC^2 = 18^2 = 324$
$\because\ AB^2 + BC^2 \neq AC^2$, $\therefore$ ABC is not a right-angled triangle.

(b) $BP^2 = 13^2 - x^2$, and $BP^2 = 15^2 - PC^2 = 15^2 - (18-x)^2$
$\therefore\ 169 - x^2 = 225 - (18-x)^2$,

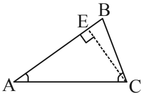
$169 - x^2 = 225 - 324 + 36x - x^2$, $36x = 169 + 324 - 225 = 268$, $x = \dfrac{268}{36} = 7.44$

$\cos\angle A = \dfrac{x}{13} = \dfrac{268}{36}\cdot\dfrac{1}{13} = \dfrac{67}{117}$, $\therefore\ \angle A = 55.1°$

51. (a) $\tan 41° = \dfrac{h}{AD}$, $AD = \dfrac{h}{\tan 41°}$, $\tan 29° = \dfrac{h}{CD}$, $CD = \dfrac{h}{\tan 29°}$

(b) AD + CD = 30, $\dfrac{h}{\tan 41°} + \dfrac{h}{\tan 29°} = 30$, $h = 30 \div \left(\dfrac{1}{\tan 41°} + \dfrac{1}{\tan 29°}\right) = 10.2$ cm

52. Draw CE ⊥ AB.
∠B = 180° – 30° – 70° = 80°, AE = x cos 30°, BE = y cos 80°
AB = AE + EB = x cos 30° + y cos 80°



53. (a) $PR = \dfrac{QR}{\tan 25°} = 2.1445QR$, $SR = \dfrac{QR}{\tan 50°} = 0.8391QR$
$\therefore$ PS = 2.1445 QR – 0.8391 QR = 1.3054 QR

PS : SR = 1.3054QR : 0.8391QR = $\dfrac{1.3054}{0.8391}$: 1 = 1.556 : 1, $\therefore\ n = 1.556$

(b) Area of ΔPQS : Area of ΔQRS = $\dfrac{1}{2}$(PS)(QR) : $\dfrac{1}{2}$(SR)(QR) = PS : SR

$\therefore$ area of ΔPQS = $\dfrac{1.536}{1} \times 30 \approx 46.7\ cm^2$

54\. (a) $\because$ Half of the water remained, $\therefore$ the cross-sectional area of the water is half of ABCD.
The water level is parallel to the horizontal, $\therefore \angle BAC = x°$ (alt. $\angle$s, // lines)
$\tan\angle BAC = \frac{BC}{AB}$, $\tan x° = \frac{50}{40}$, $x = 51.3$

(b) $\because$ one-third of the water remained,
$\therefore$ the cross-sectional area of the water $= \frac{1}{3}(40\times 50) = \frac{2\,000}{3}$
$\therefore \frac{1}{2}(BE)(40) = \frac{2\,000}{3}$, $BE = \frac{100}{3}$
The water level is parallel to the horizontal, $\therefore \angle EAB = y°$ (alt. $\angle$s, // lines),
$\tan y° = \frac{BE}{AB} = \frac{\left(\frac{100}{3}\right)}{40} = \frac{5}{6}$, $\therefore$ $y = 39.8$

55\. (a) (i) $\angle ANC = \angle BNC = 90°$ (given), CN = CN (common side), AC = BC (given),
$\therefore$ $\Delta ACN \cong \Delta BCN$ (RHS), $\therefore$ AN = BN (corr. sides, $\cong\Delta$s)

(ii) $\because$ $\Delta ACN \cong \Delta BCN$ (proved), $\therefore$ $\angle ACN = \angle BCN$ (corr. $\angle$s, $\cong\Delta$s) $= \frac{2\theta}{2} = \theta$.
In ΔACN, $\frac{AN}{AC} = \sin\angle ACN$, $AN = \ell\sin\theta$ cm
$AB = 2AN = 2\ell\sin\theta$ cm

(b) (i) In ΔPQR, $\sin 50° = \frac{QR}{PR}$, $PR = \frac{20}{\sin 50°}$ m = 26.1 m (3 sig. fig.)

(ii) $\angle RPS = 68° - 50° = 18°$, $\frac{\angle RPS}{2} = \frac{18°}{2} = 9°$.
By (a), $RS = 2(\frac{20}{\sin 50°})(\sin 9°) = 8.17$ m (3 sig. fig.)

56\. (a) (i) $AC^2 = AB^2 + BC^2$ (Pyth. Theorem)
$= (36^2 + 48^2)\text{ km}^2 = 3600\text{ km}^2$
$AC^2 + CE^2 = (3600 + 25^2)\text{ km}^2 = 4225\text{ km}^2 = 65^2\text{ km}^2 = AE^2$
$\therefore \angle ACE$ is a right angle (Converse of Pyth. Theorem)

(ii) $\angle DCE = 180° - \angle ACE = 180° - 90° = 90°$.
In ΔCDE, $\tan 35° = \frac{CD}{CE}$, CD = 25tan35° km = 17.5052 km = 17.5 km (3 sig. fig.)

(iii) In ΔCDE, $\cos 35° = \frac{CE}{DE}$, $DE = \frac{25}{\cos 35°}$ = 30.519 km = 30.5 km (3 sig. fig.)

(b) Time for Morris' journey $= \frac{36 + 48 + 17.5052}{90}\text{ h} = 1.1278\text{ h}$ (4 d.p.)
Time for Nick's journey $= (65 + 30.519) \div 84\text{ h} = 1.1371\text{h}$ (4 d.p.)
$\because$ 1.1278 < 1.1371, $\therefore$ Morris arrives at city D first. *Thus, the claim is disagreed.*

57\. (a) Let N be a point on PQ such that ON $\perp$ PQ.
Note that $\Delta OPN \cong \Delta OQN$,
$\therefore$ PN = QN (corr. sides, $\cong\Delta$s), $PN = \frac{PQ}{2} = \frac{10}{2}$ cm = 5 cm.
$ON^2 = OP^2 - PN^2$ (Pyth. Theorem)
$ON = \sqrt{13^2 - 5^2} = 12$ cm *Ans. The required vertical distance is 12 cm.*

(b) (i) Radius of the ring $= \frac{18}{2}\text{cm} = 9$ cm

$\frac{y-9}{9} = \cos(180° - 140°)$, $y = 9\cos 40° + 9 = 15.9$ (cor. to 3 sig. fig.)

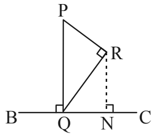
(ii) The required distance = $(y + 12 - 9)$ m = $(3 + 9\cos 40° + 9)$ m
= 18.9 m (3 sig. fig.)

58. (a) $\cos \angle PQR = \frac{QR}{PQ} = \frac{141}{235}$, $\therefore \angle PQR = 53.13°$ (4 sig. fig.)

(b) (i) Let the required distance in Position (1) be h_1 cm.

$\sin \angle PQR = \frac{h_1}{RQ}$, $\therefore h_1 = 141 \sin 53.13° = 112.8$ (4 sig. fig.)

Ans. *The required distance in Position (1) is 112.8 cm.*



Let N be a point on BC such that RN ⊥ BC.

$\angle RNC = \angle PQC = 90°$

$\therefore$ PQ // RN (corr. ∠s equal)

$\therefore \angle QRN = \angle PQR = 53.13°$ (alt. ∠s, // lines)

$\frac{RN}{QR} = \cos \angle QRN$

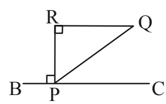
The required distance in Position (2) = RN = 141 cos 53.13°
= 84.60 cm (4 sig. fig.)

Ans. *The required distance in Position (2) is 84.6 cm.*

(ii) $PR = \sqrt{PQ^2 - QR^2}$ (Pyth. Theorem)

$= \sqrt{235^2 - 141^2} = 188$ cm > QR

Ans. *The distance is greatest when the vertex P touches BC and PR ⊥ BC.*
The required distance = PR = 188 cm.



Unit 16 Areas (2): Arcs and sectors

1. (a) Circumferences = $2\pi(15) = 94.2$ cm, area = $\pi(15)^2 = 707.9$ cm^2

(b) Circumferences = $2\pi(\frac{22}{2}) = 69.1$ cm, area = $\pi(\frac{22}{2})^2 = 380.1$ cm^2

2. (a) Radius = $\frac{100}{2\pi} = 15.9$ cm (b) Radius = $\sqrt{\frac{84}{\pi}} = 5.17$ cm

3. (a) Perimeter = $6 + 6 + 2 \times \frac{1}{2} \times \pi(6) = 30.8$ cm. Area = $6^2 - 2 \times \frac{1}{2} \times \pi(\frac{6}{2})^2 = 7.73$ cm^2

(b) Perimeter = $5 + \frac{1}{2} \times \pi(5+4) + \frac{1}{2} \times \pi(4) = 25.4$ cm

Area = $\frac{1}{2} \times \pi(\frac{5+4}{2})^2 - \frac{1}{2} \times \pi(\frac{4}{2})^2 = 25.5$ cm^2

(c) Perimeter = $\pi(4+4+4) + 2 \times \frac{1}{2} \times \pi(4+4) + 2 \times \frac{1}{2} \times \pi(4) = 75.4$ cm

Area = $\pi(\frac{4+4+4}{2})^2 - 2 \times \frac{1}{2} \times \pi(\frac{4+4}{2})^2 - 2 \times \frac{1}{2} \times \pi(\frac{4}{2})^2 = 50.3$ cm^2

(d) Perimeter = $\frac{1}{2} \times \pi(6+10) + \frac{1}{2} \times \pi(10) + \frac{1}{2} \times \pi(6) = 50.3$ cm

Area = $\frac{1}{2} \times \pi(\frac{6+10}{2})^2 + \frac{1}{2} \times \pi(\frac{10}{2})^2 - \frac{1}{2} \times \pi(\frac{6}{2})^2 = 125.7$ cm^2

4. (a) The distance travelled = π (24)(6) = 452 m

 (b) The no. of revolutions = $\frac{1\,200}{24\pi}$ = 16 (to the nearest integer)

5. (a) Radius = $95 \div 5 \div 2\pi = 3.02$ cm

 (b) $\frac{\text{no. of revolutions}}{5} = \frac{4.75\text{ m}}{95\text{ cm}}$, $\therefore$ no. of revolutions = $\frac{475}{95} \times 5 = 25$

6. Let n be the no. of revolutions of the rear wheel. π (24) (36) = π (32) n, $n = \frac{(24)(36)}{32} = 27$.

 Ans. The no. of revolutions of the rear wheel is 27.

7. Let the radius of the small circle be r cm.
 The difference = $2\pi(r + 5) - 2\pi r = 2\pi(5) = 10\pi$ cm

8. Area of the path = $\pi(\frac{30}{2}+2)^2 - \pi(\frac{30}{2})^2 = 64\pi \text{ m}^2$

9. Diameter of the small circle = $16 \div 2 = 8$ cm. Area = $\pi(\frac{16}{2})^2 - \pi(\frac{8}{2})^2 = 48\pi \text{ cm}^2$

10. Radius = $\frac{40}{2\pi} = 6.366 = 6.37$ cm. Area = $\pi(6.366)^2 = 127.3 \text{ cm}^2$

11. Radius = $\sqrt{\frac{98}{\pi}} = 5.59$ cm. Circumference = $2\pi(5.59) = 35.1$ cm

12. Radius = $\frac{6.28}{2\pi}$ m, $\therefore$ area = $\pi(\frac{6.28}{2\pi})^2 = \frac{6.28^2}{34\pi} = 3.14 \text{ m}^2$

13. (a) Arc length = $2\pi(6)(\frac{40}{360}) = 4.19$ cm. Area of sector = $\pi(6)^2(\frac{40}{360}) = 12.6 \text{ cm}^2$

 (b) Angle = $\frac{70}{2\pi(14)} \times 360 = 286.5°$. Area of sector = $\pi(14)^2(\frac{286.5}{360}) = 490 \text{ cm}^2$

 (c) Angle = $\frac{100\pi}{\pi 5^2} \times 360 = 1440°$. Arc length = $2\pi(5)(\frac{1\,440}{360}) = 40\pi$ cm

 (d) Radius = $\frac{40}{2\pi} \div \frac{120}{360} = 19.1$ cm. Area of sector = $\pi(19.1)^2(\frac{120}{360}) = 382.0 \text{ cm}^2$

 (e) Radius = $\sqrt{\frac{85}{\pi} \div \frac{200}{360}} = 6.98$ cm. Arc length = $2\pi(6.98)(\frac{200}{360}) = 24.4$ cm

14. Angle = $\frac{\pi}{2\pi(5)} \times 360 = 36°$

15. Angle = $38.5 \div [\frac{22}{7}(7)^2] \times 360 = 90°$

16. Distance = $2\pi(7)(\frac{35}{60}) = 25.7$ cm

17. Area = $(15)(7.5) - 2 \cdot \pi(7.5)^2(\frac{90}{360}) = 24.1 \text{ cm}^2$

18. (a) Area = $(7)(7) - \pi(7)^2(\frac{90}{360}) = 10.5 \text{ cm}^2$

 (b) Area = $(2+2)(2+2) - 4 \cdot \pi(2)^2(\frac{90}{360}) = 3.43 \text{ cm}^2$

 (c) Area = $\pi(5)^2(\frac{90}{360}) - \frac{1}{2}(5)(5) = 7.13 \text{ cm}^2$

 (d) Area = $2 \times [\pi(3)^2(\frac{90}{360}) - \frac{1}{2}(3)(3)] = 5.14$ sq. units

19. (a) Perimeter = $2(\frac{22}{7})(\frac{14}{2})(\frac{270}{360}) + 2 \cdot (4 + \frac{6}{2} + 6) = 33 + 26 = 59$ units

 Area = $[(\frac{22}{7})(\frac{14}{2})^2 - (6)(6)](\frac{270}{360}) = (154 - 36)\frac{3}{4} = 88.5$ sq. units

(b) Perimeter = $2\times[2(\frac{22}{7})(\frac{28}{2})(\frac{270}{360})+2\cdot(\frac{28}{2})]=2(66+28)=188$ units

Area = $2\times(\frac{22}{7})(\frac{28}{2})^2(\frac{270}{360})=924$ sq. units

20. Shaded area = $\pi[(3+2)^2-3^2]\times\frac{60°}{360°}$ = 8.38 cm^2

21. Perimeter = $2\pi(4)\times\frac{300°}{360°}+2\pi(10+4)\times\frac{360°-300°}{360°}+10\times2$ = 55.6 cm

Area = $\pi(4)^2\times\frac{300°}{360°}+\pi(10+4)^2\times\frac{360°-300°}{360°}$ = 145 cm^2

22. $2\pi r\times\frac{120°}{360°}+2r=36.84$, $3.14r\times\frac{1}{3}+r=18.42$

$3.14r+3r=55.26$, $6.14r=55.26$, $\therefore$ $r=9$

23. (a) Let r cm be the radius. $2\pi r\times\frac{1}{2}+2r=514$, $3.14r+2r=514$, $5.14r=514$,

$r = 100$ *Ans. The radius of the semi-circle is 100cm.*

(b) Enclosed area = $\pi(100)^2\times\frac{1}{2}=3.14\times5000$ = 15700 cm^2

24. Shaded area = $\pi(\frac{20}{2})^2\times\frac{1}{4}-\frac{1}{2}\times\frac{20}{2}\times\frac{20}{2}$ = 28.5 cm^2

25. The diagonal = $\sqrt{6^2+8^2}$ = 10 cm, $\therefore$ radius = $\frac{10}{2}$ = 5 cm

$\therefore$ Area of shaded region = $\pi(5)^2-6\times8$ = 30.5 cm^2

26. Area of AED = $\frac{1}{2}\times6\times6-\pi(6)^2\times\frac{45°}{360°}=3.86$ cm^2. BD = $\sqrt{6^2+6^2}=\sqrt{72}$

$\therefore$ Perimeter of AED = $2\pi(6)\times\frac{45°}{360°}+(\sqrt{72}-6)+6$ = 13.2 cm

27. (a) Perimeter = $2\pi(\frac{25}{2})+(150-25)\times2$ = 329 cm

Area = $\pi(\frac{25}{2})^2+(150-25)\times25=3615.57$ = 3620 m^2

(b) Area of the path = $\pi(\frac{25}{2}+1.5)^2+(150-25)\times(25+1.5\times2)-3615.87$

= $\pi(14)^2+125\times28-3615.87$ = 500 m^2

28. $\angle$AOB = 180° – 30° × 2 = 120°, height of ΔOAB = 8 × sin 30° = 4 cm
AB = 8 × cos 30° × 2 = 13.8564

$\therefore$ Area of shaded segment = $\pi(8)^2\times\frac{120°}{360°}-13.8564\times4\times\frac{1}{2}$ = 39.3 cm^2

29. AC = 12 cos 40°, BC = 12 sin 40°

$\therefore$ Area of shaded region = $\pi(\frac{12}{2})^2\times\frac{1}{2}-(12\cos40°)(12\sin40°)\times\frac{1}{2}$ = 21.1 cm^2

30. Let r be the original radius, then new radius = $r(1-40\%)=0.6r$

$\therefore$ Percentage decrease in area = $\frac{\pi r^2-\pi(0.6r)^2}{\pi r^2}\times100\%=(1-0.36)\times100\%$ = 64%

31. Let r_1 and r_2 be the radii of the bigger and the smaller circles respectively.

$2\pi r_1-2\pi r_2=20$, $2\pi(r_1-r_2)=20$, $r_1-r_2=\frac{10}{\pi}$

Ans. The difference between the radii is $\frac{10}{\pi}$ *m.*

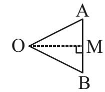
32. Time saved = $[2\pi(\frac{34}{2})\times\frac{1}{2}-29]\div 72\times 60 = 20.3$ min

33. Total length = $2\pi(1)\times 3 - 2\pi(1)\times\frac{\angle P+\angle Q+\angle R}{360^\circ} = 6\pi - 2\pi\times\frac{180^\circ}{360^\circ} = 5\pi$ cm

34. $2\pi(4)\times\frac{\angle AOB}{360^\circ} = 2$, $\frac{\angle AOB}{360^\circ} = \frac{1}{4\pi}$

∴ Area of sector = $\pi(4)^2\times\frac{\angle AOB}{360^\circ} = 16\pi\times\frac{1}{4\pi} = 4$ cm^2

35. (a) $2\pi(18)\times\frac{\angle AOB}{360^\circ} = 2\pi(5)$, $\frac{\angle AOB}{20^\circ} = 5$, ∴ ∠AOB = 100°



(b) ∠AOM = 100° ÷ 2 = 50°, ∴ distance = 18 sin 50° × 2 = 27.6 cm

36. Let r cm be the radius. $\pi r^2\times\frac{2}{2+5} = 77$, $\frac{22}{7}\times r^2\times\frac{2}{7} = 77$,

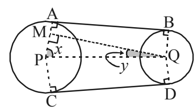
$r = 77\times\frac{49}{44} = \frac{343}{4}$, $r = \sqrt{\frac{343}{4}} = 9.26$. *Ans. The radius of the circle is 9.26 cm.*

37. Let r cm be the radius, then BD = AE = $2r$ cm and AB = ED = $2r(\frac{3}{2}) = 3r$ cm.

$3r+3r+2r+2\pi r\times\frac{1}{2} = 3\pi+24$, $8r+\pi r = 3\pi+24$, $r(\pi+8) = 3(\pi+8)$, $r = 3$

∴ Area = $\pi(3)^2\times\frac{1}{2}+(2\times3)(3\times3) = 4.5\pi+54 = 68.1$ cm^2

38. QM = $\sqrt{60^2-(16-12)^2} = \sqrt{3584}$. $\cos x = \frac{4}{60}$, $x = 86.2^\circ$

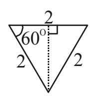


∴ Reflex ∠APC = 360° – 2(86.2°) = 187.6°. $y = 90^\circ - x = 3.8^\circ$

∴ ∠BQD = $360^\circ - 2(90^\circ) - 2(3.8^\circ) = 172.4^\circ$

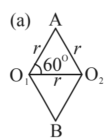
∴ Length of belt = $2\pi(16)\times\frac{187.6^\circ}{360^\circ}+2\pi(12)\times\frac{172.4^\circ}{360^\circ}+\sqrt{3584}\times 2 = 208$ cm

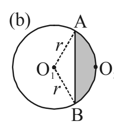
39. (a) Height of ΔABC = 2 sin 60° cm



∴ Area of ΔABC = $\frac{1}{2}\times 2\times 2\sin 60^\circ = 1.732$ cm^2

(b) Shaded area = $\pi(2)^2\times\frac{60^\circ}{360^\circ}\times 3 - 1.732\times 2 = 2.82$ cm^2





40. (a) Let $AO_1 = AO_2 = O_1O_2 = r$ cm.

$r\sin 60^\circ\times 2 = \sqrt{24}$, ∴ $r \approx 2.82843 \approx 2.83$

(b) $\angle AO_1B = 60^\circ\times 2 = 120^\circ$.

Area of segment AO_2B

$= \pi(2.82843)^2\times\frac{120^\circ}{360^\circ}-\frac{1}{2}\times\sqrt{24}\times\frac{2.82843}{2} = 4.9135$ (5 sig. fig.)

∴ Area of shaded region = 4.9135 × 2 = 9.83 cm^2 (3 sig. fig.)

41. (a) ∵ EA = AB = EB, ∴ ΔABE is an equilateral triangle and ∠EAB = 60°,

∴ ∠EAD = 90° – 60° = 30°

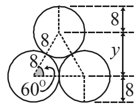
(b) $\overset{\frown}{DE} : \overset{\frown}{EB} = 2\pi S\times\frac{30^\circ}{360^\circ} : 2\pi S\times\frac{60^\circ}{360^\circ} = 1 : 2$

42. Height of ΔADB = $20\sin 45° = 20(\frac{\sqrt{2}}{2}) = 10\sqrt{2}$ cm,

$\therefore$ Area of shaded region = $\left[\pi(20)^2 \times \frac{45°}{360°} - \frac{1}{2} \times 20 \times 10\sqrt{2}\right] \times 2 = 31.3 \text{ cm}^2$

43. $\because$ OC = OD = 10 cm and $\angle$COE = 45°, $\therefore$ OE = 10 cos 45°

$\therefore$ Area of OACE = $(10\cos 45°)^2 = 50 \text{ cm}^2$



44. $y = (8+8)\sin 60° = 16\sin 60°$ cm

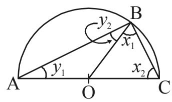
$\therefore$ Height of the stack = $8 + 16\sin 60° + 8 = 29.9$ cm

45. (a) $\because$ OA = OB = OC (radii),

$\therefore$ $x_1 = x_2$ and $y_1 = y_2$ (base $\angle$s, isos. Δ)

$x_1 + x_2 + y_1 + y_2 = 180°$ ($\angle$ sum of Δ),

$2x_1 + 2y_1 = 180°$, $x_1 + y_1 = 90°$, $\therefore$ $\angle$ABC = 90°



(b) AC = $\sqrt{m^2 + n^2}$ (Pyth. Thm.), OC = $\frac{1}{2}\sqrt{m^2 + n^2}$

$\therefore$ Area of semi-circle = $\pi(\frac{1}{2}\sqrt{m^2 + n^2})^2 \times \frac{1}{2} = \frac{\pi(m^2 + n^2)}{8}$

46. OB = 10 cm. Let OA = y cm and AB = $2y$ cm.

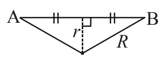
$y^2 + (2y)^2 = 10^2$, $5y^2 = 100$, $y^2 = 20$, $y = \sqrt{20}$

$\therefore$ Area of the square = $(\sqrt{20} \times 2)^2 = 80 \text{ cm}^2$

47. Let the radius of the smaller circle and the larger circle be r cm and R cm respectively.

Area of the ring = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

But $R^2 = r^2 + (\frac{20}{2})^2$ (Pythagoras' Thm.), $\therefore$ $R^2 - r^2 = 100$



$\therefore$ Area of the ring = π (100) = 100π cm^2

48. AB = $\sqrt{3^2 + 3^2} = \sqrt{18}$.

$\therefore$ Area of the crescent = area of ΔOAB + area of semi-circle − area of sector OAB

$= \frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times \pi(\frac{3\sqrt{2}}{2})^2 - \pi(3)^2 \times \frac{90°}{360°} = 4.5 + 2.25\pi - 2.25\pi = 4.5 \text{ cm}^2$

49. AB^2 + BC^2 = AC^2 (Pyth. Thm.)

$\because$ Area of the shaded region = $\frac{1}{2}\pi(\frac{AB}{2})^2 + \frac{1}{2}\pi(\frac{BC}{2})^2 + 25 - \frac{1}{2}\pi(\frac{AC}{2})^2$

$= \frac{1}{8}\pi(AB^2 + BC^2) + 25 - \frac{1}{8}\pi AC^2 = \frac{1}{8}\pi AC^2 + 25 - \frac{1}{8}\pi AC^2 = 25 \text{ cm}^2$

50. Let AB = CD = y. Arc length on AB = $2\pi(\frac{y}{2}) \times \frac{1}{2} = \frac{\pi y}{2}$

Sum of arc lengths on CD = $2\pi(\frac{y}{2n}) \times \frac{1}{2} \times n = \frac{\pi y}{2}$

Ans. They are the same.

51. (a) Area of the sector = $\pi(15)^2\left(\frac{72°}{360°}\right) = 45\pi \text{ cm}^2$

(b) Let r cm be the radius of the semi-circle. $\frac{\pi r^2}{2} = 45\pi$, $r^2 = 90$, $r = \sqrt{90}$

Length of wire A

= perimeter of the sector

$= \left[2\pi(15)\left(\dfrac{72°}{360°}\right) + 2(15)\right]$ cm = $(6\pi + 30)$ cm = 48.8496 cm (cor. to 4 d. p.),

Length of wire B

= perimeter of the semi-circle

$= \left(\dfrac{2\pi r}{2} + 2r\right)$ cm = $\left(\pi\sqrt{90} + 2\sqrt{90}\right)$ cm = 48.7774 cm (4 d. p.)

$\because$ 48.8496 cm > 48.7774 cm, $\therefore$ Wire A is longer.

52. (a) (i) $\because$ OP = OQ, $\therefore$ $\angle$OQP = $\angle$OPQ = 15° (base $\angle$s, isos. Δ),

$\angle$QON = $\angle$OQP + $\angle$OPQ = 15° + 15° = 30° (ext. $\angle$ of Δ)

(ii) In ΔQON, $\dfrac{QN}{OQ} = \sin 30°$, QN = $r \sin 30°$ cm = $\dfrac{r}{2}$ cm

Area of ΔPOQ = $\dfrac{1}{2}(OP)(QN) = \dfrac{1}{2}(r)\left(\dfrac{r}{2}\right)$ cm^2 $= \dfrac{r^2}{4}$ cm^2

(b) $\angle$POQ = 180° – 15° – 15° = 150° ($\angle$ sum of Δ)

reflex $\angle$POQ = 360° – 150° = 210°

Area of segment PQR = $\dfrac{r^2}{4} + \pi r^2\left(\dfrac{210°}{360°}\right) = \left(\dfrac{1}{4} + \dfrac{7\pi}{12}\right)r^2$

$\therefore$ $\left(\dfrac{1}{4} + \dfrac{7\pi}{12}\right)r^2 = 1200$, $r^2 = 1200 \div \left(\dfrac{1}{4} + \dfrac{7\pi}{12}\right)$, r = 24.0 (3 sig. fig.)

(c) In ΔQON, $\dfrac{QN}{PQ} = \sin \angle OPQ$, PQ = $\dfrac{r}{2} \div \sin 15°$ cm = $\dfrac{r}{2\sin 15°}$ cm

Perimeter of the segment PQR

$= \left[2\pi r(\dfrac{210°}{360°}) + \dfrac{r}{2\sin 15°}\right]$ cm = $\left(\dfrac{7}{6}\pi r + \dfrac{r}{2\sin 15°}\right)$ cm = 134 cm (3 sig. fig.)

53. (a) (i) $\because$ OA = OB = OC = OD = OE (radii)

and AB = BC = CD = DE = AE (regular polygon),

$\therefore$ ΔAOB, ΔBOC, ΔCOD, ΔDOE and ΔAOE are congruent to each other. (SSS)

(ii) $\angle$COD = $\angle$AOB = $\angle$BOC = $\angle$DOE = $\angle$AOE (corr. $\angle$s, $\cong\Delta$s)

$\therefore$ $\angle$COD $= \dfrac{360°}{5}$ = 72°

(b) Let N be a point on CD such that ON $\perp$ CD. Note that ΔONC $\cong$ ΔOND.

$\therefore$ CN = DN (corr. sides, $\cong\Delta$s) $= \dfrac{12}{2}$ cm = 6 cm,

and $\angle$CON = $\angle$DON (corr. $\angle$s, $\cong\Delta$s) $= \dfrac{72°}{2} = 36°$.

In ΔCON, $\dfrac{6}{r} = \sin 36°$, $r = \dfrac{6}{\sin 36°}$ = 10.2 (3 sig. fig.)

Ans. The radius of the circle is 10.2 cm.

(c) In ΔCON, $\tan \angle CON = \dfrac{CN}{ON}$.

ON= $\dfrac{6}{\tan 36°}$ cm = 8.25829 cm (5 d.p.)

Area of ΔCOD = $\dfrac{(CD)(ON)}{2} = \dfrac{(12)(8.25829)}{2}$ cm^2 = 49.54974 cm^2

Area of shaded region = πr^2 – 5(49.54974) = 79.6 cm^2 (3 sig. fig.)

54. (a) Area of ΔABC = $\dfrac{(AB)(BC)}{2} = \dfrac{(4)(4)}{2}$ = 8 cm^2

(b) Area of semi-circle BCD = $\pi\left(\frac{BC}{2}\right)^2\left(\frac{1}{2}\right) = \pi\left(\frac{4}{2}\right)^2\left(\frac{1}{2}\right) = 2\pi \text{ cm}^2$

$\angle BAC = \angle ACB$ (base $\angle$s, isos. Δ) = $\frac{180° - 90°}{2}$ ($\angle$ sum of Δ) = 45°,

area of sector ABE = $\pi(4)^2\left(\frac{45°}{360°}\right) = 2\pi \text{ cm}^2$. $\therefore$ The claim is agreed.

(c) Area of shaded region
= area of semi-circle BCD – (area of ΔABC – area of sector ABE)
= $[2\pi - (8 - 2\pi)]$ cm^2 = $(4\pi - 8)$ cm^2

Unit 17 Volumes (2): Cylinders

1. Volume = $\pi(\frac{14}{2})^2(0.4) = 61.6 \text{ cm}^3$

2. Area = $2\pi(5)(12) + \pi(5)^2 = 145\pi \text{ cm}^2$

3. Largest possible volume = $\pi(\frac{12}{2})^2(12) = 432\pi \text{ cm}^3$

   Total surface area = $2\pi(\frac{12}{2})(12) + 2\pi(\frac{12}{2})^2 = 216\pi \text{ cm}^2$

4. Let r cm be the base radius. $\pi r^2(10) = 471$, $r = \sqrt{\frac{471}{10\pi}} = 3.87$

   Ans. *The base radius is 3.87 cm.*

5. Let h cm be the height. $2\pi(\frac{14}{2})h = 1056$, $h = 1056 \times \frac{1}{14} \times \frac{7}{22} = 24$

   Ans. *The height is 24 cm.*

6. Let r cm be the base radius. $2\pi r(4) = 704$, $r = 704 \times \frac{1}{8} \times \frac{7}{22} = 28$

   Ans. *The base radius is 28 cm.*

7. Diameter = 60 cm = 0.6 m, $\therefore$ the area covered = 8 × π (0.6) (1.6) = 24.1 m^2

8. (a) Volume = $\pi(\frac{35}{2})^2(280) \times \frac{1}{2} = 134\,700 \text{ cm}^3$ = 134.7 L (4 sig. fig.)

   Ans. *The trough can hold 134.7 L of water.*

   (b) Total internal surface area = $\pi(35)(28) \times \frac{1}{2} + 2 \times \frac{1}{2} \times \pi(\frac{35}{2})^2 = 2\,501.5 \text{ cm}^2$

9. Total surface area = $2\pi(\frac{18}{2})(27) \times \frac{1}{2} + \pi(\frac{18}{2})^2 + 6 \times 18 \times 2 + 6 \times 27 \times 2 + 18 \times 27 = 2\,040 \text{ cm}^2$.

   Volume = $\pi(\frac{18}{2})^2(27) \times \frac{1}{2} + 6 \times 18 \times 27 = 6\,350 \text{ cm}^3$

10. Total surface area = $2\pi(9) \times \frac{300}{360} \times 27 + \pi(9)^2 \times \frac{300}{360} \times 2 + 9 \times 27 \times 2 = 2\,180 \text{ cm}^2$ (3 sig. fig.)

11. (a) External curved surface area = $2\pi(12)(50) = 1\,200\pi \text{ cm}^2$

    (b) Volume = $\pi(12^2 - 10^2)(50) = 2\,200\pi \text{ cm}^3$

12. Let y cm be the length. $\pi[(\frac{4}{2})^2 - (\frac{1}{2})^2] \times y = 30 \times 10 \times 6$, $3.75\pi \times y = 1800$,

    $y = \frac{480}{\pi}$ = 152.8. *Ans.* *The length of the pipe is152.8 cm.*

13. Internal radius = $\frac{9}{2} - 0.6 = 3.9$ cm, internal height = 10 – 0.6 = 9.4 cm

$\therefore$ Internal surface area = $2\pi(3.9)(9.4) + \pi(3.9)^2 = 278.1\text{ cm}^2$

14. Let h cm be the depth of water. $\pi(8)^2(h) = 250$, $h = \frac{125}{32\pi}$

$\therefore$ The required area = $2\pi(8)(\frac{125}{32\pi}) + \pi(8)^2 = 263.6\text{ cm}^2$

15. Let h cm be the depth of water. $\pi(\frac{8}{2})^2(h) = \pi(\frac{4}{2})^2(12)$, $h = \frac{4\times 12}{16} = 3$

Ans. The depth of water is 3 cm.

16. Let h cm be the rise in water level. $\pi(4)^2(h) = (1.5)^3 \times 8$, $h = \frac{27}{16\pi}$

$\therefore$ New water level = $6 + \frac{27}{16\pi} = 6.54$ cm

17. Let h cm be the rise in water level.

$\pi(\frac{16}{2})^2(h) = \pi(\frac{8}{2})^2(0.5)\times 50$, $64h = 400$, $h = 6.25$

Ans. The rise in water level is 6.25 cm.

18. Volume of water collected = $\pi(\frac{8}{2})^2 \times 6 \times 12 = 1\,152\pi\text{ cm}^3$

19. (a) Total wet surface area = $2\pi(\frac{10}{2})(8) + \pi(\frac{10}{2})^2 = 330\text{ cm}^2$

(b) Let h cm be the rise in water level. $\pi(\frac{10}{2})^2(h) = 2\times 3\times 6$, $h = \frac{36}{25\pi}$

$\therefore$ Original water level = $8 - \frac{36}{25\pi} = 7.54$ cm

20. Height of equilateral triangle = $4\sin 60° = 3.4641$ cm.
Let r cm be the base radius.

$\pi r^2(12) = \frac{4\times 3.4641}{2}\times 18$, $\pi r^2 = 10.3923$, $r = \sqrt{\frac{10.3923}{\pi}} = 1.82$

Ans. The base radius is 1.82 cm.

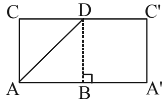
21. Let r cm and h cm be the base radius and the height of original cylinder.
$\because$ $2\pi rh = 100$, $\therefore$ new curved surface area = $2\pi(2r)(2h) = 4(2\pi rh) = 4(100) = 400\text{ cm}^2$

22. Original surface area = $2\pi(3r)(2r) + \pi(3r)^2 \times 2 = 30\pi r^2$

New surface area = $2\pi(3r)(2r) + 2\pi(r)(2r) + \pi[(3r)^2 - r^2]\times 2 = 32\pi r^2$

$\therefore$ Percentage change = $\frac{32\pi r^2 - 30\pi r^2}{30\pi r^2}\times 100\% = 6.67\%$

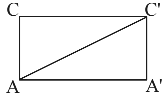
23. (a) The shortest possible length is the diagonal of the rectangle formed by flattening the semi-circular curved surface ABDC.



$\overset{\frown}{AB} = 2\pi(\frac{10}{2})\times\frac{1}{2} = 5\pi$ cm

$\therefore$ Shortest possible length = $\sqrt{(5\pi)^2 + 12^2} = 19.8$ cm (Pyth. Thm.)

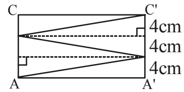
(b) The shortest possible length is the diagonal of the rectangle formed by flattening the curved surface of the cylinder.



Circumference = $2\pi(\frac{10}{2}) = 10\pi$

$\therefore$ Shortest possible length = $\sqrt{(10\pi)^2 + 12^2} = 33.6$ cm (Pyth. Thm.)

(c) Shortest length = $3\times\sqrt{(10\pi)^2+(\frac{12}{3})^2}$ = 95.0 cm (Pyth. Thm.)



24. Time taken = $(15\times10\times5)\div(\pi\times0.05^2\times10)\div3\,600=2.65$ = 2.65 hours

25. Capacity of swimming pool = $\frac{(1.2+2.4)\times50}{2}\times18$ = 1620 m^3

$\therefore$ Time taken = $1\,620\div[\pi\times(\frac{0.06}{2})^2\times15]\div3\,600$ = 10.6 hours

26. When the base circumference is AB, volume = $\pi(\frac{AB}{2\pi})^2\cdot BC=\frac{AB^2\cdot BC}{4\pi}$

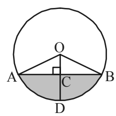
When the base circumference is BC, volume = $\pi(\frac{BC}{2\pi})^2\cdot AB=\frac{BC^2\cdot AB}{4\pi}$

Their ratio of volumes = $\frac{AB^2\times BC}{4\pi}:\frac{BC^2\times AB}{4\pi}$ = AB : BC = 3 : 2 [Or: 2 : 3]

27. Let the original water level be x cm. $\pi\times9^2\times x=\pi\times9^2\times11-\pi\times4^2\times11$,
$81x=715$, $x=8.83$. *Ans. The water level was 8.83 cm.*

28. Let the rise of water level be h cm.
$\pi(10^2)(14+h)=\pi(10^2)(14)+\pi(4^2-7^2)(14+h)$,
$1400+100h=1400+33(14+h)$, $100h=462+33h$, $67h=462$, $h=6.90$
Ans. The minimum height of the cylindrical vessel = 6.90 + 14 = 20.9 cm

29. (a) OC = 30 – 12 = 18 cm, OA = radius = 30 cm,



$\therefore$ AC = $\sqrt{30^2-18^2}=24$ cm (Pyth. Thm.)

$\cos\angle AOD=\frac{OC}{OA}=\frac{18}{30}$, $\angle AOD=53.13°$

Area of segment ADB = $\pi(30^2)\cdot\frac{53.13\times2}{360}-\frac{1}{2}(24)(18)\times2\approx402.564$ cm^2

Volume of water = $\frac{402.564\times200}{1000}\approx80.5128$ L $\approx$ 80.5 L

(b) When the depth is 30 cm, the water will be in the form of a semi-cylinder.

$\therefore$ Water added = $(\frac{\pi\cdot30^2}{2}\times200)\div1\,000-80.5128$ L $\approx$ 202.2 L

30. (a) Base radius $=\frac{30}{2}$ cm = 15 cm, base area = $\pi(15^2)$ cm^2 = 225π cm^2

(i) Volume of water = $225\pi(8)$ cm^3 = 1800π cm^3
(ii) The required area = $[225\pi+20\pi(8)]$ cm^2 = 465π cm^2

(b) (i) Rise in water level = $\frac{(2^3)(15)}{225\pi}=\frac{8}{15\pi}$ cm

(ii) Increase in the area = $30\pi\left(\frac{8}{15\pi}\right)$ cm^2 = 16 cm^2

31. (a) (i) $\angle COD=120°-90°=30°$, OC = (45 – 21) cm = 24 cm

Area of figure AODCB = $\left[\pi(45^2)\left(\frac{120°}{360°}\right)-\pi(24^2)\left(\frac{30°}{360°}\right)\right]$ cm^2 = 627π cm^2

(ii) OD = OC, $\therefore$ Perimeter of figure AODCB

$=\left[2\pi(45)\left(\frac{120°}{360°}\right)+45\times2+2\pi(24)\left(\frac{30°}{360°}\right)\right]$ cm = $(34\pi+90)$ cm

(b) (i) Volume of the prism = $(627\pi)(80)$ cm^3 = $50\,160\pi$ cm^3
(ii) Total surface area = $[(34\pi+90)(80)+627\pi(2)]$ cm^2 = $(3\,974\pi+7\,200)$ cm^2

32. (a) (i) Let h cm be the height of the cylinder.
$\pi(4)^2(h) = 144\pi$, $h = 9$, $\therefore$ the height of the cylinder is 9 cm.
(ii) Total surface area = $[2\pi(4)(9) + \pi(4)^2(2)]$ cm^2 = 104π cm^2
(b) (i) Length of arc PR = $2\pi(4)\dfrac{135°}{360°} = 3\pi$ cm
(ii) Imagine that the curved surface PQR can be flattened, and when QR is shortest, PQR is a right-angled triangle in which PQ = 9cm and PR = 3π cm.
By Pyth. Theorem, the shortest length of the line
$= \sqrt{(3\pi)^2 + 9^2}$ cm = 13.0 cm (3 sig. fig.)

33. (a) (i) $\angle BCF = 90°$, $\angle FCG = \angle BCG - \angle BCF = 120° - 90° = 30°$,
$\angle BAC = \angle FCG = 30°$ (corr. $\angle$s, DF // AE).
In ΔABC, $\dfrac{BC}{AB} = \tan\angle BAC$, $AB = \dfrac{\sqrt{3}}{\tan 30°} = 3$ cm
(ii) Arc BFG $= 2\pi\left(\sqrt{3}\right)\left(\dfrac{120°}{360°}\right)$ cm $= \dfrac{2\sqrt{3}}{3}\pi$ cm
$\dfrac{BC}{AC} = \sin\angle BAC$, $AC = \dfrac{\sqrt{3}}{\sin 30°} = 2\sqrt{3}$ cm
Perimeter $= \left(\dfrac{2\sqrt{3}}{3}\pi + 3 + 2\sqrt{3} + \sqrt{3}\right)$ cm $= \left(\dfrac{2\sqrt{3}}{3}\pi + 3 + 3\sqrt{3}\right)$ cm
(b) Base area of the prism
= area of ΔABC + area of sector BFGC
$= \dfrac{(3)(\sqrt{3})}{2} + \pi(\sqrt{3})^2\left(\dfrac{120°}{360°}\right)$ cm^2 $= \left(\dfrac{3\sqrt{3}}{2} + \pi\right)$ cm^2
Volume of the prism $= \left(\dfrac{3\sqrt{3}}{2} + \pi\right)(2)$ cm^3 $= \left(3\sqrt{3} + 2\pi\right)$ cm^3

34. (a) Volume of water = $30 \times 12 \times 18 - \pi(6^2) \times 18$ cm^3 = 4444.2 cm^3 (1 d.p.)
(b) (i) Let h cm be the new depth of water.
$4444.2 + \pi(6^2)(h) = 30(18)(h)$, $h = \dfrac{444.2}{540 - 36\pi} = 10.4$ (1 d.p.)
Ans. The new water depth is 10.4 cm.
(ii) Area not getting wet
$= \pi(6^2) + 2\pi(6)(18 - h) = 399.2$ cm^2 < 400 cm^2
$\therefore$ The claim is disagreed.

Unit 18 More about statistical charts

1. (a) Lower class limit = \$400, upper class limit = \$490
(b) Lower class boundary = \$95, upper class boundary = \$195
(c) Class mark = $\dfrac{200 + 290}{2}$ = \$245
(d) Class width = \$195 – \$95 = \$100
(e) 100 students
(f) The class with most students is \$300–\$390. Percentage = $\dfrac{30}{100} \times 100\% = 30\%$

2. (a)

Weight (kg)	40 – 44	45 – 49	50 – 54	55 – 59	60 – 64
Tally	𝍸	𝍸 𝍸 𝍸 /	𝍸 𝍸 ////	𝍸 ////	////
Frequency	5	16	14	9	4

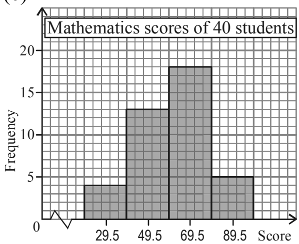
(b) Percentage of girls = $\frac{16+14}{48}\times 100\% = \frac{30}{48}\times 100\% = 62.5\%$

3. (a)

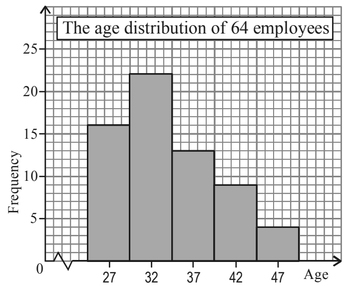
Score	Class boundaries	Class mark	Frequency
20 – 39	19.5 – 39.5	29.5	4
40 – 59	39.5 – 59.5	49.5	13
60 – 79	59.5 – 79.5	69.5	18
80 – 99	79.5 – 99.5	89.5	5

(b) Class width = 39.5 – 19.5 = 20

(c)

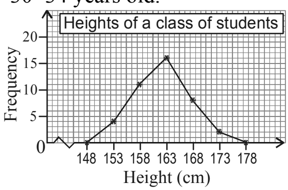


4. (a)



4. (b) None of the employees are younger than 24.5 years old or older than 49.5 years old. Besides, the ages are more concentrated in the first two classes, especially the class of 30–34 years old.

5. (a)



(b) Lower boundary = 168+2.5 = 170.5 cm, upper boundary = 173+2.5 = 175.5 cm

(c) Its lower class boundary = 150.5 cm, upper class boundary = 155.5 cm

∴ its lower class limit = 151 cm, upper class limit = 155 cm

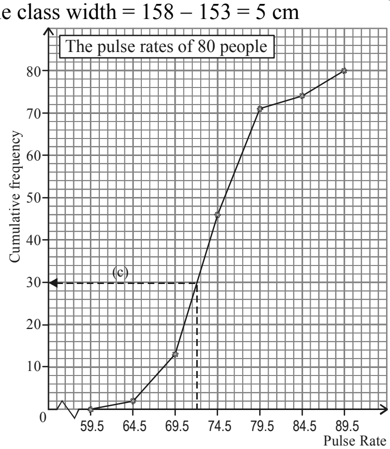
Unit 18

(d) The class width = 158 – 153 = 5 cm

6. (a)

Pulse rate less than	Cumulative frequency
59.5	0
64.5	2
69.5	13
74.5	46
79.5	71
84.5	76
89.5	80

(b)



(c) From the graph, 30 people have pulse rates less than 72.

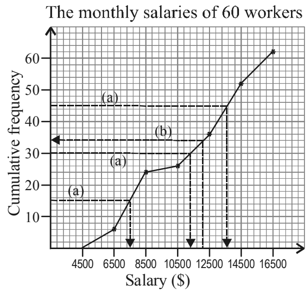
7. (a) $60 \times \frac{1}{4} = 15$, $60 \times \frac{1}{2} = 30$, $60 \times \frac{3}{4} = 45$

From the graph,
the lower quartile = 6 500 +1 000 = \$7 500,
the median = 10 500 + 800 = \$11 300,
the upper quartile ≈ 12 500 +1 100 = \$13 600.

(b) From the graph, 34 workers have salaries less than \$12 100.

∴ Percentage of experienced workers

$$= \frac{60-34}{60} \times 100\% = \frac{26}{60} \times 100\% = 43.3\%$$



8. (a) There is space between the bars of a bar chart, but there is no space between the bars of a histogram. Besides, only one axis of a bar chart is a number line and has a scale, but both axes of a histogram are number lines and have a scale.

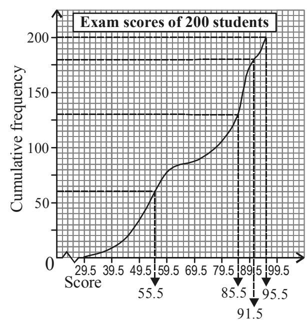
(b) Set A: It should be presented by a histogram. There are 100 data and have to be grouped into classes to show the pattern or characteristics of the set of data.

Set B: It should be presented by a bar chart. There are only 5 data, and they should be treated as discrete data (離散的、不連續的數據).

Set C: A bar chart is better than a histogram. Since there are only 12 data, the pattern or characteristic of the set of data can be observed easily in a bar chart. [A histogram can also be drawn, but it is neither meaningful nor necessary to group 12 data into several classes.]

9. No. of grade D students = 200 × 30% = 60
No. of grades D or C students = 200 × (30%+35%) = 130
No. of grades D, C or B students = 200 × (1−10%) = 180
From the graph, 60 students have scores less than 55.5, 130 students have scores less than 85.5, and 180 students have scores less than 91.5. And none of the students have scores less than 29.5 or above 95.5.

∴ Grade D: scores 30–55,
Grade C: scores 56–85,
Grade B: scores 86–91,
Grade A: scores 92–95

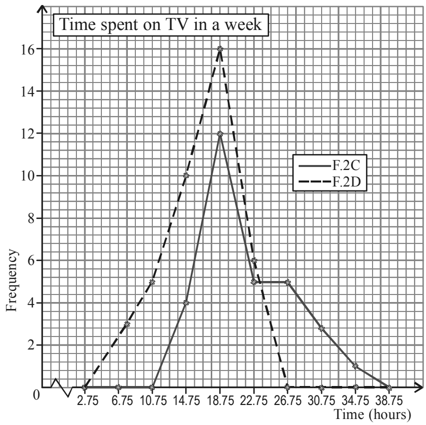


Unit 18

10. (a) No. of students of F.2C = 4 + 12 + 5 + 5 + 3 + 1 = 30
No. of students of F.2D = 3 + 5 + 10 + 16 + 6 = 40

Time spent on TV in a week (hours)	Class boundaries	Class mark	Frequency	
			F.2C	F.2D
5 – 8.5	4.75–8.75	6.75	0	3
9 – 12.5	8.75–12.75	10.75	0	5
13 – 16.5	12.75–16.75	14.75	4	10
17 – 20.5	16.75–20.75	18.75	12	16
21 – 24.5	20.75–24.75	22.75	5	6
25 – 28.5	24.75–28.75	26.75	5	0
29 – 32.5	28.75–32.75	30.75	3	0
33 – 36.5	32.75–36.75	34.75	1	0

(b)



(c) Both frequency polygons have the greatest frequency for the class mark of 18.75 hours.

(d) The frequency polygon of F.2D lies more to the left, while the frequency polygon of F.2C lies more to the right. This shows that on average students of F.2D spend less time on TV in a week than students of F.2C.

(e) The numbers of students are different in both classes, so we have to compare the percentages.

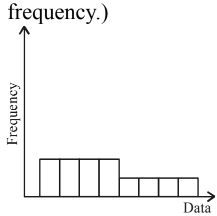
Percentage of F.2C students in this interval = $\frac{5}{30}\times 100\% = 16.7\%$

Percentage of F.2D students in this interval = $\frac{6}{40}\times 100\% = 15\%$

Ans. *F.2C has a greater proportion of students in the interval of 21 hours to 24.5 hours.*

11. Figure 11C is its corresponding cumulative frequency polygon. The horizontal line segment in the middle matches the class of zero frequency in the given histogram. (Figure 11B is wrong because a cumulative frequency polygon must not go downward. Figure 11D is wrong because it does not have any horizontal line segment matching with the class of zero frequency.)

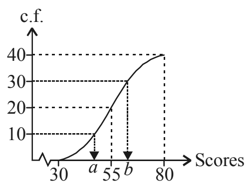
12.



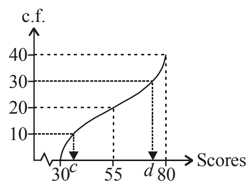
The cumulative frequency polygon is composed of 2 line segments. This shows that the first 4 classes are of equal frequencies and the last 4 classes are of equal frequencies. Besides, the total frequency of the first four classes is about twice of that of the last four classes, so the frequency of each of the first four classes should be twice of that of the last four classes.

Unit 18

13. (a) In the figures below, *a* and *b* are the lower quartile and the upper quartile of Class A, while *c* and *d* are the lower quartile and the upper quartile of Class B. Since $c < a$, the average performance of the bottom 25% of Class B is poorer than that of Class A. Since $b < d$, the average performance of the top 25% of Class A is not so good as that of Class B. In sum, there are more students with low scores and high scores in Class B than in Class A.

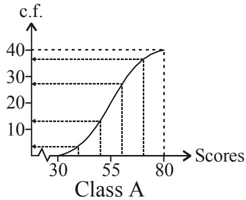


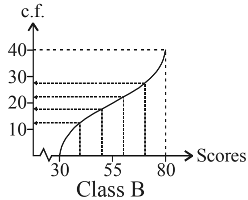
Class A



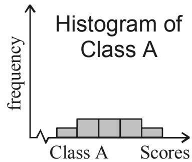
Class B

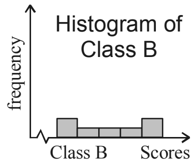
(b) The figures on the right show that in both Class A and Class B, the frequencies of the three class intervals in the middle are of similar frequencies.





The histograms of the two classes are as follows:

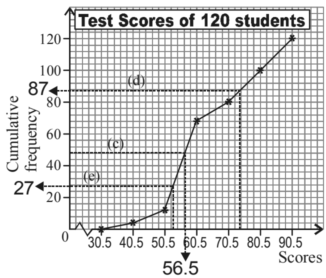




14. (a)

Scores less than	Cumulative frequency
30.5	0
40.5	4
50.5	12
60.5	68
70.5	80
80.5	100
90.5	120

Score interval	Frequency
31 – 40	4
41 – 50	8
51 – 60	56
61 – 70	12
71 – 80	20
81 – 90	20



(b) The class with most students is of scores 51–60.

(c) No. of students failed the test = 120 × (1 – 60%) = 48
From the graph, the passing mark = 56.5.

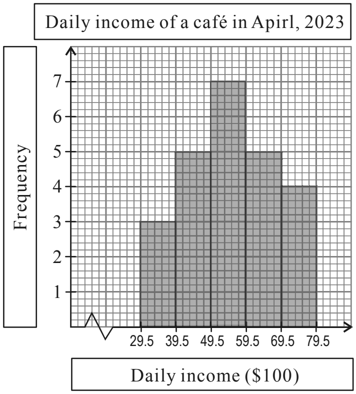
(d) From the graph, 87 students have scores *less than* 74.
Percentage of students with scores *greater than* 74 = $\dfrac{120-87}{120}\times 100\% = 27.5\%$

(e) From the graph, 27 students have scores less than 52.
Percentage of students who need to take the re-test = $\dfrac{27}{120}\times 100\% = 22.5\%$

15. (a)

Daily income ($100)	Class boundaries ($100)	Class mark ($100)	Frequency
30 – 39	29.5 – 39.5	34.5	3
40 – 49	39.5 – 49.5	44.5	5
50 – 59	49.5 – 59.5	54.5	7
60 – 69	59.5 – 69.5	64.5	5
70 – 79	69.5 – 79.5	74.5	4

(b)



(c) Number of days in which the daily incomes are less than $4 950 = 3 +5 = 8

Percentage of days in April, 2023 in which the daily incomes are less than $4 950

$= \frac{8}{3+5+7+5+4} \times 100\%$

$= \frac{8}{24} \times 100\% = 33\frac{1}{3}\% > 30\%$

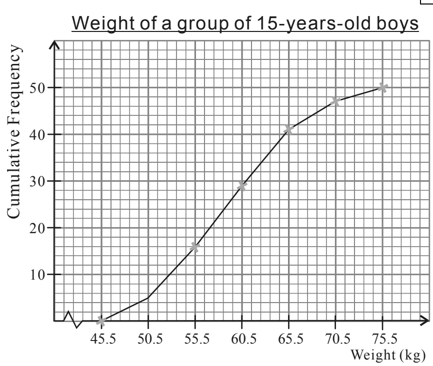
∴ The claim is agreed.

16. (a)

Frequency distribution table		
Weight (cm)	Class Mark (cm)	Frequency
46 – 50	48	5
51 – 55	53	11
56 – 60	58	13
61 – 65	63	12
66 – 70	68	6
71 – 75	73	3

Cumulative frequency table	
Weight less than (cm)	Cumulative frequency
45.5	0
50.5	5
55.5	16
60.5	29
65.5	41
70.5	47
75.5	50

(b)



(c) (i) Number of boys less than or equal to 63 kg = 35

∴ The required percentage $= \frac{50-35}{50} \times 100\% = 30\%$

(ii) The 20th percentile = 53 kg (accept 52.5 kg)
The required difference = (68 – 53) kg = 15 kg (accept 15.5 kg)

(iii) Number of overweight boys = 2

∴ The required percentage $= \frac{2}{50} \times 100\% = 4\%$