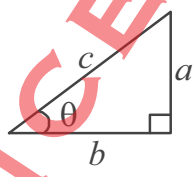
Unit 15 Introduction to trigonometric ratios

⚠ Important facts ⚠

1. Key terms

- sine ratio (正弦), cosine ratio (餘弦), tangent ratio (正切)
- opposite side (對邊), adjacent side (鄰邊), hypotenuse (斜邊)

2. Basic knowledge



(a) Trigonometric ratios

$$\sin\theta = \frac{\text{opp. side}}{\text{hyp.}} = \frac{a}{c}, \quad a = c\cdot\sin\theta, \quad c = \frac{a}{\sin\theta}$$

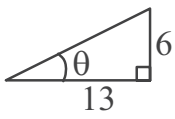
$$\cos\theta = \frac{\text{adj. side}}{\text{hyp.}} = \frac{b}{c}, \quad b = c\cdot\cos\theta, \quad c = \frac{b}{\cos\theta}$$

$$\tan\theta = \frac{\text{opp. side}}{\text{adj. side}} = \frac{a}{b}, \quad a = b\cdot\tan\theta, \quad b = \frac{a}{\tan\theta}$$

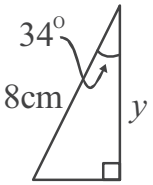
(b) The above ratios are applied to right-angled triangles only.

3. Find the unknown marked values in the given figures

(a)

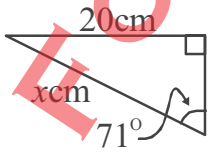


$\tan\theta = \frac{6}{13}$, $\therefore\ \theta = 24.8°$



(b) $\cos 34° = \frac{y}{8}$, $y = 8\cos 34° = 6.63$ cm

(c)



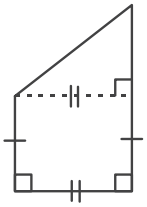
$\sin 71° = \frac{20}{x}$, $x\sin 71° = 20$,

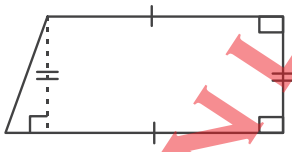
$$x = \frac{20}{\sin 71°} = 21.2$$

4. Trapezium (梯形)

Given a right-angled trapezium, divide it into a rectangle and a right-angled triangle. Then the unknown angle(s) or length(s) can be found.

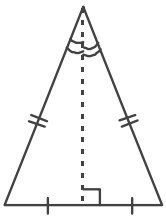
Examples:





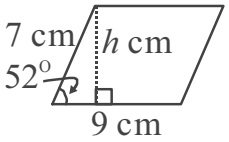
5. Isosceles triangle (等腰三角形)

Draw a perpendicular line from the vertical angle to the base of an isosceles triangle. This perpendicular line bisects the vertical angle as well as the base. Then the unknown angle(s) or length(s) can be found.



6. Areas of plane figures

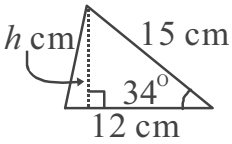
(a) **Question**: Find the area of the given parallelogram.



Solution: the height $h = 7 \times \sin 52°$

$\therefore$ area of the parallelogram $= 9 \times 7 \times \sin 52° = 49.6 \text{ cm}^2$

(b) **Question**: Find the area of the given triangle.

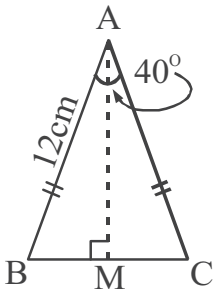


Solution: the height $h = 15 \times \sin 34°$

$\therefore$ area of the triangle $= \frac{1}{2} \times 12 \times (15 \sin 34°)$

$= 50.3 \text{ cm}^2$ (3 sig. fig.)

(c) **Question**: Find the area of the given isosceles triangle.



Solution: $\angle BAM = \frac{40°}{2} = 20°$

$AM = 12 \times \cos 20°$, $BM = 12 \times \sin 20°$

$\therefore$ area of $\Delta ABC = 2 \times (\frac{1}{2} \times BM \times AM)$

$= 12 \cos 20° \times 12 \sin 20° = 46.3 \text{ cm}^2$ (3 sig. fig.)

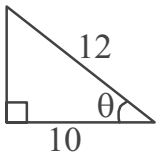
In this exercise, give the answers to 3 significant figures or 1 decimal place when appropriate.

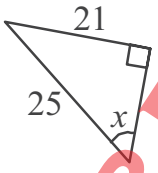
(I) Warm-up items, No.1-26

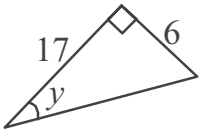
1. Find the values of θ in the following.

(a) $\tan\theta = \frac{14}{5}$ (b) $3\cos\theta = 2$ (c) $5\sin\theta - 4 = 0$

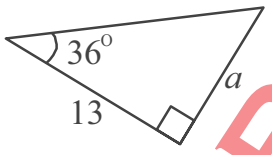
2. In the following figures, find the values of the unknown marked angles.

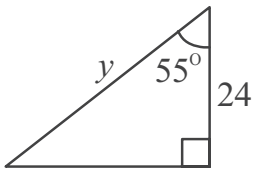
(a) 

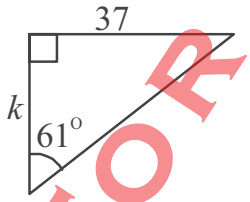
(b) 

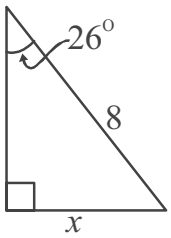
(c) 

3. In the following figures, find the values of the unknown marked lengths.

(a) 

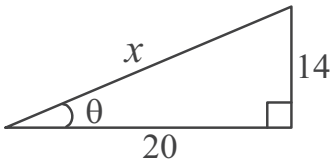
(b) 

(c) 

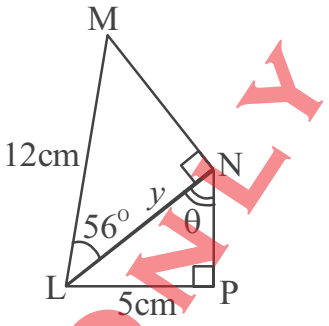
(d) 

4. Find the unknown marked lengths and angles in the following figures.

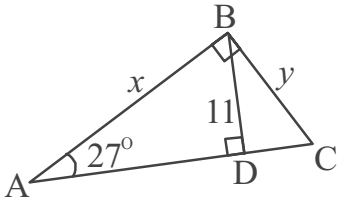
(a)



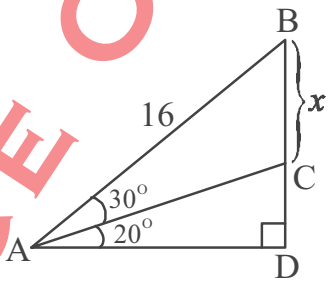
(b)



(c)



(d)



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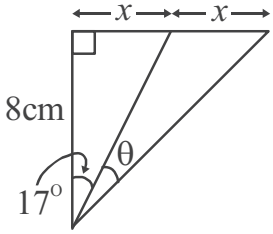

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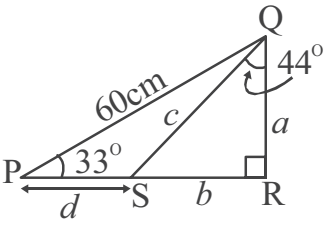
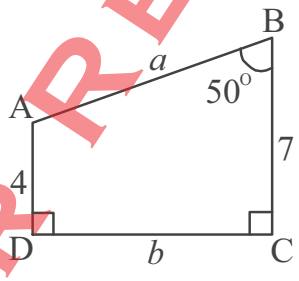
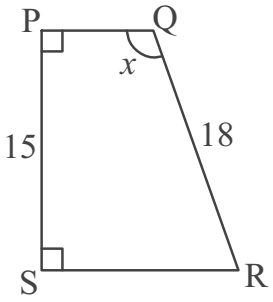


5. Find the unknown marked lengths and angles in the following trapeziums.

(a)

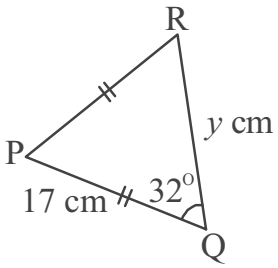


(b)



6. Find the unknown marked lengths and angles in the following isosceles triangles.

(a)



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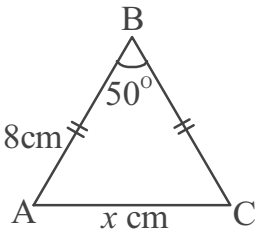

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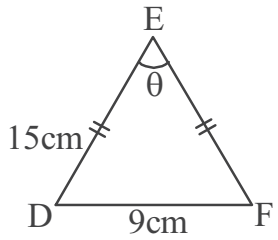
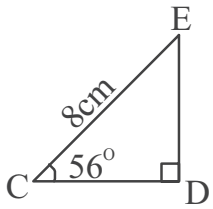
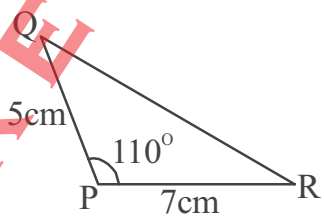


7. Find the areas of the following figures.

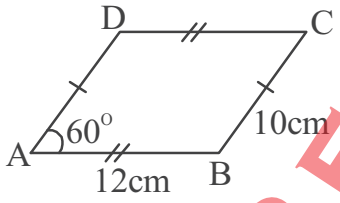
(a)



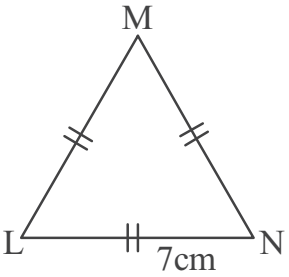
(b)



(c)



(d)



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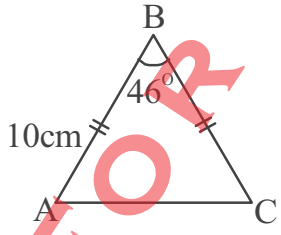

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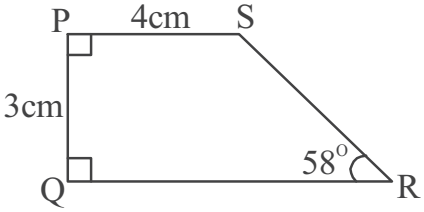
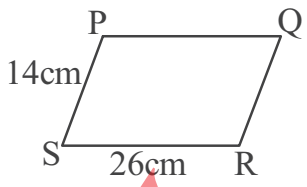


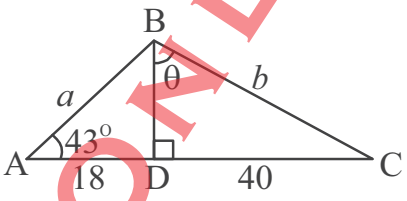
8. The base angles of an isosceles triangle are both 30°. If the base is 7 cm, what is the area of the triangle?

9. The area of the given parallelogram is 160 cm^2. Find $\angle Q$.



10. (a) Find the area of ΔABC.

(b) Find the values of a, b and θ.

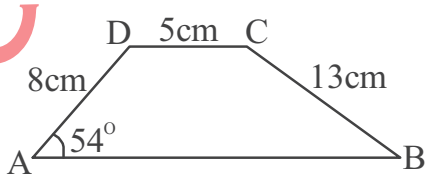


11. In the figure, ABCD is a trapezium.

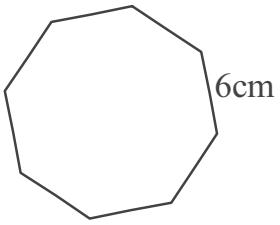
(a) Find $\angle B$ to 2 decimal places.

(b) Find AB.

(c) Find the area of ABCD.

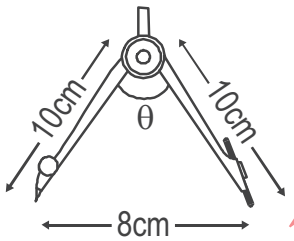


12. Find the area of a regular octagon of sides 6 cm.

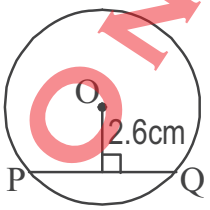


13. The tops of two vertical poles of heights 2.5 m and 4 m are joined by a rope which makes an angle of 72° with the vertical poles. Find the length of the rope.

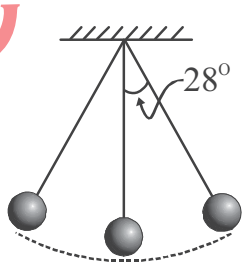
14. The legs of a pair of compasses are each 10 cm long, and they are used to draw a circle of radius 8 cm. Find the angle between the legs.



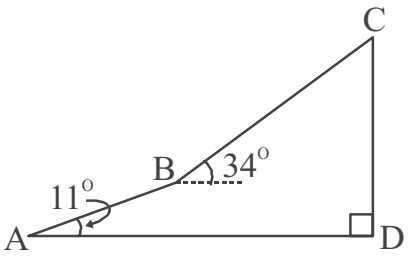
15. In the figure, O is the centre of the circle whose radius is 3 cm. If the distance from O to the chord PQ is 2.6 cm, find the angle at the centre to the nearest degree.



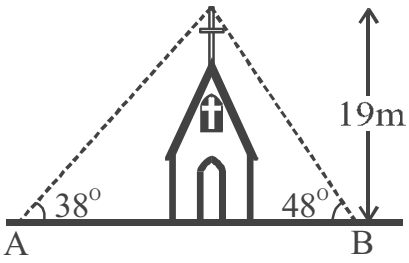
16. A pendulum 24 cm long swings through an angle of 28° on each side of the vertical. Find the greatest height that the tip rises above its lowest position.



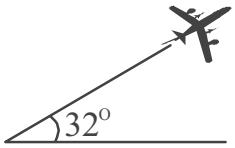
17. A man walks 450 m up a slope of 11° and then 200 m up a slope of 34°. Find the vertical distance he has risen.



18. Two tight ropes are tied from the top of a church to the opposite sides of the ground. If the church is 19 m high, and the ropes make angles of 38° and 48° with the horizontal, find the distance between the two ends of the ropes.



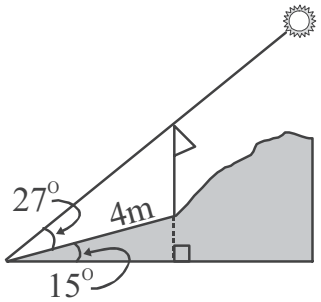
19. A model plane takes off at a fixed angle of 32° to the horizontal. If the speed of the model plane is 900 m per minute, find the time it takes to reach a vertical distance of 1.5 km.



(Correct the answer to 2 decimal places.)

20. The shadow of a vertical post is 3.8 m long when the sun makes an angle of 40° with the horizontal. What is the length of the shadow when the angle becomes 28°?

21. When the sun makes an angle of 27° with a slope, the shadow of a vertical pole is 4 m on the slope. If the slope makes an angle of 15° with the horizontal, find the length of the vertical pole.

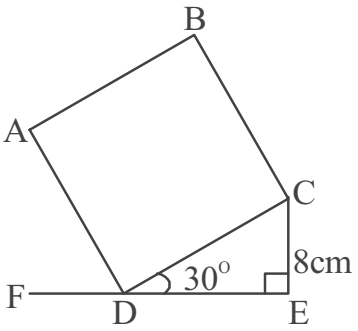


22. ABCD is a square. Find:

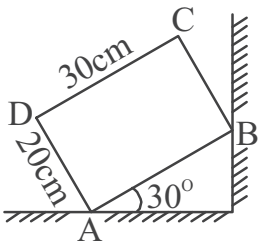
(a) the area of ΔCDE,

(b) the area of square ABCD,

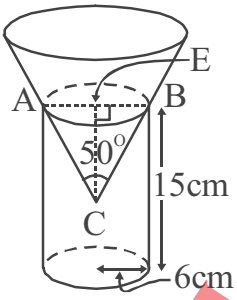
(c) the vertical distance from A to DE.



23. ABCD is a rectangle with sides 20 cm by 30 cm and AB makes an angle of 30° with the horizontal. Find the vertical distance from C to the horizontal.



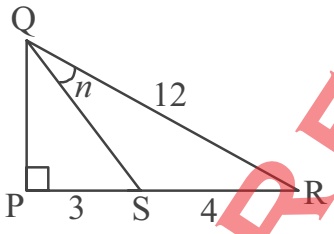
24. A conical funnel (錐形漏斗) is placed over a cylinder such that their circular bases are parallel to each other. The cylinder is 15 cm long, and it base radius is 6 cm. If the vertical angle of the funnel is 50°, how far is its vertex from the bottom of the cylinder?



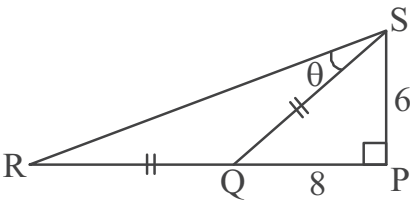
25. In the figure, there is a big box near the door, and so the door, which is 0.8 m wide, can only be opened up to 50°. Later, the box is pushed x cm away from the door, and the door can just be opened completely (see the figure). Find the value of x.

26. Find the unknown marked lengths and angles in the following figures.

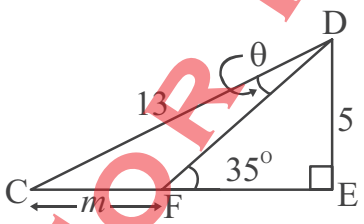
(a)



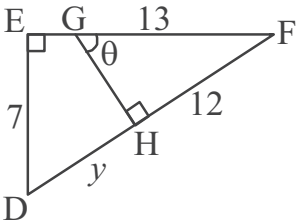
(b)



(c)



(d)



(II) Stimulating items, No. 27-54

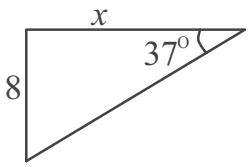
27. Find the values of θ in the following.

(a) $\sin 3\theta = 0.9$

(b) $\tan(\theta + 18°) = \frac{1}{2}\tan 70°$

(c) $5\cos(2\theta - 15°) = 1$

28. In the figure, is it correct that $x = \frac{8}{\tan 37°}$? Explain your answer.

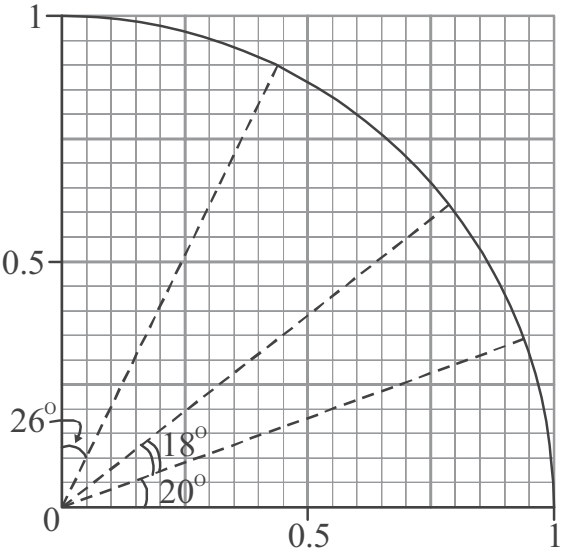


29. The figure shows a quadrant of a circle whose radius is 1 unit.

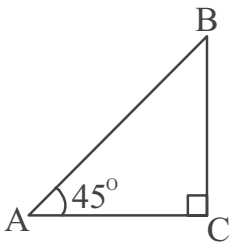
(a) Find from the graph the values of sin 20°, sin 38°, sin 64°.

(b) Find from the graph the values of cos 20°, cos 38°, cos 64°.

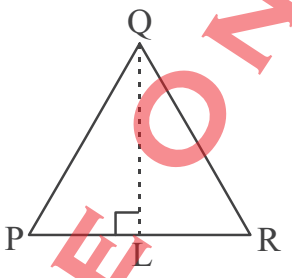
(c) If $\cos\theta > \sin\theta$, find the range of values of θ. Explain your answer.



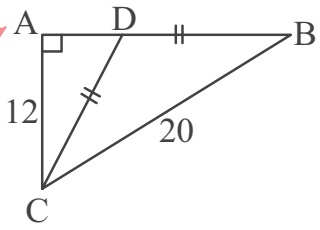
30. Using the given right-angled triangle, find the value of cos 45° without using the calculator. *(Leave the* $\sqrt{}$ *sign in your answer when appropriate.)*



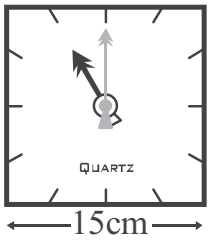
31. PQR is an equilateral triangle, and QL ⊥ PR. Making use of ΔPQR, find the values of sin 30° and tan 60° without the calculator. *(Leave the* $\sqrt{}$ *sign in your answer when appropriate.)*



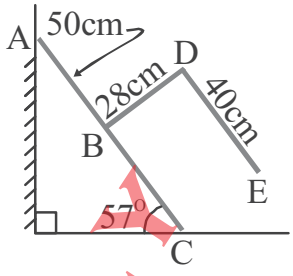
32. If AC = 12, BC = 20 and CD = DB, find the value of ∠ DCB.



33. The figure shows a clock face in the shape of a square of sides 15 cm. What is the distance between the 11-o'clock mark and the 12-o'clock mark?



34. In the figure, a chair leans against a vertical wall.



(a) Find the vertical distance from E to the ground.

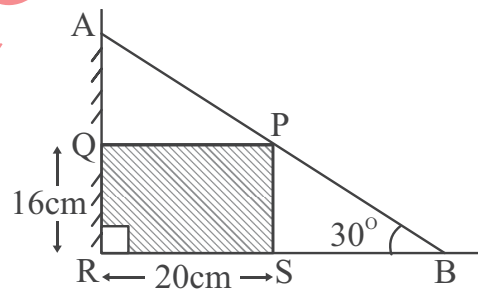
(b) Find the vertical distance from D to the ground.

(c) Find the horizontal distance from D to the wall.

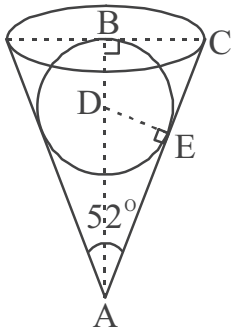
35. In the figure, AB is a rod leaning against a vertical wall and it touches a rectangular block at P. The dimensions of the block is 20 cm × 16 cm, and the rod makes an angle of 30° with the horizontal.

(a) Find the length of the rod.

(b) Now the rod slides down until its angle with the horizontal becomes 20°. How far does tip B slide?



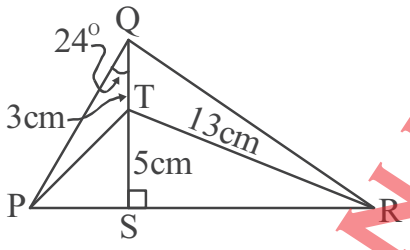
36. A sphere is put inside a sealed right cone so that the highest point of the sphere touches the base of the cone. The height of the cone is 8 cm and its vertical angle is 52°. Find the radius of the sphere.



37. In the figure, QS is an altitude of ΔPQR, and it makes an angle of 24° with PQ. T is a point on QS such that QT = 3 cm, TS = 5 cm and TR = 13 cm.

(a) Find the area of ΔPQT.

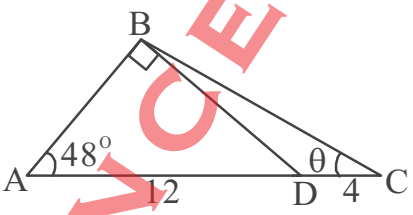
(b) Find ∠QRT.



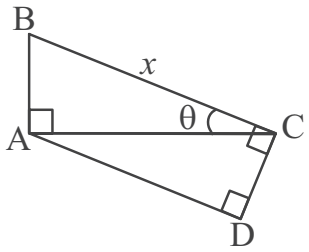
38. In the figure, AD = 12, DC = 4, ∠ABD = 90°, and ∠A = 48°.

(a) Find BD.

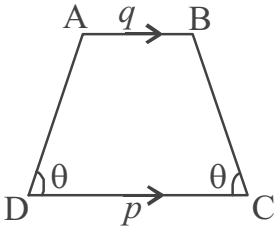
(b) Find the value of θ.



39. In the figure, ∠BAC = ∠BCD = ∠ADC = 90°, ∠BCA = θ and BC = x. Find CD in terms of x and θ.



40. In the figure, ABCD is a trapezium in which AB // DC, ∠C = ∠D = θ, CD = p and AB = q. If $p^2 = q^2 + 12$, find the area of the trapezium in terms of θ.

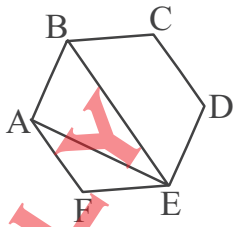


41. ABCDEF is a regular hexagon with sides 6 cm.

(a) Find ∠F and AE.

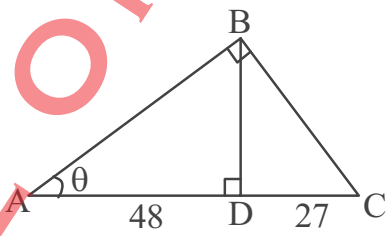
(b) Prove that ABE is a right-angled triangle.

(c) Find the area of ΔABE.

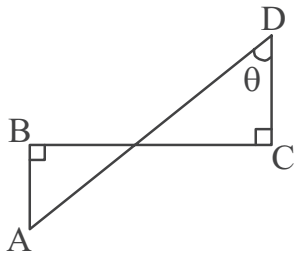


42. (a) Prove that ΔABD ~ ΔBCD.

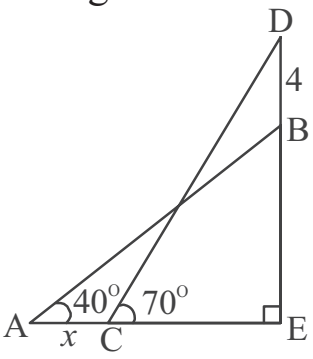
(b) Find the value of θ.



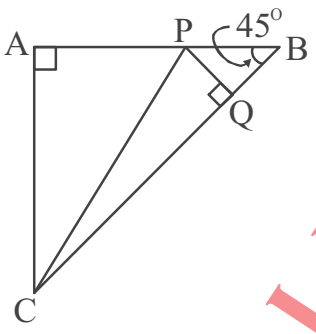
43. If AB = 15, CD = 18 and BC = 56, find the value of θ.



44. If AB = CD, find the value of x in the figure.



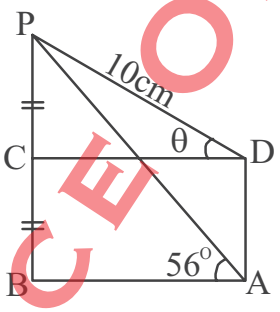
45. ΔABC is right-angled at A. $PQ \perp BC$, $\angle B = 45°$, and AP : PB = 2 : 1. Find $\angle CPQ$.



46. In the figure, ABCD is a rectangle, and BC is produced to P. PC = CB, and $\angle PAB = 56°$.

(a) Find θ, correct to 2 decimal places.

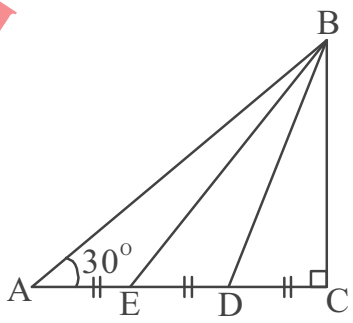
(b) If PD = 10 cm, find PA.



47. In the figure, $\angle C = 90°$, $\angle A = 30°$. D and E are points on AC such that AE = ED = DC.

(a) Find $\angle BDC$.

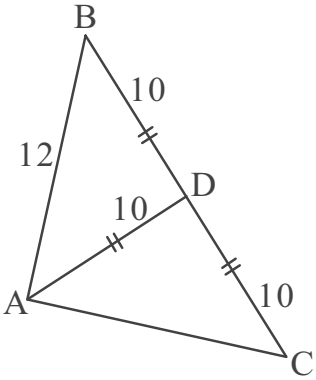
(b) Prove DA = DB.



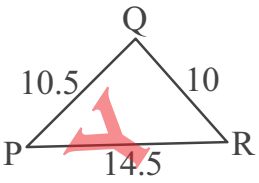
48. In ΔABC, BD = DC = DA = 10, and AB = 12.

(a) Prove $\angle BAC = 90°$.

(b) Hence find $\angle C$.



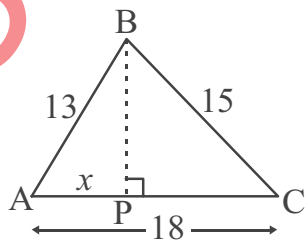
49. In the figure, QP = 10.5, PR = 14.5 and RQ = 10.



(a) Show whether PQR is a right-angled triangle.

(b) Find ∠P, ∠Q and ∠R.

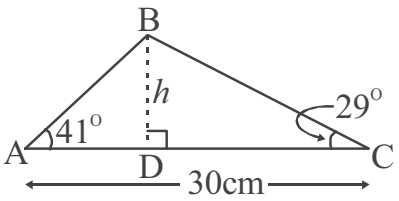
50. In the figure, AB = 13, BC = 15 and CA = 18.



(a) Show whether ABC is a right-angled triangle.

(b) From B, draw BP ⊥ AC. Let AP = x. Find the values of x and ∠A.

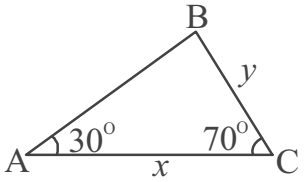
51. In the figure, BD ⊥ AC.



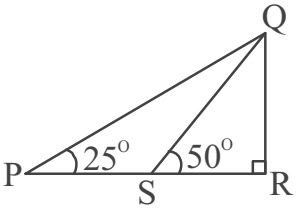
(a) Express AD and CD in terms of h.

(b) Hence find the value of h.

52. In the figure, find AB in terms of x and y.



53. In the figure, QR ⊥ PR.



(a) If PS : SR = n : 1, find the value of n, correct to 4 significant figures.

(b) If the area of ΔQRS is 30 cm^2, find the area of ΔPQS.

54. A rectangular tank with dimensions 50 cm × 40 cm × 120 cm was originally filled with water. The tank was tilted (傾斜) up so that only one of the base edges touched the ground, and the water poured out slowly.

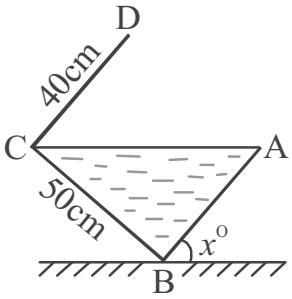


Figure A

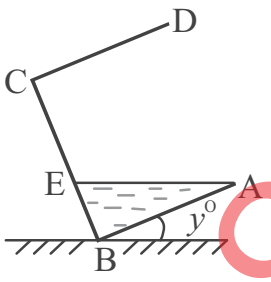


Figure B

(a) When half of the water had poured out, the edge AB made an angle of $x°$ with the ground (see Figure A). Find the value of x.

(b) When one third of the water remained in the tank, the edge AB made an angle of $y°$ with the ground (see Figure B). Find the value of y.

(III) Exam Practice, No. 55-58

55.

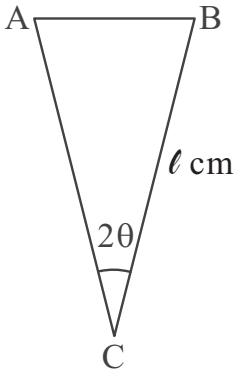


Figure (a)

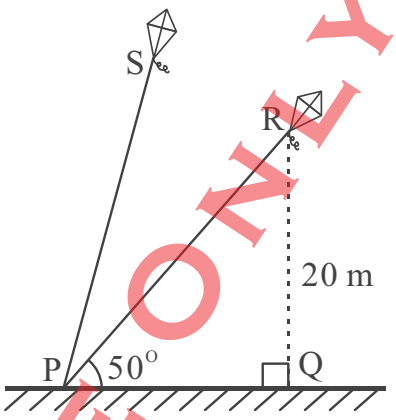


Figure (b)

(a) Figure (a) shows an isosceles triangle ΔABC with AC = BC = ℓ cm and ∠ACB = 2θ.

(i) Let N be a point on AB such that AB ⊥ CN. By considering ΔACN and ΔBCN, prove that AN = BN.

(ii) Hence, prove that AB = $2\ell \sin\theta$ cm.

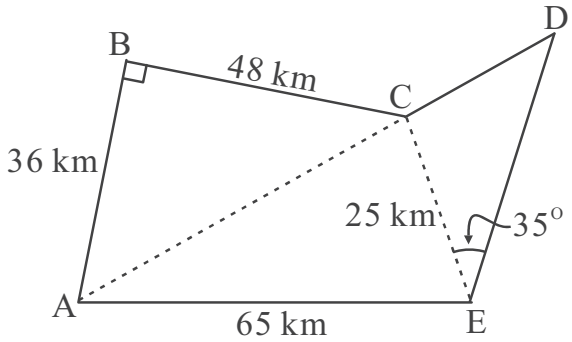
(b) In Figure (b), a kite is tied by a string to point P on the horizontal ground. When the kite is at position R, which is 20 m above the ground, the string is taut and makes an angle of 50° with the horizontal.

(i) Find the length of the taut string PR.

(ii) The kite is then raised to a new position S and the string is still taut. If the angle between the string and the ground becomes 68°, using the results of (a), or otherwise, find the distance between R and S.

[Give the answers correct to 3 significant figures.]

56. The figure shows part of a map. A, B, C, D and E are five cities. A, C and D lies on the same straight line.



(a) (i) Determine whether ∠ACE is a right angle. Explain your answer.

(ii) Find the distance between C and D.

(iii) Find the distance between D and E.

[Give the answers correct to 3 significant figures.]

(b) Morris drives at an average speed of 90 km/h from city A to city D via city B and city C. On the other hand, Nick drives at an average speed of 84 km/h from city A to city D via city E. Morris and Nick leave city A simultaneously. Nick claims that he would arrive at city D first. Do you agree? Explain your answer.

57. Figure (a) shows a decoration on a horizontal table. The decoration is composed of a revolving circular ring and a triangular stand OPQ. Laser light can be emitted from point A on the ring. The diameter of the ring is 18 cm, and O is its centre. OP = OQ = 13 cm and PQ = 10 cm.

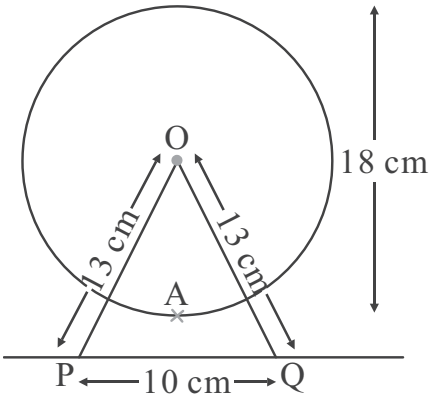


Figure (a)

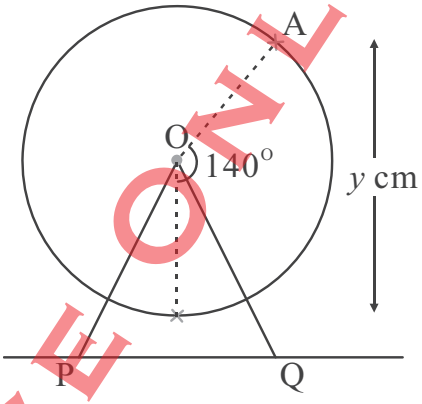


Figure (b)

(a) Find the vertical distance from the centre O to the table.

(b) At the beginning, A is right below the centre O. Then the ring is rotated 140° anti-clockwise, and the height that A rises above its original position is y m [see Figure (b)].

(i) Find the value of y.

(ii) Find the distance from A to the table.

[Give the answers correct to 3 significant figures.]

58. Figure (a) shows the top view of a room. AB and BC are vertical walls perpendicular to each other, and PQR is a triangular table right-angled at R. PQ = 235 cm and QR = 141 cm.

(a) Find ∠PQR.

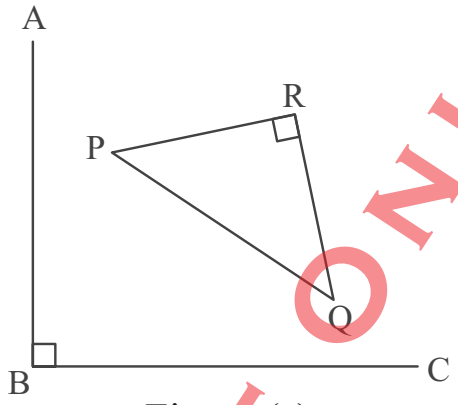


Figure (a)

(b) The table will be placed in the room such that it has to touch the wall BC.

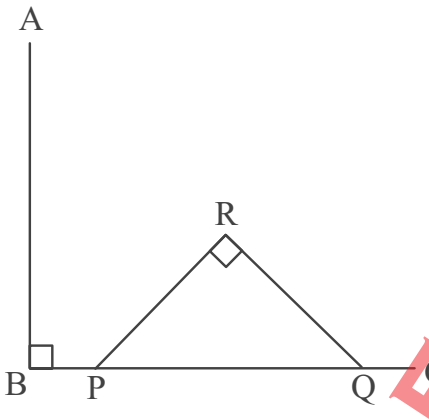


Figure (b): Position 1

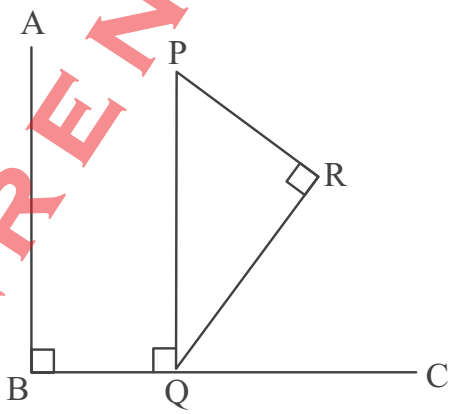


Figure (c): Position 2

(i) Figures (b) and (c) show two possible positions of the table. Find the distances from R to the wall BC in each of these two positions.

(ii) How should the table be placed so that the distance from R to BC would be the greatest? And what is the greatest distance.

[Give the answers correct to 4 significant figures.]